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THE PROPAGATION OF ANOMALOUS SOUND

FROM LARGE EXPLOSIONS

by

Erhard R. Reinelt

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES
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FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read,
and recommend to the Faculty of Graduate Studies
for acceptance, a thesis entitled THE PROPAGATION
OF ANOMALOUS SOUND FROM LARGE EXPLOSIONS, submitted
by Erhard R. Reinelt in partial fulfilment of the
requirements for the degree of Doctor of Philosophy.

THE PROPAGATION OF ANOMALOUS SOUND FROM LARGE EXPLOSIONS

ABSTRACT

The introductory chapter of this thesis traces the history of anomalous sound from the time of its discovery, at the turn of the century, to the end of 1963. In the second chapter, the theory of sound propagation in moving air is developed from simple postulates of geometrical acoustics, and a new equation for the ray path is obtained by exact integration of Rayleigh's equation for sound moving parallel to the wind. The third chapter reviews theoretical aspects of sound absorption in the atmosphere, with particular reference to the work of Stokes, Rayleigh, Schrödinger and Lindsay.

The last two chapters describe the recording and evaluation of atmospheric pressure disturbances initiated by a series of test explosions at the Defence Research Board Experimental Station at Suffield, in Southeastern Alberta. The largest of these tests, an explosion of 100 tons of TNT, provided most of the experimental data. The atmospheric wind and temperature distribution to 60 kilometers is deduced from the observed travel time curve and the pattern of acoustic arrivals. Moderately strong easterly winds above 40 kilometers limited reception of anomalous signals to the western half plane, with marginal reception to the north and south. The observed first zone

of audibility consists of an annular sector to the west of the shot point. The sector subtends an angle of some 180 degrees, and has a mean inner radius of about 200 kilometers, in good agreement with theoretical computations, and observations taken in other parts of the world.

The absorption along the ray path is computed for propagation due west, and angle of emergence intervals of one half degree. The resulting pattern of absorption near the inner edge of the first anomalous zone is found to be very irregular, with great fluctuations of intensity from point to point. Beyond 210 kilometers, the absorption proceeds more smoothly, but frequencies above 20 cycles per second are attenuated very rapidly. Power spectrum analysis is used to examine the frequency content of the anomalous waves. The general shape of the spectrum is in good agreement with existing theory, but important structural details (lines and bands) remain unexplained. The spectra are not corrected for instrumental response.

Ray parameters are computed for the newly revised U.S. Standard Atmosphere, extended to 700 kilometers. It is suggested, on the basis of these results, that the increase of high frequency content with distance, first noted by Cox (1949), could be at least in part a consequence of ray geometry. A simple method is also suggested for taking account of the effect of sloping layers in the upper atmosphere.

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I. A BRIEF HISTORY OF THE DISCOVERY AND INVESTIGATION OF THE ANOMALOUS PROPAGATION OF SOUND IN THE ATMOSPHERE.

1.1 The Discovery

The curious and often portentous roll of anomalous sound was first recognized in England, at the very end of an illustrious era. In January of 1901, during Queen Victoria's funeral, the rhythmic boom of the minute guns at Spithead was heard clearly not only in the adjoining borough of Portsmouth, but also at London and Oxford, some 70 miles away, and at other points far to the north. This notable incident was made all the more puzzling by the fact that places at intermediate distances much closer to Portsmouth did not hear the guns at all. Mysterious rumbles from far-off volcanoes and detonating meteors must have startled unwary observers like bolts from the blue since time immemorial, but there are few records to suggest that the ultimate cause of such an event was ever correctly identified on previous occasions.

Names first given to new discoveries and imperfectly understood events often turn out, on closer inspection, to have been misnomers. And so it went with anomalous propagation: investigations soon showed that this mode of transmission was neither anomalous nor rare, and not too difficult to observe if only one knew where and when to listen. Terms such as "indirect" or super-refracted sound would be more appropriate today on physical grounds, but since the original designation has become part of acoustical usage, it will be

retained without further ado, despite its semantically "un-sound" derivation. Terminology aside then, what is meant by anomalous propagation of sound, and similar phrases? E. F. Cox^{*}, one of the foremost workers in this field, describes it succinctly as follows:

"An observer situated 200 km or farther from a large explosion may or may not hear the boom at the time he would normally expect the noise to reach him. It is more probable that he will hear a much delayed report, one which appears to travel from source to receiver with a velocity some ten to 20 per cent lower than normal sound. In this instance, normal sound velocity is defined as that given by the Laplace equation ... A sound signal is generally considered normal if the distance from source to observer, divided by the signal travel time, gives an apparent velocity within three per cent of the Laplace value computed with the mean ground-level temperature."

Sometimes meteorological conditions cause both normal- and anomalous-velocity sound signals to reach an observer, who thus hears two distinct and separate booms from a single explosion. Cox himself has recorded an excellent example of such a double boom: five hundred and fifty-nine seconds after detonation of 125 tons of TNT the normal sound signal arrived at a microbarograph recorder located 182 kilometers from the site of the explosion. After the normal sound had died away completely, a second, equally loud anomalous signal reached the recorder fully 77 seconds after the arrival of

^{*}E. F. Cox et al., J. Meteor., Vol. 6, pp.300 (1949)

the first pulse of normal sound. Double booms of this kind are comparatively rare events, however. On most occasions, the normal sound is unable to reach the distant observer, and only the anomalous signal is heard distinctly at the later time.

In addition to the normal signal and the principal anomalous signal, there are various related phenomena, discovered in the past few decades. But before we discuss the complications of later years we must return to the early days of discovery and surmise.

1.2 Early Theories of Anomalous Propagation

The accidental discovery that sound waves could bridge great distances was made all the more baffling by the presence of a shadow zone, i.e. a zone at intermediate distances where the sound could not be heard at all. It had long been known that sounds are generally heard much better in the direction of the wind than against the wind, but this fact was not explained until 1857, when Stokes pointed out that winds aloft must needs interfere with the rectilinear propagation of sound waves. Thus, when the wind increases with height (the usual state of affairs) a ray travelling against the wind will be bent upwards, and pass over the head of an observer; rays travelling with the wind, on the other hand, are bent downwards and returned to ground, if the angle of incidence is not too small. Other things being equal, a "skipping" ray of this type should be heard much better, and over greater distances, than a horizontal, rectilinear ray, for it would

avoid not only many physical obstacles, but much of the dissipative effects of the turbulent friction layer as well. By the same token, the existence of shadow zones could thus be explained quite simply by postulating suitable wind gradients in the high atmosphere.

Within a few years the wind refraction hypothesis got into serious difficulties, however, and had to be discarded, when it was found that the zone of silence and the zones of anomalous audibility could, on occasion, form complete rings around the site of an explosion. This discovery clearly ruled out wind shear refraction as the primary cause of anomalous propagation, for the circular symmetry of such patterns could not possibly be effected by the highly directional mechanism of wind vectors.

In 1910 G. von dem Borne proposed a scalar theory of anomalous propagation, based on the assumption that the necessary increase in sound velocity with height could be brought about by a change in composition of the atmosphere. Thus if lighter gases, particularly hydrogen and helium, were to become predominant at higher levels, the molecular weight of the air would decrease, and the velocity of sound would increase. This theory seemed most attractive for a time, for it was known, of course, that sound travelled much faster* in the lighter gases than in air of normal composition. Doubts arose, however, when Wiechert showed that it would be necessary to assume a 25 per cent concentration of hydrogen

* e.g. velocity of sound in hydrogen at 0°C. is 1269.5 m/sec.; in air 331.6 m/sec.

at a height of only 40 kilometers. This requirement was difficult to reconcile with the experimental evidence at hand, for it was known by then that the composition of the air was essentially constant to heights of at least 20 kilometers. Early upper air soundings had detected a slight increase in the relative helium content above 20 km, but the measured rate of increase was far too small to produce the required 25 per cent concentration at 40 kilometers. Nevertheless, this qualitatively reasonable theory was quite popular for almost 20 years, and did not lack in prominent support: A. Wegener held to it stubbornly in the face of mounting evidence, and defended it almost literally to the very end.

At about the same time the suggestion was made that changes in the ratio of the specific heats of air might be responsible for the observed anomalous phenomena. Since air in the lower atmosphere consists largely of diatomic gases, this ratio is very nearly 1.41, but for monatomic gases, such as might be present at great altitudes, the ratio is 1.67. This theory failed also, and for much the same reason as von dem Borne's theory: the required presence of large concentrations of monatomic gases at relatively low levels conflicted with kinetic theory, and could not be reconciled with experimental evidence. Faced with such difficulties, the very notion that sound might be transmitted through the upper air at all came under attack from some quarters; it was even suggested that the transmission depended on a new and yet to be discovered principle.

1.3 The Work of Schrödinger

Viewed from the vantage point of 1963, the historically most fruitful, if erroneous, objection was probably made by Schmidt (1916), who rejected upper air transmission altogether on energetical grounds. Arguing that the amplitude of an ascending wave could not possibly increase with height, but would, at best, remain constant in the absence of all damping, Schmidt showed that the intensity of the wave would decrease in direct proportion to the decrease in atmospheric density. Postulating an equal attenuation for the descending wave by invoking the principle of reciprocity, Schmidt found the total attenuation of the refracted wave to be proportional to the square of the ratio of the atmospheric density at the wave apex and the earth's surface. Since this quantity is vanishingly small for apex heights in excess of 40 km, the returning wave would be of exceedingly low intensity, and thus quite beyond detection.

At this juncture, Schrödinger (1917) became interested in the problem and promptly produced an incisive paper of enduring merit. Schrödinger disposed of Schmidt's objections by showing conclusively that the amplitude of an ascending wave must, indeed, increase with height. In the course of this proof he derived also the dispersion relation for plane waves propagating vertically in an isothermal atmosphere. Schrödinger's greatest contribution to anomalous propagation theory is contained, however, in the last sections of his paper, added, it almost seems, as if by happy afterthought.

Here, in a few columns, Schrödinger deduces the absorption equation for sound waves travelling horizontally through an isothermal atmosphere and presents, on the very last page, a miniature set of graphs, the celebrated absorption curves. These little graphs created new and serious difficulties for the hydrogen theory, and clearly suggested an impasse. Schrödinger was of course well aware of this, but he did not, as far as is known, pursue the matter any further. Atomic theory had begun to engage his attention, and in the end he came to forsake the upper air for the more rarified realms of quantum mechanics.

1.4 A Bright Flash in the Darkness

The first new clue to the mystery of anomalous propagation came to light some five years later, in 1922. This clue to an acoustic puzzle turned up in rather unexpected quarters - astrophysics. Lindemann and Dobson, investigating the brightness of meteors, found they could not account for the observed brightness without assuming implausibly high atmospheric temperatures, at levels of 50 to 80 kilometers.

By and large, the requirements of the astrophysicists were modest enough: all that was needed was a warm layer of air in the high stratosphere, at temperatures of the order of 300 degrees absolute. But this demand was difficult to reconcile with common sense and all contemporary aerological evidence. It was known, of course, that atmospheric temperatures would on occasion increase with height, but such in-

versions were always quite shallow and impermanent. The stratosphere, discovered by de Bort in 1899, had been found to be a region of low temperature to the very limit of sporadic penetration by early sounding balloons. Surely, there was little reason to suppose that the temperature would not continue to drop monotonously to the absolute cold of the infinite void beyond! Even so, there was no a priori reason for being unnecessarily dogmatic about either side of the question: the controversial region was still well beyond the ken of direct observation, and the evidence was largely imperfect and circumstantial. But then, here was a new and startling piece of evidence which, sooner or later, had to be placed in the proper context of related facts - and anomalous propagation was just such a fact.

When F. J. W. Whipple (1923) read Lindemann and Dobson's paper, he immediately grasped its revolutionary implications: here then, at last, was the long-awaited explanation of anomalous sound phenomena!

Since the refraction of sound in a gas depends largely on the gradient of the absolute temperature, Whipple realized that sound could be returned to ground by appropriate positive temperature gradients in the high atmosphere. His first rough calculations lent strong support to this view: temperature refraction was not only feasible but eminently reasonable, and the observed time delay and skip phenomena could be explained easily in a most natural way.

1.5 Controversy and Computations

It must not be thought that Whipple's new theory gained immediate and universal acceptance. For one thing, the question as to why a part of the upper atmosphere should be so much warmer than the frigid stratosphere remained unanswered in 1923, and for another, men's minds just balked a little at the sheer magnitude of the required temperature reversal. A modest 10 or 20 degree rise in temperature would have been accepted perhaps with little dissent, but the required 80 to 100 degree inversion proved too much of a good thing for some investigators. Alfred Wegener, for instance, a creative mind of many parts, thought nothing of rejecting the new theory outright, if not out of hand; as late as 1925 he could state flatly* "Temperature has nothing whatsoever to do with it ... To say more is useless and a waste of words". Waste or not, Wegener was not to wait long for some pointed rejoinders: writing in a subsequent issue of the Meteorologische Zeitschrift, and armed with the latest figures, Joseph K lzer** rose to the attack for the opposite camp: "The categorical tenor of the assertion does not make it any the more correct for all that. Es fehlt auch ein mathematischer Beleg" ... December 1925, the year was almost over, but not quite.

Gutenberg and Wiechert, well versed in the methods of seismology, had independently tackled the riddle of the upper air by turning the conventional geophysical problem upside-

* Met. Zeitsch., Vol. 42, p.261 (1925)

** Ibid. p. 461

down. Conjecture gave way to Computation: they announced their results at a meeting of the German Geophysical Society in December 1925. The temperatures of the upper air had been computed for the first time.

1.6 Shock Waves

Whipple's theory was now the leading contender: the scales had dipped in its favour, and it began to be accepted widely if not altogether unanimously. Still, in the absence of proof positive a considerable area of doubt remained; the search had to go on, and other possibilities had to be explored to the full, before the issue could be decided on the basis of existing indirect evidence alone.

One possible mechanism which has received considerable attention from time to time involves the speed of shock waves in rarified air. The derivation of the familiar equation for the speed of sound (Laplace's equation) is based on the assumption that the pressure perturbation p is several orders of magnitude smaller than the ambient pressure P of the transmitting medium. If, on the other hand, the perturbation pressure is an appreciable fraction of the ambient pressure, the Laplace relation is no longer valid, and the resulting shock wave will travel faster than "normal" sound, a result first derived theoretically by Riemann and well confirmed by experiment. Specifically, compressions of finite amplitude travel with supersonic speed, and rarefactions proceed with subsonic speed. An N-shaped wave results*, whose steep

*See J. W. M. DuMond et al., J. Acoust. Soc. Am. 18,97 (1946)

front travels at a supersonic speed V given by the equation

$$V = C \left[1 + \beta(\gamma + 1)/2\gamma \right]^{\frac{1}{2}} \quad (1 - 1)$$

where C is the normal (Laplace) speed of sound, γ is the ratio of the specific heats of the transmitting medium, and β is the pressure ratio p/P .

It seems conceivable, therefore, that a large pressure disturbance from an explosion might behave as a shock wave on reaching the low pressure regions of the upper atmosphere. If then the ambient pressure continued to decrease while the perturbation pressure remained high, the increasing speed of the shock wave would lead to refraction, and return the wave to ground.

This problem was investigated in 1925 by Wiechert, who pointed out that the governing principle is energy conservation, and not pressure conservation. As the pressure perturbation p advances into the upper atmosphere (of ambient pressure P) the sound intensity $p^2/\rho C$ will decrease in accordance with the inverse square law. Since the ambient density ρ is proportional to the ambient pressure P , and since the sound velocity in the upper levels differs at most by a few per cent from the velocity at the earth's surface, it is evident that

$$p^2/\rho C = ap^2/P = b/\Delta^2 \quad (1 - 2)$$

where a and b are constants, and Δ is the distance from the source. Evaluating the constants from experimental data, Wiechert showed that the ratio $\beta = p/P$ decreased to about $1/75$ at a height of 45 km. He concluded, therefore, that shock

wave phenomena could not account for the anomalous propagation of sound.

Cox (1948) has studied this problem by considering the actual geometric divergence of acoustic rays in the troposphere, and the ensuing conical wave fronts. On this basis, the pressure ratio would have to be greater than 0.3 before the wave could be returned to ground at the appropriate distance. Sound refracted in this manner would produce peak perturbation pressures of the order of four millibars in the first anomalous zone, but measurements carried out by Cox show actual pressures to be less than 3 per cent of this value.

Perhaps the most telling argument against shock wave refraction is this: explosions ranging in strength from nuclear bombs to the muzzle blast of an 8-inch naval gun all seem to produce anomalous zones of the same general diameter. This is obviously not in accord with the theoretical requirement that the distance from the explosion to the anomalous zone boundary decrease as the size of the explosion increases.

Wegener (1925) has found that the radius of the inner boundary of the zone varies with the season, ranging from about 105 km in winter to 200 km in summer. The inner radius depends also strongly on short term meteorological factors, and it may well depend also on latitude; but it appears to be completely independent of the energy input at the site of the explosion.

1.7 Ozone

Solar radiation penetrates the atmosphere to varying depths and suffers maximum absorption at different altitudes. Wavelengths shorter than 1800 Å are largely absorbed above 100 km. Radiation with wavelengths between about 1800 and 2900 Å penetrates to about 35 km, and the still longer waves in the visible and infra-red reach the earth's surface. The absorption at any particular level is often largely due to a specific atmospheric constituent. Thus molecular oxygen absorbs strongly between 1300 and 2000 Å at heights of about 100 km, but below 70 km, the absorption of ultraviolet radiation by ozone becomes increasingly important, particularly between 2000 and 2900 Å. X-rays will penetrate to about 80 km, but, by and large, there is little absorption of any kind at levels near 80 km. In the lower regions of the atmosphere, certain bands in the infra-red are absorbed strongly by water vapor, carbon dioxide and also, to some extent, by ozone.

Little such information was, of course, available to the investigators of the nineteen-twenties. The upper atmosphere was still almost as much beyond the reach of direct exploration as the mantle of the earth is now. But this was, nevertheless, a time of great advance and triumph. It was the challenge of those early days to obtain the answers through observations taken from a distance and, if at all possible, by the simplest means, used with the greatest of skill and imagination.

Atmospheric ozone has long held the special attention of research workers from several fields: geophysics, astrophysics, meteorology and biophysics, to name the most important. But this is hardly the place to tell the story of ozone: suffice it to say that it is a trace substance of almost incredible importance to life on earth and, by the same good fortune, to anomalous propagation of sound. The absorption coefficients of ozone are so large that most of the solar energy in the ultraviolet is absorbed between 40 and 60 km, well above the level of maximum concentration. The main ozone layer, located between 15 and 35 km, can thus absorb only weakly in a few bands in the visible and near ultra-violet region.

The presence of a strong absorber in the upper air must have a profound effect on the temperature regime of the atmosphere. This was realized early by E. H. Gowan, and led him to suggest that the high temperature layer in the upper stratosphere might well be due to the absorption of solar radiation by ozone. Gowan's first computations of 1928-1930, and the later series of 1947 have not only confirmed this view, but are in rather good agreement with the results of direct measurements carried out in recent years.

The postulated high temperatures in the refraction layer were now explained and established beyond all reasonable doubt. Whipple's theory reigned supreme. But it must not be assumed that this ended the matter altogether, for now a new kind of objection was raised in some quarters. Thus Arabadzhi (1946) has argued that the speed of sound in the ozone layer should

decrease because ozone is a triatomic gas. This decrease in speed, amounting to over 20 per cent in pure ozone, would be far from trivial if the absolute concentration of ozone in the upper layers were at all high. But the total content of ozone in the entire atmosphere is so low - a layer less than 3 mm thick when reduced to standard conditions - that ozone, in this one respect at least, is just another impurity, whose intrinsic effect on sound propagation must needs be deemed to be negligibly small.

1.8 Early Empiricism

The discovery of ozone in the upper air, as much as the ready availability of large quantities of explosives, gave new impetus to the study of anomalous sound propagation. Many extensive tests were carried out in several countries, often in connection with the destruction of ammunition left over from the First Great War.

The first comprehensive analysis of data available up to 1925 was made by Wegener in the course of his work on the shadow zone, referred to earlier. This study included many reports of anomalous audibility of gun fire on the Western Front. These melancholy observations were made notable by the fact that the anomalous din of battle was best heard on the French side in summer, and on the German side in winter. It was suggested early on that such seasonal variations must be largely due to changes in the upper wind circulation, a supposition well confirmed by later investigations.

The years between the wars were marked by vigorous and extensive experimentation in Europe, which yielded much new observational material. Notable in this respect are contributions by Angenheister (1924), Maurain (1924), Hergesell and Duckert (1929), and Meisser (1934). Gutenberg published numerous papers and review articles on the subject, first in Germany (1926-1932) and later in the United States.

Whipple (1931) carried out some of the first systematic observations in England during the years 1929-1930. These observations enabled him to deduce the temperature distribution between 30 and 60 km, on the assumption that wind effects could be neglected. Whipple (1935) later endeavoured to separate the wind and temperature effects, but his attempts were not altogether successful for lack of sufficiently complete sets of data. He was able to show, however, that the changes in upper air circulation from winter to summer were such as to produce the observed seasonal reversals of anomalous audibility.

In the winter of 1932-33, Wölken carried out the first anomalous propagation studies in the Arctic. The tests were made in Novaya Zemlya, latitude 76°N , and showed the presence of a reflecting layer even during the rigours of the Arctic night. The presence of a warm ozonosphere in midwinter poses interesting questions, for it is difficult to see how a photochemical ozone balance could be maintained in the prolonged absence of all sunlight. Whipple (1935) has suggested that the warm layer could be sustained by direct meridional transfer of ozone from sunlit latitudes.

1.9 Cacophony 1939 - 1945

The Hell of the Second World War should have been pure Paradise to atmospheric acousticians, but, as far as is known and perhaps to their eternal credit, they took not the slightest advantage of the countless heaven-sending opportunities proffered with callous indifference, and on a monstrous scale. The list of papers that could have been written is endless.

On the aberrance of airwaves observed during the destruction of Coventry.

Beitrag zur Beobachtung der durch die Vernichtung der Dresdener Altstadt erzeugten Schallwellen.

Sur la propagation des ondes aériennes lors de destruction de Caen.

... Warsaw ... Rotterdam ... Monte Cassino ... Hiroshima
The list is endless.

1.10 Atmospheric Acoustics since 1945

When the promiscuous cacophony of war had at last abated, the "controlled" experiments got under way again with hardly a pause, and with a vengeance: for one thing, large "left-over" stocks of munitions were available again for the asking, and for another, there was The Bomb. The Curtain came down, and it became fashionable to speak of two kinds of explosives: the old-fashioned, chemical kind, and the nuclear new.

The largest and best publicized single chemical blast of recent memory - Operation 'Big Bang' - involved the planned destruction of the fortifications of Helgoland in 1947, with 5000 tons of high explosive. Several scientific groups took

part in recording this venture, notably E. F. Cox (1947, 1949) and his associates from the U.S. Office of Naval Research. Making use of expressly built low-frequency equipment, this group obtained not only a good set of signals from the mesosphere, but succeeded also in first recording acoustic returns from the ionosphere. These ionospheric signals are infra-sonic waves with periods of the order of one to ten seconds, which have been refracted to ground by the high temperature inversion existing at heights above 80 km. Waves with periods of under one second suffer strong attenuation in the highly rarified upper air and are, as a rule, below the threshold of detection when they return to ground - in complete agreement with Schrödinger's predictions of 1917.

A. P. Crary (1950, 1952, 1953) of the U.S.A.F. Cambridge Research Laboratories, carried out extensive tests between 1946-1951. Most of the tests were conducted from bases in the Canal Zone, Florida, Hawaii, and Bermuda. A bombing plane was used to drop 500 lb bombs into the sea, at varying azimuths and distances from a single fixed recording site. Another series of tests, made in Alaska, was rather more conventional in that it used 200 lb surface bursts and several recording sites. Crary's tests yielded a wealth of information on winds and temperatures in the 30 to 60 km region, but attempts to obtain reflections from the ionosphere were unsuccessful, probably because the charges used were rather small.

Richardson and Kennedy (1952) conducted an important test series in Colorado, designed to study the seasonal changes of

wind and temperature over an eleven month period. Variations in azimuth and distance from shot point to the recording sites permitted the separation of wind and temperature components of the observed sonic velocity structure to heights of about 50 km. They found that the wind field over Colorado was generally strong and westerly during the winter, and light easterly in summer; wide changes in speed and direction were encountered in the spring and fall months.

Johnson and Hale (1953) performed an interesting series of tests in California and Arizona. The complete program consisted of ten test periods extending over some 14 months. The blast waves were produced by surface detonation of two navy depth charges, a combined weight of 1200 lb of TNT. The waves were detected at distances of 100 to 320 km by an array of microphones located at intervals of $6\frac{1}{2}$ km. Johnson and Hale recorded many signals which had been returned to ground by the high temperature inversion at the base of the ionosphere. These signals were better received to the east of the blast in summer, and to the west in winter, just opposite to the seasonal direction pattern for waves from the mesosphere. Careful analysis of the many records showed that the arrivals from the mesosphere were much more complex than first expected. Several wave groups of different speed and intensity were identified, some with proper speeds in excess of the normal speed of sound, an experimental fact which still awaits theoretical explanation.

The information which can be obtained from ground-based experiments is, of course, largely limited to heights at which reflection occurs. The use of sound propagation can nevertheless be extended to other regions by transporting grenades in high altitude rockets to heights beyond the reflection layer. The grenades are ejected from the rocket and detonated at the desired level; the arrival of the sound wave is monitored by an array of stations on the ground.

Rocket grenades were first employed at Whitesands in 1950, borne aloft in Aerobee rockets (Brasefield 1954). Such experiments have been repeated since on many occasions, e.g. at Fort Churchill as part of the program for the International Geophysical Year (Stroud et al, 1960).

The development of meteorological rocket systems in the U.S. and elsewhere (Webb 1959, 1962) has led to a decline in conventional acoustic propagation studies of the upper air. However, recent factual reports (see The Press) seem to suggest that the rocket system may face serious competition from a newly discovered acoustic research tool: the earth-bound ton of TNT.

SCIENCE

METEOROLOGY

Mapping the Air by Sound

At Queen Victoria's funeral in 1901, cannons in London boomed a ceremonial farewell—and villagers 90 miles away were startled by the rumbling volley. Yet not a shot was heard in towns halfway between. What caused the funereal boom to leapfrog?

Turn-of-the-century scientists theorized that the cannons' ascending roar had been bent by a freak atmospheric condition that sent it tumbling back to earth. But not until men began probing the upper atmosphere with instrumented rockets could the conditions that caused this sound bounce be fully understood. Early this month, scientists at the White Sands Missile Range used a phenomenon like that at Victoria's funeral to help them chart a region of the upper atmosphere.

Since 1958, Army Meteorologist Marvin Diamond had fired more than 1,000 rockets deep into the atmosphere above White Sands. Probing high above the maximum altitude of sounding balloons, his investigative missiles dropped metalized parachutes carrying temperature-measuring devices and providing tracking radars with easily detectable targets. By charting their drift, Diamond hoped to map the weather through which the U.S. must fire its growing family of space vehicles.

On the way up, Diamond's rockets passed through the 125-knot "jet stream," which boots airliners along from west to east some 35,000 ft. above the earth. Far above that, they found a speedier "upper jet stream," which reversed its direction with the changing seasons. During the fall and winter, it zooms out of the west at some 150

knots. In spring and summer, it slows to 100 knots and drives from east to west. In either direction, its altitude seems to be about 150,000 ft. Significantly, the upper jet stream is a warm wind, ideal for refracting sound waves.

Diamond knew that the speed of sound is greater in warm air than in cold air. If a sound wave, rising through the sub-zero temperatures below the upper jet stream, suddenly hit a layer nearly as warm as the earth's surface, the top of the wave front, he figured, would accelerate. The whole front would then bend back earthward and rumble down. Diamond figured that he might be able to bounce a boom off the upper stream, predict its course, and record the boom as it came back to earth, thus helping to confirm his rocket data.

Finally he got the weather he was waiting for. The still, windless desert air was shattered by the roar of a 5,000-lb. explosive charge. Ten miles away, in his White Sands headquarters, Diamond saw only the flash. The sound waves traveling along the surface had been muffled by the dense lower air. Yet 200 miles to the north, four microphones caught the roar of the explosion after it caromed back to earth—in much the same way that the cannons' roar was bent at Victoria's funeral.

1.12 Aeroclysms

No description of anomalous sound phenomena would be complete without at least a passing reference to atmospheric gravity waves. Any large explosion (a so-called "big bang") excites two types of oscillation: sound waves, and gravity waves. Most of the energy released by big bangs goes into gravity waves, a train of travelling waves that can always be observed at ground level, and at very large distances from the site of the explosion, a thousand miles or more.

When a gravity wave passes an observer, the whole mass of the atmosphere above him will heave simultaneously. The wave is propagated largely within a spherical wave guide bounded by the concentric surfaces of the earth and the first high temperature inversion. The wave front is thus essentially vertical, and there are no shadow zones. Gravity waves usually travel with a horizontal velocity which is roughly equal to the average velocity of sound within the wave guide, or typically about 320 m/sec. relative to the air; this is lower than the speed of sound at the ground (about 340 m/sec) and faster than sound in the stratosphere (roughly 300 m/sec). The speed of the wave relative to the ground may differ from the "air-speed" by as much as 20 to 30 m/sec, depending on the prevailing wind circulation. The long gravity waves show noticeable dispersion, and a short initial pulse is soon dispersed into an ever lengthening wave train of many minutes duration.

Scorer (1958) has suggested that the largest "big bangs" which rock the entire atmosphere be given the grander name of aeroclysm (a flooding of air). Such atmospheric tidal waves are of truly global proportions: they oscillate with periods of half an hour or more, and circle the earth repeatedly for several days. (Domand Ewing 1962).

The aeroclysm was, until quite recently, a very rare event. The obliteration of Krakatoa in 1883 was probably the only event of sufficient magnitude to qualify in all recorded history, but the detonation of the Great Siberian Meteorite of 1908 was violent enough to deserve at least passing recognition. Hunt (1960) has estimated that the energy released by the meteorite was equivalent to about 10^{16} calories, or ten megatons of TNT; its flash outshone the morning sun, the primeval forests were flattened for miles around, and the pressure waves were detected in London, some 6000 km away. But man has since learned to do rather better with 57 megatons: the Great Siberian Super Bomb of 1961 was of the same order of violence as Krakatoa. A sobering thought.

II. THE THEORY OF PROPAGATION OF SOUND RAYS IN THE ATMOSPHERE.

2.1 Introduction

The theory of sound propagation in the atmosphere is a special case of the general theory of wave propagation in moving, inhomogeneous, viscous media. More specifically, the motion of sound waves through the atmosphere is also an aerodynamical problem, beset with most of the difficulties of the general theory of fluid dynamics. Most problems arising in practice can therefore be solved only subject to certain simplifying assumptions.

Temperature and wind, the main factors governing sound transmission in the atmosphere, vary about one hundred times less rapidly in the horizontal than in the vertical direction. Horizontal temperature and wind variations are hence comparatively small and usually negligible; practical cases do arise, however, where it may not be safe to neglect them, e.g. in the vicinity of frontal surfaces and jet streams. Furthermore, many of the observed local irregularities in sound patterns are, no doubt, largely due to horizontal temperature and wind gradients.

In the following discussion all horizontal variations of wind and temperature will be neglected. The atmosphere will be considered to be a horizontally stratified medium, with wind and temperature continuous functions of height only.

Since the distance over which sound can be transmitted is rarely greater than a few hundred kilometers, the curvature of the earth's surface will be neglected, and the atmospheric layers will be assumed to be bounded by parallel planes, rather than concentric spheroidal shells.

Many problems of atmospheric acoustics can be solved with sufficient accuracy by the familiar methods employed in geometric optics. Analogous to light, a sound wave will be propagated approximately rectilinearly, provided that the wave length is small in comparison with the dimensions of apertures and obstacles encountered by the wave. This condition is readily fulfilled in the free atmosphere, but it usually limits the geometric propagation of infrasonic waves near the surface of the earth.

2.2 The Speed of Sound in Air

The speed of sound C in a pure gas is given by Laplace's equation:

$$C^2 = \frac{\gamma RT}{M} \quad (2-1)$$

where $\gamma = c_p/c_v$, the ratio of the specific heats of the gas at constant pressure and constant temperature,

R = the universal gas constant ($= 8.314 \times 10^7$ erg/deg^oK mole)

T = absolute temperature

M = molecular weight of the gas

Laplace's equation is valid also for a mixture of gases if M is defined as the effective "molecular" weight of the mixture, and γ the ratio of the weighted specific heats of the components. Substituting the values $\gamma = 1.403$ and $M = 28.966$ gm/mole for dry air^{*}, the local speed of sound is given by

$$C = 20.06 \sqrt{T} \text{ m/sec.} \quad (2-2)$$

Laplace's equation is obtained most directly from the expression for the speed of longitudinal waves in a homogeneous medium of density ρ , and Lamé constants λ and μ :

$$C^2 = (\lambda + 2\mu)/\rho \quad (2-3)$$

Since the bulk modulus $B = \lambda + 2/3\mu$ and the modulus of rigidity μ is very nearly zero in gases, $B = \lambda$. Equation (2-3) thus reduces to

$$C^2 = B/\rho \quad (2-4)$$

The adiabatic bulk modulus must be used in this expression, since the propagation of sound proceeds adiabatically. B is readily obtained from the relation describing adiabatic processes in a gas: if p is the pressure and v the volume, then

$$pv^\gamma = \text{const}$$

$$\left(\frac{\partial p}{\partial v}\right)_s = -\frac{\gamma p}{v}$$

$$B = -v\left(\frac{\partial p}{\partial v}\right)_s = \gamma p$$

^{*}This value of M for dry air is the defining molecular weight of the proposed U.S. Standard Atmosphere, assumed constant from mean sea level to a height of 90 km. See Sissenwine (1962).

Laplace's equation (2-1) follows directly by substitution for p/ρ from the equation of state written in the form

$$\frac{p}{\rho} = \frac{RT}{M} \quad .$$

Experiments show that this equation holds with great accuracy for wide ranges of frequency and temperature, provided that the amplitude of the sound is not too large (Hardy et al, 1942).

When the air is moist, the speed of sound is slightly higher than in dry air. Gutenberg (1942) showed that this increase ΔC is approximately given by

$$\Delta C = 0.14 C_e / p \quad (2-5)$$

where e is the vapor pressure of water at temperature T .

If the air is very warm and moist, the speed may increase by as much as 5 m/sec.

This effect may be readily allowed for by the introduction of the virtual temperature T^* in the Laplace equation. The virtual temperature of a volume of moist, unsaturated air is defined as the temperature at which dry air of the same pressure would have the same density as the moist air. Dry air is heavier than moist air! It can be shown (e.g. Hewson and Longley, 1943) that

$$T^* = \frac{T}{1 - \frac{0.378e}{p}} \quad (2-6)$$

With this substitution, the speed of sound C_m in moist air is

$$C_m = 20.06 \sqrt{T^*} \quad (2-7)$$

The virtual temperature is easily obtained from standard meteorological observations of temperature and humidity. The air masses encountered on the Canadian Prairies are usually so dry, however, that humidity corrections are negligibly small.

2.3 Sound Rays in Quiet Air

As in optics, the ray equation for sound follows from Fermat's "Principle of Least Time", or, more directly, from Snell's Law derived from it. Consider three contiguous thin layers of a horizontally stratified medium such as the atmosphere, at temperatures T_1 , T_2 and T_3 . Since the speed of sound in each layer is a function of the temperature only, we have, in view of Snell's Law and (2-2)

$$\frac{\sin i_1}{\sin i_2} = \frac{C_1}{C_2} = \left\{ \frac{T_1}{T_2} \right\}^{\frac{1}{2}}$$

$$\frac{\sin i_2}{\sin i_3} = \frac{C_2}{C_3} = \left\{ \frac{T_2}{T_3} \right\}^{\frac{1}{2}},$$

i_1 , i_2 , i_3 being the respective angles of incidence, i.e. the angles between the ray and the vertical.

Combining the above expressions, we obtain

$$\frac{\sin i_1}{C_1} = \frac{\sin i_2}{C_2} = \frac{\sin i_3}{C_3} = P \quad (2-8)$$

where P is a constant for each ray, the ray parameter of seismological terminology. When the velocity of sound varies continuously in the medium

$$\frac{\sin i}{C} = \frac{\sin i}{20.06 T^{\frac{1}{2}}} = P \quad (2-9)$$

If the temperature varies discontinuously, and jumps from T_1 to $T_2 = T_1 + \Delta T$, the ray may suffer total reflection. The critical angle i_c is given by

$$\frac{\sin i_c}{C_1} = \frac{1}{C_2}$$

$$\text{or } \sin i_c = \frac{C_1}{C_2} = \left\{ \frac{T_1}{T_2} \right\}^{\frac{1}{2}} \approx 1 - \frac{\Delta T}{2T_1} \quad (2-10)$$

For $T_1 = 273^\circ\text{K}$

and temperature jumps $\Delta T = 2^\circ \quad 5^\circ \quad 10^\circ$
the critical angles are $i_c = 85.1^\circ \quad 82.3^\circ \quad 79.2^\circ$.

Neglecting jumps in speed and temperature, the general equation of the ray as a function of height is obtained from (2-9). If we denote distance along the ray by s , the horizontal displacement by Δ , and displacement along the local vertical by z (positive up), we have, for a ray moving in the $\Delta - z$ plane:

$$\frac{d\Delta}{ds} = \sin i = PC \quad (2-11a)$$

$$\frac{dz}{ds} = \cos i = (1 - P^2 C^2)^{\frac{1}{2}} \quad (2-11b)$$

$$\frac{d\Delta}{dz} = \tan i \quad (2-11c)$$

$$\text{and } {}_1\Delta_2 = \int_1^2 \tan i \, dz = \int_1^2 \frac{PC \, dz}{(1 - P^2 C^2)^{\frac{1}{2}}} \quad (2-12)$$

To obtain the associated travel time t we note that

$$dt = ds/C = \frac{dz}{C \cos i}$$

and

$$t_1^2 = \int_1^2 \frac{dz}{C \cos i} = \int_1^2 \frac{dz}{C(1 - P^2 C^2)^{\frac{1}{2}}} \quad (2-13)$$

The problem now is to find a general expression for the absolute temperature as a function of altitude which, when substituted into Snell's Law, and then into (2-12) and (2-13) will give functions which can be readily integrated. Cox (1947) has given the general solution of this problem, but two special solutions were used in an earlier work by Wiechert (1925).

From Snell's Law in the form of (2-9)

$$\sin^2 i = P^2 C^2 = aT, \quad (2-14)$$

where a is a constant. Denoting differentiation with respect to z by a prime, we obtain from this equation:

$$dz = \frac{2 \sin i \cos i \, di}{aT'} \quad (2-15)$$

Substituting into (2-12) and making use of (2-14) gives

$$\Delta = \frac{2}{a} \int \frac{\sin^2 i \, di}{T'} = 2 \int \frac{T}{T'} \, di \quad (2-16)$$

Similarly, from (2-13) and (2-14) we obtain for the travel time

$$\begin{aligned} t &= 2 \int \frac{\sin i \cos i \, di}{a T' C \cos i} \\ &= \frac{2P}{a} \int \frac{di}{T'} \end{aligned} \quad (2-17)$$

The last two expressions can be easily integrated if

$$\frac{dT}{dz} = T' = b_n T^{n/2}, \quad (2-18)$$

where n can have any value from $-\infty$ to $+\infty$.

Since $(\sin^2 i/a)^{n/2} = T^{n/2}$, the horizontal distance integral becomes

$$\begin{aligned}\Delta &= 2 \int \frac{T}{T} di \\ &= \frac{2}{a} \int \frac{\sin^2 i di}{b_n T^{n/2}} \\ &= \frac{2a^{(n/2-1)}}{b_n} \int (\sin i)^{2-n} di \quad (2-19)\end{aligned}$$

Similarly, the travel time integral becomes

$$\begin{aligned}_1 t_2 &= \frac{2P}{a} \int_1^2 \frac{di}{b_n T^{n/2}} \\ &= \frac{2Pa^{(n/2-1)}}{b_n} \int_1^2 \operatorname{cosec}^n i di \quad (2-20)\end{aligned}$$

where $T^{n/2} = \sin^n i/a^{n/2}$ follows from (2-14).

Integrating (2-18) we find

$$\frac{T^{(1-n/2)}}{1-n/2} = b_n z + \text{const}$$

If $T = T_1$ at $z = z_1 = 0$, then

$$\begin{aligned}\text{const} &= \frac{2T_1^{(1-n/2)}}{2-n} \\ \text{and } T &= T_1 \left\{ 1 + \frac{2-n}{2} \cdot \frac{b_n z}{T_1^{(1-n/2)}} \right\}^{2/(2-n)} \quad (2-21)\end{aligned}$$

for all values of n except $n = 2$. For $n = 2$ it is readily shown that $T = T_1 \exp(b_2 z)$.

Cox (1947) has worked out a number of distance and travel time integrals for various values of n ; the resulting functions are related to the Beta function.

The simplest solutions are obtained for $n = 0$, corresponding to a linear temperature distribution:

$$T = T_1 + b_0 z$$

$$\text{or } T = T_1 - \Gamma z \quad (2-22)$$

where $\Gamma = -b_0 = -\partial T / \partial z$, the lapse rate of meteorological usage.

Putting $n = 0$ in (2-19), the horizontal distance is given by

$$\begin{aligned} {}_1\Delta_2 &= \frac{2}{ab_0} \int_1^2 \sin^2 i \, di \\ &= \frac{1}{ab_0} \left[i - \cos i \sin i \right]_1^2 \end{aligned}$$

But since $a = \sin^2 i_1 / T_1$, we have finally,

$${}_1\Delta_2 = \frac{T_1}{\Gamma \sin^2 i_1} \left[\cos i \sin i - i \right]_1^2 \quad (2-23)$$

For the travel time integral we obtain similarly

$$\begin{aligned} {}_1t_2 &= \frac{2P}{ab_0} \int_1^2 di \\ &= \frac{-2 T_1}{V_1 \Gamma \sin i_1} (i_2 - i_1) \end{aligned} \quad (2-24)$$

In the present work the simple linear solutions are used for ray computations in quiet air. It is, of course, debatable whether linear temperature gradients in the atmosphere are, a priori, more likely to occur, or whether such simplicity results merely from wistful applications of Occam's razor. There is little doubt, however, that many atmospheric

processes, e.g. turbulent mixing, heating and cooling from below, etc., lead to temperature distributions which, when sampled with present day standard equipment, seem in fact linear. In any event, it is, of course, always possible to approximate any observed distribution by means of short sections of constant lapse rate.

2.4 The Curvature of Sound Rays in Quiet Air

It is often useful to know the curvature K of a ray produced by a given temperature lapse rate. If i , the angle of incidence, denotes again the angle between the tangent to the ray at a point P_1 and the vertical, then the angle between the radius of curvature (normal to the ray at P_1) and the horizontal is also i . Thus $ds = R di$, and

$$K = \frac{1}{R} = \frac{di}{ds}$$

Evaluating di/ds we find from (2-9) that

$$\cos i \frac{di}{ds} = P \frac{dC}{ds}$$

and

$$\frac{di}{ds} = \frac{P}{\cos i} \frac{dC}{dz} \frac{dz}{ds}$$

or, making use of (2-11b)

$$K = P \frac{dC}{dz} \quad (2-25)$$

$$= \frac{dT}{dz} \frac{\sin i}{2T} \quad (2-26)$$

It is seen that the sign of K depends only on the sign of the vertical temperature gradient. In the troposphere $dT/dz \approx -6^\circ\text{K/km}$ and $T \approx 300^\circ\text{K}$. The curvature is, therefore

negative*, i.e. concave upward, and the radius of curvature of the ray is about 100 km when $i = 90^\circ$.

If the speed of sound varies linearly with height, K is constant and the rays will be circular arcs, but, if the lapse rate is constant, the ray paths will be cycloids. It is also worth noting that the curvature of the ray does not depend on the magnitude of C , but only on the velocity gradient dC/dz .

2.5 The Range of a Ray and the Temperature at the Ray Apex

If the atmospheric temperature distribution is such that a ray is returned to the ground, the horizontal range Δ of a ray from a source at $z = 0$ follows directly from (2-12):

$$\Delta = 2 \int_0^{z_m} \tan i \, dz = 2 \int_0^{z_m} \frac{PC \, dz}{(1 - P^2 C^2)^{\frac{1}{2}}} \quad (2-27)$$

where z_m is the maximum height or apex attained by the ray. The ray will be returned to ground if the speed of sound at the apex ($i = 90^\circ$) is

$$C_m = P^{-1} = \frac{C_0}{\sin i_0}, \quad (2-28)$$

and the absolute temperature at the apex is

$$T_m = \left\{ \frac{C_m}{20.06} \right\}^2 \quad (2-29)$$

The temperature distribution in the atmosphere up to about 30 km is usually known from routine observations made twice

* The curvature is negative as seen from the z -axis, but positive in the conventional sense with respect to the Δ -axis. Note that i is measured clockwise from the z -axis.

daily with radio-sonde balloons. The ray parameter P is readily obtained from the travel time curve, a plot of the travel time t versus the horizontal range Δ of the ray. The speed C_0 at ground level and C at any height in the troposphere and lower stratosphere can thus be evaluated from the observed temperatures, and the angle of incidence i_0 can be calculated from the slope of the travel time curve. Thus

$$\frac{\sin i_0}{C_0} = \left(\frac{\delta t}{\delta \Delta} \right)_0 = P \quad (2-30a)$$

The travel time curve is obtained in the following manner: consider a number of observing stations located, for the sake of convenience, along a straight line through an intense source of sound, such as an explosion. The travel time for a particular ray landing at the point O_1 , say, is determined by recording the time of arrival of the wave front at O_1 , and subtracting from it the time of the "shot instant", i.e. the time at which the explosion was set off. Since the distance between the source and observer is also known, a travel time curve can be plotted from a series of such observations.

The angle of incidence of a returning ray is obtained in a similar manner. In Fig. 1 let O_1 and O_2 be two neighbouring observations points in a line of detectors, and consider the rays returning to the earth's surface at O_1 and O_2 .

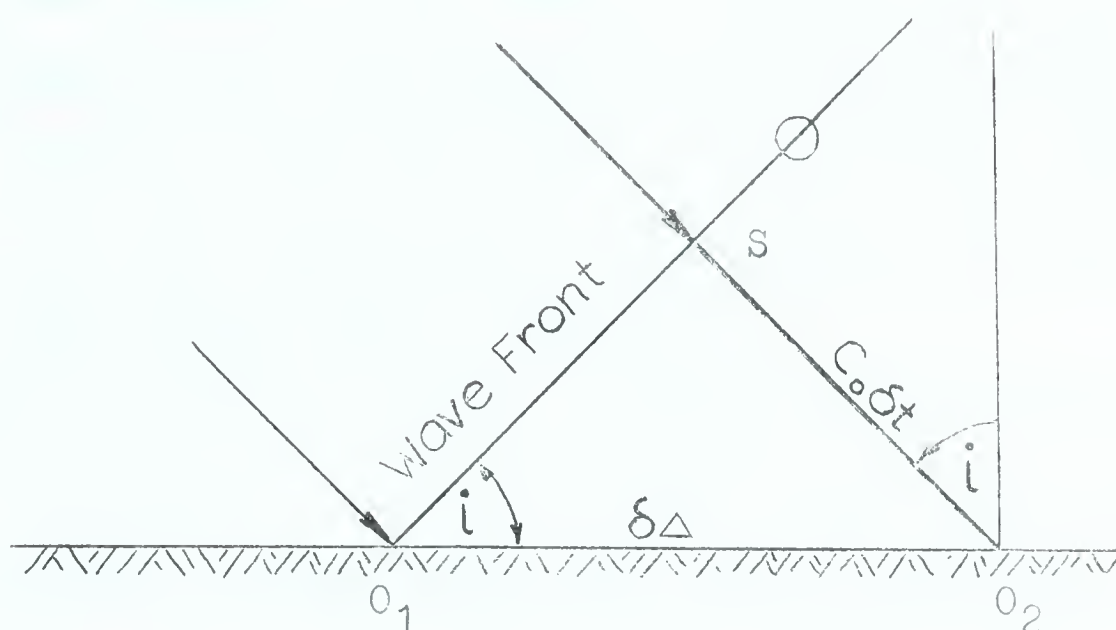


Fig. 1. Determination of the Angle of Incidence and the Apparent Velocity V_a .

When the wave front reaches the surface of the earth at O_1 , it still has to travel a distance SO_2 before reaching O_2 . If δt is the time interval between the arrivals of the wave at O_1 and O_2 , and if C_0 is the speed of sound in the region of O_1 and O_2 , then SO_2 is equal to $C_0 \delta t$. With $\delta\Delta = O_1O_2$, we find

$$\delta\Delta = \frac{C_0 \delta t}{\sin i_0},$$

$$\text{or } \sin i_0 = C_0 \left(\frac{\delta t}{\delta\Delta} \right)_0 \quad (2-30b)$$

Since C_0 , the speed of sound at ground level, can be measured directly, or deduced from the surface temperature, the angle of incidence can be calculated from equation (2-30b).

The quantity $(\delta\Delta/\delta t)_0 = V_a$ is called the apparent or characteristic velocity of the sound wave. The apparent velocity is the phase velocity at which the wave front

sweeps along the surface of the earth. Moreover, as seen from (2-28) and (2-30a), the measured value of V_a is also equal to the speed of the sound wave at the ray apex. If V_a is known, the apex temperature is also known, provided, of course that wind effects can be neglected.

The apparent velocity should, normally, be greater than or equal to the ambient velocity of sound at the surface, but there is increasing evidence that such is not always the case: phase velocities lower than normal sound velocity have been observed in the last few years by a number of investigators. Such velocities violate Snell's law and can thus not be explained in terms of simple ray theory. It is obvious that a plane wave, moving across a linear array of detectors, could not have, on this basis, an apparent velocity less than its inherent velocity.

This peculiar phenomenon will be discussed later in more detail.

2-6. The Determination of Ray Paths and Temperatures above 30 km.

Discounting satellites and sporadic rocket probes, present day aerological observations end, for all practical purposes, at about the 30 km. level. To be sure, a few sounding balloons penetrate a little higher, but most routine ascents terminate below 30 km. Subject to certain assumptions, it is still possible, however, to trace a ray to its apex, and calculate the corresponding temperature

distribution at heights beyond the reach of conventional sounding balloons. Several methods are now available to deduce the ray path entirely from ground based observations.

(a) The Method of Gutenberg and Wiechert

The oldest method historically, and the most elegant in principle, is based on the Herglotz-Wiechert-Bateman variational equation, originally developed for use in Seismology; this equation was first applied to aerological problems by Gutenberg and Wiechert in 1925. The application of this method presupposes that the wave front is normal to the ray, and that the speed of sound increases with height. The first requirement is strictly fulfilled only when the air is at rest, but light winds or weak wind gradients are admissible, and may have but little effect. The second condition requires, of course, an increase in temperature with height, and this occurs in general only in the upper stratosphere. It is necessary, therefore, to select a reference level at a height at which the temperature begins to increase, and apply the equation to the part of the ray above the reference level. Below this level the aerological conditions are usually known, and the ray can be traced directly by the methods given earlier. The appropriate portions of the horizontal distance, and the corresponding travel time can thus be computed below the reference level, and deducted from the horizontal range of the ray, and the total travel time. Next, a travel time curve for ray paths above the

reference level is plotted from the residual values of travel time and distance.

A derivation of the Herglotz-Wiechert-Bateman equation can be found in most books on seismology and will not be given here. The equation is usually stated as

$$H = \frac{1}{\pi} \int_0^{\bar{\Delta}} \cosh^{-1} \left(\frac{\bar{V}_a}{V_a} \right) d\Delta \quad (2-31)$$

where H is the height of the ray apex, and $\bar{\Delta}$ is the range of the ray above the reference level;
 $\bar{V}_a = \frac{\delta \bar{\Delta}}{\delta t}$, the apparent velocity at $\bar{\Delta}$
 $V_a = \frac{\delta \Delta}{\delta t}$, the apparent velocity anywhere between 0 and $\bar{\Delta}$.

Since V_a and $\bar{V}_a$ can be obtained from the slope of travel time curve, the integration can be carried out graphically. Obviously, the true height of the ray apex is obtained by adding H to the height of the reference level.

(b) Whipple's Method

This method is, in essence, a trial and error method, first used by Whipple in 1923 to estimate upper air temperatures in support of his theory of anomalous propagation. The method is based on measurement of the angles of incidence and the corresponding apparent velocities. Use is made again of all available aerological data to trace a given ray directly from the surface to as great a height as possible, the ray starting off from the ground in the direction

of the observed angle of incidence. The path of the ray above this height is then calculated for two extreme cases of assumed temperature distribution. In the first case the temperature is held constant to the apex of the ray, the last measured temperature being used for this purpose; this means, evidently, that the ray path is straight to the apex. At this point the temperature is postulated to increase abruptly so that the ray suffers specular reflection. In the second instance the assumption is made that the temperature rises uniformly to the summit. The actual path of the ray will usually lie somewhere between these limiting cases. Travel times may now be computed for various assumed temperature distributions, and compared with the observed travel time until a satisfactory match is obtained. The solution so obtained is not, of course, unique, but the choice of likely temperature distribution is surprisingly limited. A schematic outline of the method is given on Page 41.

(c) The Cut-and Try Method

This method is, undoubtedly, the least elegant of the three, but also the most efficient, provided that a high speed computer is available for the calculations. Basically, ray paths with angles of incidence between 50 and 90 degrees are computed up to the highest level for which aerological data are available. Above this level, the rays are computed for various assumed temperature distributions.

A "synthetic" travel time curve is plotted from the calculated values of time and horizontal distance, and compared directly with the observed travel-time curve. Since the curves must match in slope, length and intercept, the choice of likely temperature distributions is narrowed down considerably. Of course, the number of constraints which can be placed on the match are again insufficient to determine the solution uniquely.

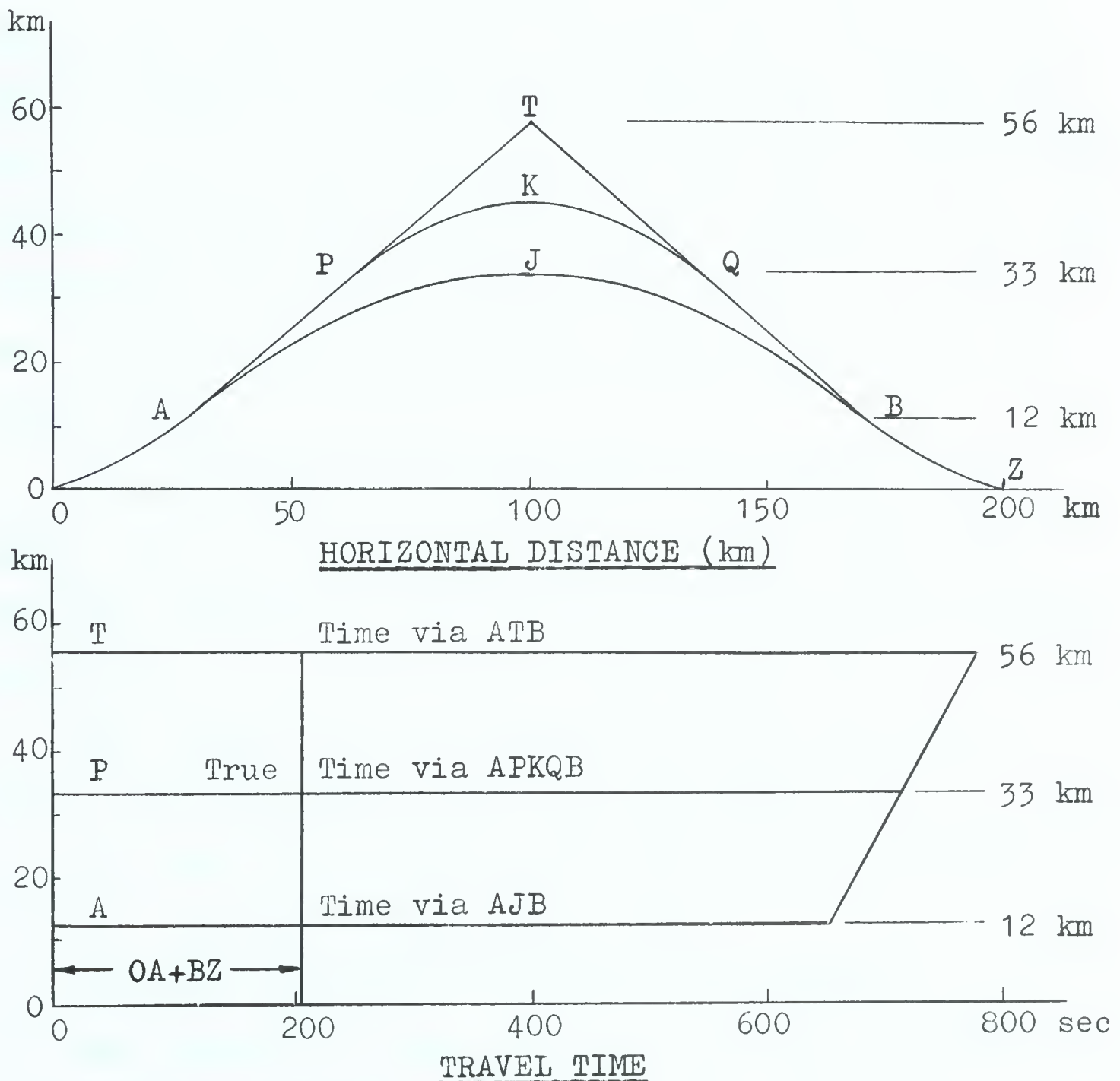


Fig. 2 : Whipple's Method of Estimating the Apex Height of Anomalous Sound Rays.

2.7 The Propagation of Sound in the Presence of Wind. Rayleigh's Equation.

One of the first theoretical treatments of the effect of wind on the propagation of sound was given by Rayleigh in 1896.

Rayleigh's equation

$$\frac{C}{\sin i} + w = \text{const.} \quad (2-32a)$$

holds when both the velocity of sound C and the horizontal wind velocity w are functions of height only. Rayleigh derived this equation initially for the special case of two contiguous horizontal streams of air of different speed and temperature, but quite general and rigorous derivations have been made in recent years by Blokhintzev (1946) and Groves (1955).

The derivation to be given here is largely due to Emden (1918) and Benndorf (1929). It is a simple, geometrical extension of Snell's Law, subject to the following postulate:

The velocity of an element of the wave front at a given point in space at time t is the sum of

- (1) the local velocity of sound in the direction of the wave front normal, and
- (2) the horizontal velocity of the medium.

The validity of this postulate has been confirmed by experiment on numerous occasions.

The Rayleigh equation will be derived with the help of the following simplifying assumptions:

- (1) The atmosphere is horizontally stratified, and air temperature is a function of height only.
- (2) The origin of the cartesian co-ordinate system is coincident with the source of sound. The x-axis is positive toward the east, the y-axis is positive to the north, and the z-axis positive upward.
- (3) The wind direction does not change with height: it is westerly at all levels, i.e. the wind blows from west to east, in the positive direction of the x-axis.
- (4) The wind speed u is a function of height only.
- (5) The direction of the ray will be given in terms of the angle of emergence e , rather than the angle of incidence; e is the angle between the direction of the wave front normal and the horizontal, and thus $e = 90 - i$, the complement of the angle of incidence. This choice of angle greatly simplifies explanatory diagrams, since the ray is referred directly to the horizontal layers of the medium.
- (6) The sound rays are confined to planes parallel to the x-z plane.

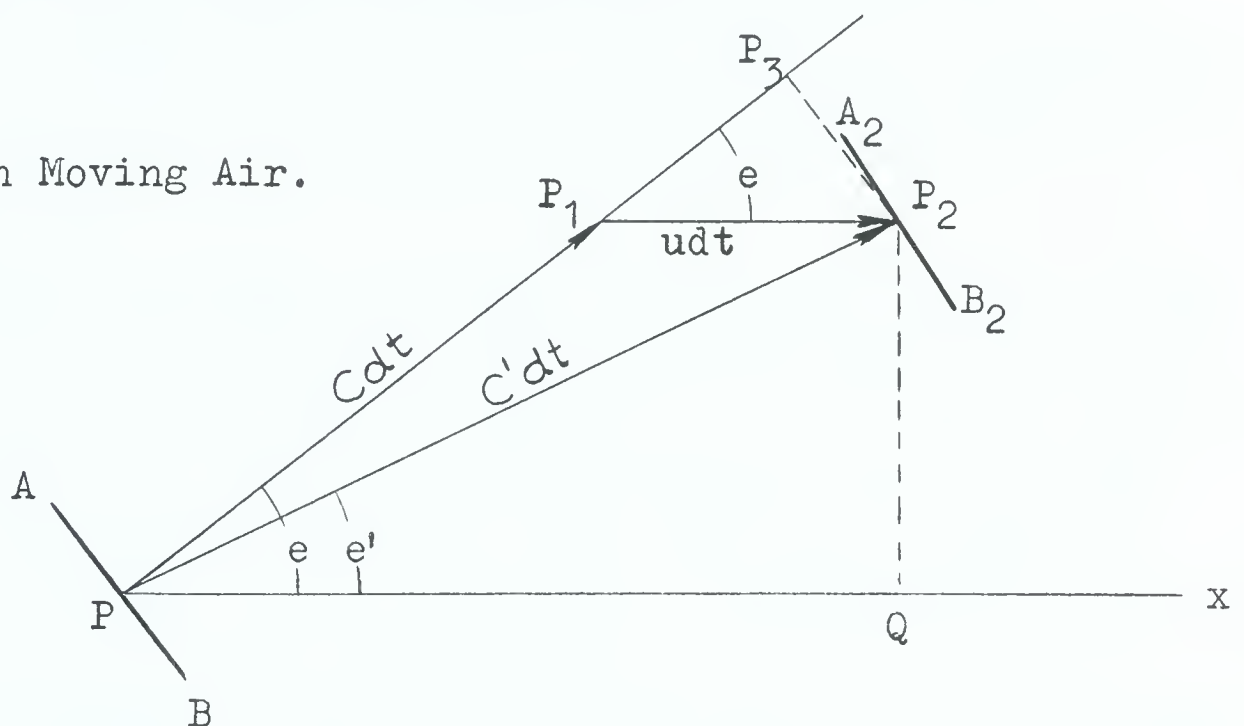
Writing (2-32a) in terms of this new notation, Rayleigh's equation becomes

$$\frac{c}{\cos e} + u = \text{const.} \quad (2-32b)$$

Consider first a small section AB of the wave front at time t , intersecting a ray at the point $P(x, z)$. (See Fig.3) The element AB will move in the direction of the wave-normal with the speed of sound C , and in the x -direction with the speed of the wind, u . Hence, at time $t + dt$, the midpoint of AB will reach $P_2(x + dx, z + dz)$. The ray is evidently propagated along PP_2 ; we denote this direction by e' , the angle between the ray and the horizontal.

Fig. 3

Sound Rays in Moving Air.



From the geometry of Fig. 3, we obtain immediately the following relations:

The speed of the ray, C' is seen to be

$$C' = (C^2 + u^2 + 2uC \cos e)^{\frac{1}{2}} ; \quad (2-33)$$

The speed C_n of the wave front normal is the speed at which the element AB is moving toward the point P_3 ; the corresponding displacement is $C_n dt$, represented by PP_3 . The speed of the wave normal is, therefore, given by

$$C_n = C + u \cos e \quad (2-34)$$

The angle of emergence e' of the ray is related to the angle e of the wave normal by the equation

$$\begin{aligned}\cot e' &= \frac{PQ}{QP_2} = \frac{C \cos e + u}{C \sin e} \\ &= \cot e + \frac{u}{C} \cdot \csc e\end{aligned}\quad (2-35)$$

It is evident that the angle between the ray and the wave normal ($e - e'$) depends largely on the ratio u/C . Since u/C is, in practice, usually less than 0.1, the difference $e - e'$ is hardly ever greater than 5 degrees, and frequently much less.

After these preliminaries, Rayleigh's equation is derived easily with the help of Fig. 4 (Page 46). The regions marked I, II, III and IV are four horizontal layers of air of equal thickness. The speeds of sound, wind, and wave normal in layer I are denoted by C_1 , u_1 , and $C_{n1} = C_1 + u_1 \cos e_1$, as shown in the diagram; similarly for the other layers. Furthermore, let $R, R_1, R_2 \dots$ and $R', R'_1, R'_2 \dots$ represent two adjacent rays of sound. Let $aR_1, a'R'_1$ be the corresponding wave normals in layer I; in layer II the normals are denoted similarly by R_1b and cR_2 , and R'_1b and $c'R'_2$, respectively; and so on.

Consider now the section of the wave front passing through the points R_1, P_1 : as the front advances from P_1 to R'_1 in layer I, it will, in the same time dt , move from R_1 to Q_2 in layer II.

Thus

$$\frac{C_{n1}dt}{C_{n2}dt} = \frac{P_1R'_1}{R_1Q_2}$$

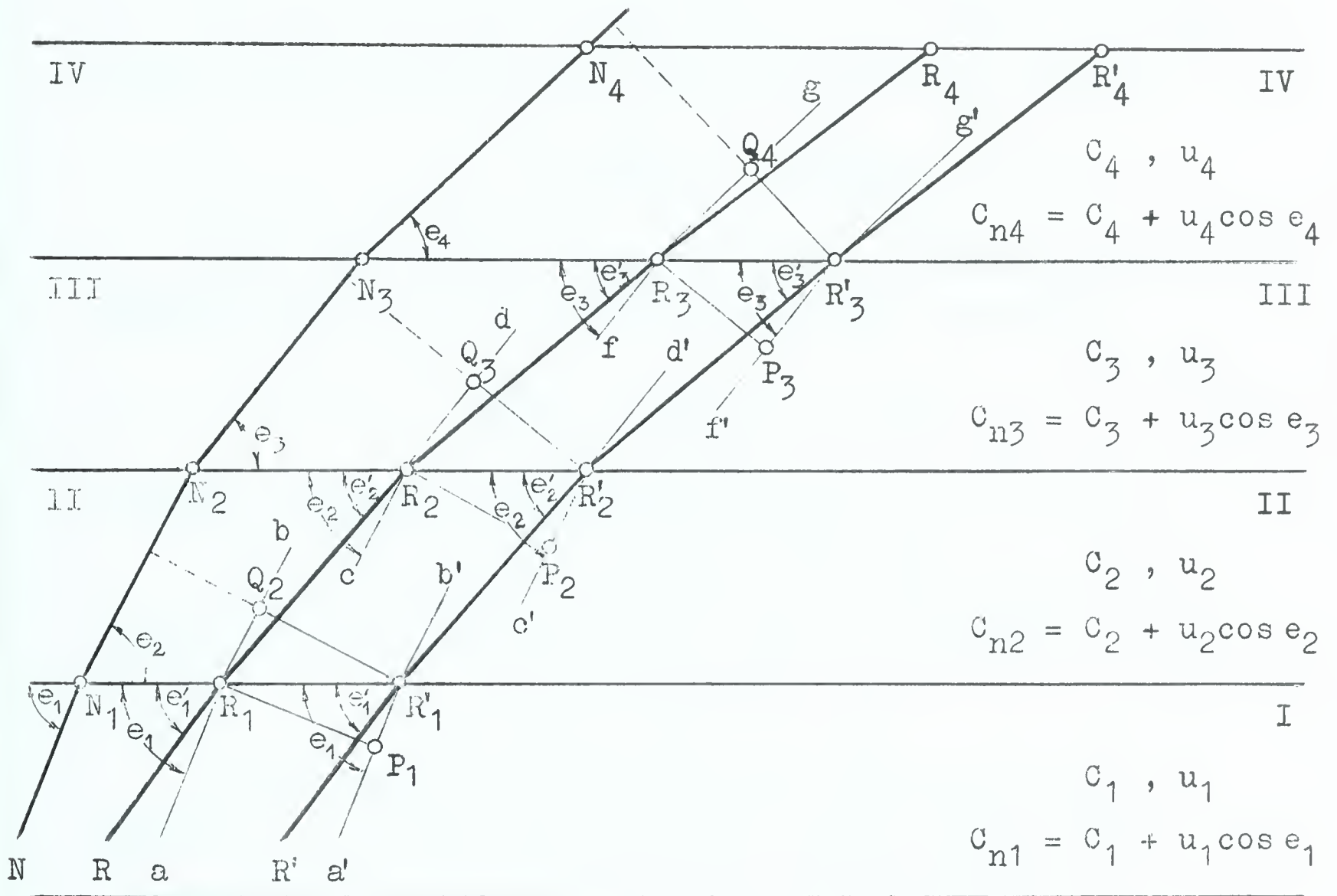


Fig. 4

Sound rays in a horizontally stratified, moving medium :
 Derivation of Rayleigh's Equation. (After Benndorf.)

But, since $P_1 R'_1 = R_1 R'_1 \cos e_1$, and $R_1 Q_2 = R_1 R'_1 \cos e_2$, we obtain:

$$\frac{C_{n1}}{\cos e_1} = \frac{C_{n2}}{\cos e_2} \quad (2-36)$$

It is quite obvious from Fig. 4 that similar relations hold for the interfaces of the other layers as well:

$$\frac{C_{n2}}{\cos e_2} = \frac{C_{n3}}{\cos e_3}, \text{ etc.}$$

The ratio $C_n / \cos e$ is, therefore, constant along the ray. Making use of (2-34), we obtain Rayleigh's equation:

$$\frac{C_n}{\cos e} = \frac{C}{\cos e} + u = \frac{1}{P}, \quad (2-37)$$

where P is again the ray parameter.

This generalization of Snell's Law could have been obtained directly from (2-8) by formal substitution of C_n for C .

The extension to a continuously varying medium is obvious. It should be emphasized, however, that this equation applies to the wave normal intersecting the ray, and not to the ray itself. The appropriate relation for the ray could be obtained, of course, by substituting for $\cos e$ in terms of e' , from (2-35). But it is hardly advisable to do so explicitly, for the resulting expression is quite complicated, and awkward to use. It is usually more convenient to work with Rayleigh's formula for the wave normals, and calculate only the ray angles from (2-35) as required.

Shown also in Fig. 4, is the curve $N, N_1, N_2 \dots$, constructed of segments drawn parallel to the wave normals $aR_1, R_1b \dots$ etc. This curve will be called the train of wave normals associated with the ray $R, R_1, R_2, \dots$, or, simply, the normal train. Rayleigh's formula applies, evidently, without modification to the normal train. This curve may be thought of as the orthogonal trajectory of the planes tangent to the wave fronts, at the points pierced by a given ray: in the present case $R, R_1, R_2 \dots$. The normal train is of little physical significance, but it simplifies the analysis, and is easier to work with.

2.8 The Curvature of Sound Rays parallel to the Wind, and the Condition for Total Reflection.

Analogous to (2-25), the curvature of the normal train is given by

$$K = \frac{1}{R} = -\frac{\cos e}{C_n} \frac{dC_n}{dz} = -P \frac{dC_n}{dz} \quad (2-38)$$

The difference in sign is due to the use of the angle e in place of the angle of incidence. The curvature of the normal train will thus be negative, or concave down, when C_n increases with height, and positive when C_n decreases; the same is true of the curvature of the associated ray.

The tangent to the normal train will be horizontal when $e = 0, \cos e = 1$; in this case

$$C_n = C + u = \frac{1}{P} = \frac{C_0}{\cos e_0} + u_0, \quad (2-39)$$

where the subscript $_0$ refers to quantities at the origin or the sound source. But when $e = 0$, the ray angle e' is also

zero by (2-35), and the ray will reach precisely the same height as the associated normal train.

A ray may suffer total reflection if C_n increases suddenly beyond a certain critical value. The condition for total reflection is obtained from (2-37). Let C_1, u_1 be the speeds below the discontinuity, and C_2, u_2 the corresponding speeds above. It is evidently necessary that

$$C_2 + u_2 = \frac{C_1}{\cos e_1} + u_1 ,$$

or

$$u_2 - u_1 = \Delta u = \frac{C_1}{\cos e_1} - C_2 \quad (2-40)$$

where Δu denotes the jump in the wind speed across the discontinuity. If we assume that the speed of sound does not vary across the discontinuity, so that $C_1 = C_2 = 330$ m/sec, say, we may calculate the critical values of Δu . A few typical values are given below:

e	5°	10°	15°	20°	
Δu	1.25	4.95	11.6	21.8	m/sec.

It would seem that total reflection due to a jump in the wind field is not likely to occur for angles of emergence much greater than 5°.

2.9 The Differential Equation of Sound Rays Parallel to the Wind.

The differential equation of the normal train follows directly from Rayleigh's equation (2-37):

Since $\frac{dx}{ds} = \cos e = \frac{PC}{1 - Pu}$, (2-40a)

and $\frac{dz}{ds} = \sin e = \left[1 - \left(\frac{PC}{1 - Pu} \right)^2 \right]^{\frac{1}{2}}$ (2-40b)

the equation of the normal train is given by

$$\frac{dz}{dx} = \tan e = \left[\left(\frac{1 - Pu}{PC} \right)^2 - 1 \right]^{\frac{1}{2}} \quad (2-41)$$

The differential equation of the sound ray is now easily obtained from (2-35) by substituting for $\sin e$ and $\cos e$:

$$\frac{dx}{dz} = \cot e' = \frac{1 + \frac{u}{C} \cdot \frac{1 - Pu}{PC}}{\left[\left(\frac{1 - Pu}{PC} \right)^2 - 1 \right]^{\frac{1}{2}}} ,$$

Or, after some simplification, and with $\frac{dx}{dz} = \text{ctg } e'$

$$\frac{dx}{dz} = \frac{u + P(C^2 - u^2)}{C \left[(1 - Pu)^2 - (PC)^2 \right]^{\frac{1}{2}}} \quad (2-42)$$

This equation will be integrated later for linear variations of wind and speed of sound.

If $C = C(z)$, and $u = u(z)$ are known, the height z_m of the ray apex may be deduced directly from (2-39), without recourse to the differential equation.

Since $C(z_m) + u(z_m) = \frac{1}{P}$, (2-43)

the height of the apex may be obtained by solving the equation for z_m .

2.10. The Propagation of Sound in Wind Fields Turning with Height.

In order to treat the more complicated case of sound propagation in wind fields turning with height, we assume again that the temperature is a function of height only, and that the vertical wind component is zero. We admit now, however, two horizontal wind components u and v , both functions of z only, and parallel to the x - and y -axis, respectively, the orientation of the co-ordinate axes being the same as in section 2.7.

We shall derive the necessary equations with reference to Fig. 5 below (Page 52). Consider first the upper half of the diagram, the V_φ plane: Let AB be again an element of the wave front (seen on edge) intersecting a ray at the point $R_1(x, y, z)$, and let R_1N be the corresponding wave normal. (Note that R_1N is entirely in the plane of the paper). We now pass a vertical plane V_φ through R_1 and the wave normal, and denote the angle between V_φ and the positive x -axis by φ . The angle φ will be called the bearing of the plane V_φ . This angle is shown in the lower half of Fig. 5, which represents the horizontal xy -plane: MM is the line of intersection of V_φ with the xy -plane.

Furthermore, let $\vec{w} = \vec{i}u + \vec{j}v$, be the resultant wind velocity at R_1 , and let w_p and w_n be the components of w , resolved parallel and normal to the plane V_φ :

These components are evidently given by

$$w_p = u \cos \varphi + v \sin \varphi \quad (2-44a)$$

$$w_n = u \sin \varphi - v \cos \varphi \quad (2-44b)$$

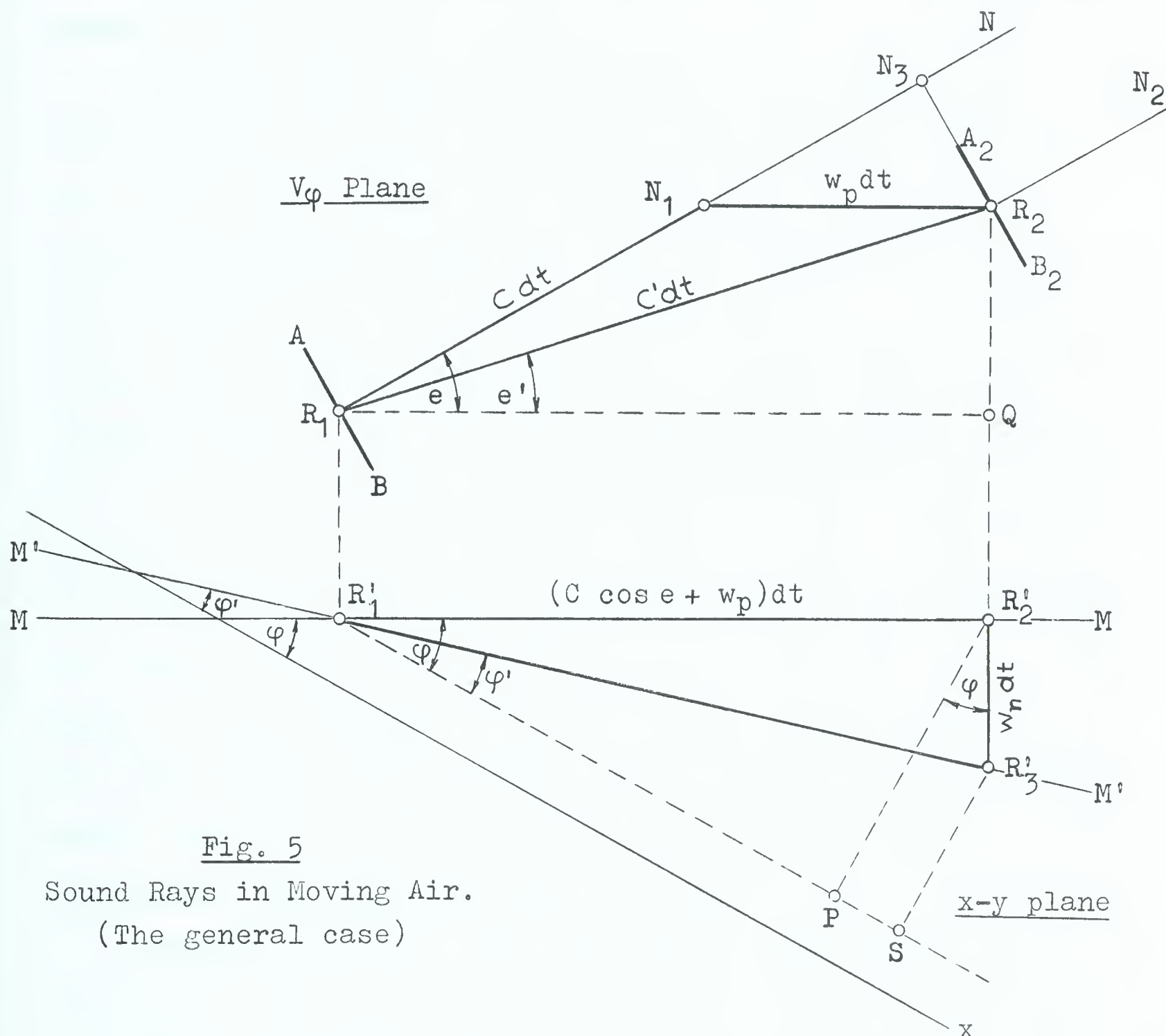


Fig. 5

Sound Rays in Moving Air.
(The general case)

When $w_n \neq 0$, the element ds' of the ray through R_1 will not be confined to the V_φ plane, but emerge in the direction of the normal component w_n . Evidently, the sound ray will be no longer a plane curve, and the bearing of the element ds' is thus not equal to φ ; to fix its

direction we pass a vertical plane through R_1 and ds' . We denote this plane by V_φ' , and the angle between the plane and x-axis by φ' . $M'M'$, the projection of V_φ' on the xy-plane, is shown in the lower half of Fig. 5.

Consider now again AB, an element of the wave front. In the absence of wind this element would move in the direction of the wave normal R_1N , with the speed C , reaching N_1 after an interval of time dt . However, the presence of a wind component w_p in the plane V_φ will displace the element to R_2 , and a normal component w_n will further translate the element to R_3 . Note that the points R_2, R_3 happen to coincide in the upper figure, but their projections R_2', R_3' on the xy-plane are shown in the proper relation in the lower half. In essence, the component w_n displaces the element AB parallel to itself: normal to the plane of the figure, and out of the paper. It is evident, therefore, that the orientation of the wave normal through AB will not be changed by the "cross-wind" component w_n , but note that the ray itself will be deflected, and turned into the plane V_φ' .

Consider once more the lower half of the figure: R_1' and R_2' are the projections of R_1 and R_2 ; the segment $R_1'R_3'$ is the projection of the ray element ds' , and $M'M'$ is the trace of the plane V_φ' in the xy-plane. The distance $R_1'R_2'$ is evidently equal to $(C \cos e + w_p)dt$, and

$$\begin{aligned} \tan \varphi' &= \frac{SR_3'}{R_1'S} = \frac{(C \cos e + w_p) \sin \varphi - w_n \cos \varphi}{(C \cos e + w_p) \cos \varphi + w_n \sin \varphi} \\ &= \frac{C \cos e \sin \varphi + v}{C \cos e \cos \varphi + u} \end{aligned} \quad (2-45)$$

using the relations (2-44). The bearing angle φ' of the ray elements ds' may thus be computed if e and φ are known.

Table I shows a number of typical values of φ' for a plane wave propagating toward the east under various wind conditions: the wave front is assumed to be vertical, so that $e = 0$; the wave normal is thus horizontal and bears due east, with bearing $\varphi = 0$. See Fig. 6 for a plot of φ' versus wind direction.

Wind*	Ratio w/C						
Degrees	0.01	0.03	0.05	0.10	0.15	0.20	0.30
15	-0.55	-1.67	-2.80	-5.66	-8.57	-11.52	-17.44
30	-0.50	-1.51	-2.54	-5.21	-7.99	-10.89	-17.00
45	-0.41	-1.24	-2.10	-4.35	-6.77	-9.35	-15.07
60	-0.29	-0.88	-1.50	-3.13	-4.93	-6.90	-11.46
75	-0.15	-0.46	-0.78	-1.64	-2.60	-3.67	-6.24
90	0 00	0.00	0.00	0.00	0.00	0.00	0.00
105	0.15	0.46	0.78	1.64	2.60	3.67	6.24
120	0.29	0.88	1.50	3.13	4.93	6.90	11.46
135	0.41	1.24	2.10	4.35	6.77	9.35	15.07
150	0.50	1.51	2.54	5.21	7.99	10.89	17.00
165	0.55	1.67	2.80	5.66	8.57	11.52	17.44
180	0.57	1.72	2.86	5.71	8.53	11.31	16.70
195	0.55	1.65	2.73	5.38	7.94	10.41	15.05
210	0.49	1.47	2.42	4.71	6.89	8.95	12.73
225	0.40	1.19	1.96	3.78	5.48	7.06	9.93
240	0.28	0.84	1.37	2.63	3.80	4.87	6.79
255	0.15	0.43	0.71	1.35	1.94	2.48	3.44
270	0.00	0.00	0.00	0.00	0.00	0.00	0.00
285	-0.15	-0.43	-0.71	-1.35	-1.94	-2.48	-3.45
300	-0.28	-0.84	-1.37	-2.63	-3.80	-4.87	-6.79
315	-0.40	-1.19	-1.96	-3.78	-5.48	-7.06	-9.93
330	-0.49	-1.47	-2.42	-4.72	-6.89	-8.95	-12.73
345	-0.55	-1.65	-2.73	-5.38	-7.94	-10.41	-15.05
360	-0.57	-1.72	-2.86	-5.71	-8.53	-11.31	-16.70

*Direction of wind measured clockwise from North.

TABLE I.

Bearing Angle φ' of Ray for Various Wind Conditions.

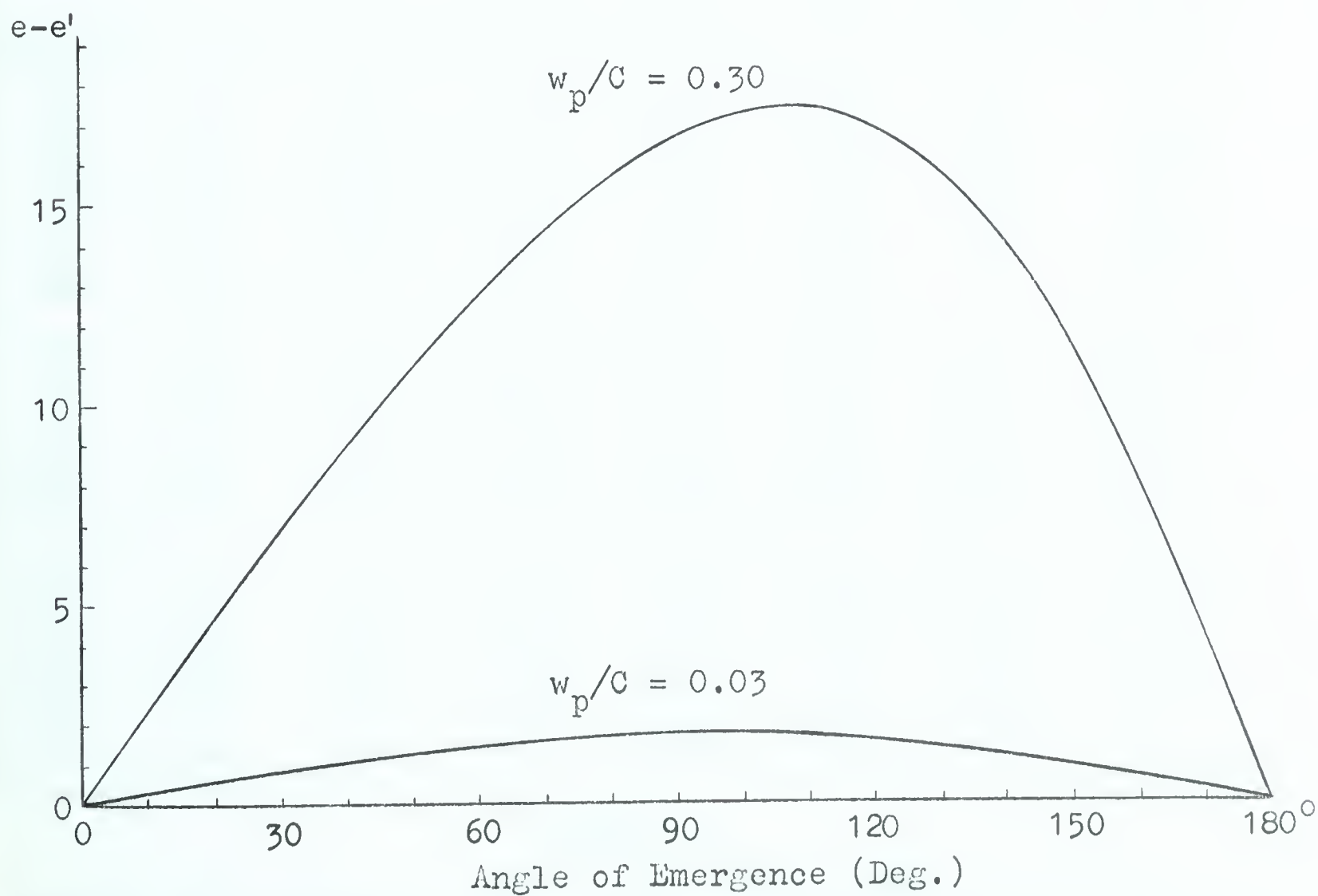
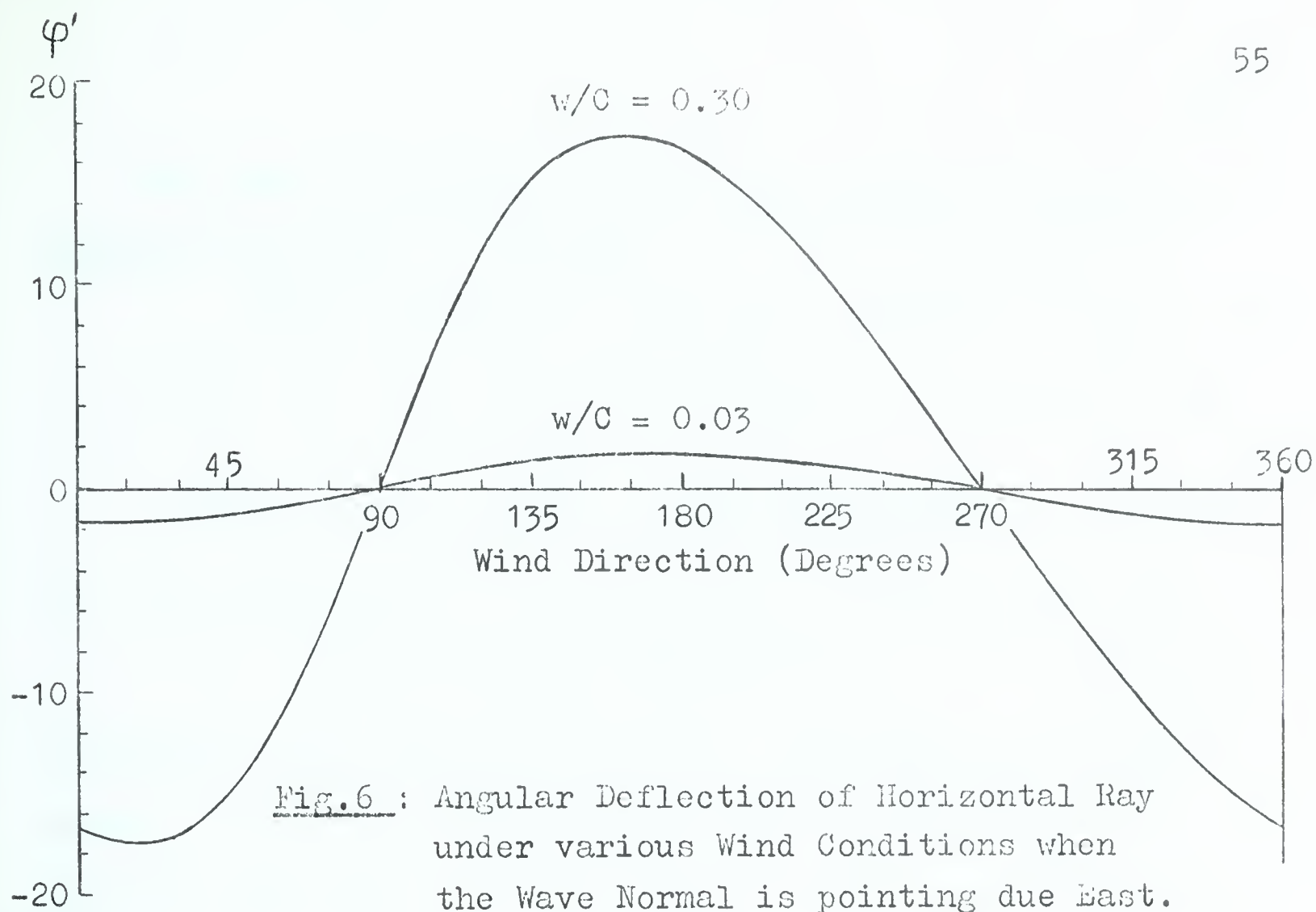


Fig.7 : Angular Separation of Ray and Wave Normal for $\varphi - \varphi' = 0$.

The angle of emergence e' of the element ds' is also obtained from Fig. 5: we have

$$R_1' R_3' \cos(\varphi - \varphi') = (C \cos e + w_p) dt$$

$$\text{and} \quad R_2 Q = C \sin e dt$$

$$\text{whence } \cot e' = \frac{R_1' R_3'}{R_2 Q} = \frac{C \cos e + w_p}{\cos(\varphi - \varphi') C \sin e} \quad (2-46)$$

Since the angle $\varphi - \varphi'$ is usually small, $\cos(\varphi - \varphi') \approx 1$, and

$$\cot e' \approx \cot e + \frac{w_p}{C} \csc e, \quad (2-47)$$

an equation very similar to (2-35). The angle e' is thus largely a function of the parallel wind component w_p , as might be expected.

Angle e	Ratio w_p/C						
	0.01	0.03	0.05	0.10	0.15	0.20	0.30
00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	0.10	0.29	0.47	0.91	1.30	1.66	2.30
20	0.20	0.57	0.94	1.79	2.58	3.30	4.58
30	0.29	0.84	1.37	2.64	3.80	4.87	6.79
40	0.37	1.08	1.77	3.42	4.94	6.36	8.91
50	0.44	1.29	2.13	4.12	5.99	7.73	10.91
60	0.50	1.47	2.42	4.72	6.89	8.95	12.73
70	0.54	1.60	2.65	5.20	7.64	9.98	14.34
80	0.57	1.69	2.80	5.53	8.20	10.78	15.69
90	0.58	1.72	2.87	5.71	8.53	11.31	16.70
100	0.56	1.70	2.84	5.72	8.62	11.54	17.32
110	0.54	1.63	2.73	5.55	8.45	11.40	17.44
120	0.50	1.51	2.54	5.21	7.99	10.89	16.99
130	0.44	1.34	2.26	4.68	7.24	9.97	15.89
140	0.37	1.13	1.91	3.98	6.21	8.63	14.05
150	0.29	0.88	1.50	3.13	4.92	6.89	11.45
160	0.20	0.60	1.03	2.16	3.42	4.81	8.13
170	0.10	0.31	0.52	0.10	1.75	2.48	4.23
180	0.00	0.00	0.00	0.00	0.00	0.00	0.00

TABLE II.

Angular separation $e - e'$ of ray and wave normal for various angles of emergence e , and $\varphi - \varphi' = 0$.

Table II shows the angular separation $e-e'$ of ray and wave normal for various values of e , w_p/C , and $\varphi-\varphi' = 0$. The relation is also depicted graphically for two values of w_p/C , in Fig. 7. Tables for other values of $\varphi-\varphi'$ are given in Appendix I.

It is important to note from the foregoing that the wave normals associated with the individual points of a certain ray are all parallel to the V_φ plane. We are able, therefore, to construct the associated normal train of the ray, either in the plane V_φ itself, or, better still, in a new plane V_{φ_0} , parallel to V_φ , and passing through the origin. The normal train has the obvious advantage of being a plane curve; besides, Rayleigh's equation (2-37) still holds with but slight changes in notation. Replacing u by w_p , we obtain:

$$\frac{C_n}{\cos e} = \frac{C}{\cos e} + w_p = \frac{C_o}{\cos e_o} + w_{po} = \frac{1}{P} \quad (2-48)$$

where the subscript o refers to initial values at the origin, and $w_{po} = u_o \cos \varphi_o + v_o \sin \varphi_o$. The constant P is again the ray parameter, and the parameter of the associated normal train.

2.11. The Differential Equation of Sound Rays when the Wind Direction changes with Height.

The equations of the ray path proper will now be derived with the aid of the simpler normal train. We begin with a normal train emerging from the origin with angle e_0 and bearing φ_0 ; its parameter is given by (2-48) or, more explicitly, by

$$\frac{1}{P} = \frac{C_0}{\cos e_0} + u_0 \cos \varphi_0 + v_0 \sin \varphi_0 \quad (2-49)$$

The angle of emergence of the normal train at the height z follows also directly from (2-48), or

$$\cos e = \frac{PC}{1 - Pw_p}, \quad (2-50)$$

whence
$$\cot e = \left[\left(\frac{1 - Pw_p}{PC} \right)^2 - 1 \right]^{-\frac{1}{2}} \quad (2-51)$$

From Fig. 5, we readily deduce the following relations pertinent to the normal train:

$$dz = C \sin e \, dt \quad (2-52a)$$

$$dy = C \cos e \sin \varphi_0 \, dt \quad (2-52b)$$

$$dx = C \cos e \cos \varphi_0 \, dt \quad (2-52c)$$

We thus obtain the following set of equations for the normal train:

$$\frac{dx}{dz} = \cot e \cos \varphi_0 = \cos \varphi_0 \left[\left(\frac{1 - Pw_p}{PC} \right)^2 - 1 \right]^{\frac{1}{2}} \quad (2-53a)$$

$$\frac{dy}{dz} = \cot e \sin \varphi_0 = \sin \varphi_0 \left[\left(\frac{1 - Pw_p}{PC} \right)^2 - 1 \right]^{\frac{1}{2}} \quad (2-53b)$$

$$\frac{dx}{dy} = \cot \varphi_0 = \text{constant} \quad (2-53c)$$

The geometry of the normal train may evidently be determined by integration of (2-53) if u , v , C are given functions of z .

Referring again to Fig. 5 we deduce another set of relations pertaining to the ray path itself:

$$dz = C' \sin e' dt \quad (2-54a)$$

$$dy = C' \cos e' \sin \varphi' dt \quad (2-54b)$$

$$dx = C' \cos e' \cos \varphi' dt \quad (2-54c)$$

whence
$$\frac{dx}{dz} = \cot e' \cos \varphi' \quad (2-55a)$$

$$\frac{dy}{dz} = \cot e' \sin \varphi' \quad (2-55b)$$

$$\frac{dx}{dy} = \cot \varphi' \quad (2-55c)$$

These are the differential equations of the ray in their simplest, parametric form. The problem is solved in principle: explicit expressions in terms of u , v , C may be obtained by substituting for the trigonometric functions, but little is gained by this procedure in practice. Such expressions cannot be integrated, as a rule, by standard analytical means; they are so complicated, even for the simplest distributions of u , v , C , that recourse must be had to numerical methods of integration. However, the simpler equations for the normal train may be integrated directly for linear changes in wind and sound velocity, in a manner similar to that given below in Section 2.12.

Wind components parallel to the direction of ray propagation will be considered only in the present investigation.

Since horizontal wind gradients are assumed to be zero, the effect of the cross-wind component is usually small, for ray curvature is largely dependent on wind shear, and not on wind speed per se.

2.12. Ray Equations for Propagation of Sound Parallel to the Wind.

The calculations of a complete ray path in an atmosphere consisting of 30 or more horizontal moving layers was, until quite recently, at best a tedious, time-consuming procedure. It is not surprising, therefore, that many approximate methods have been devised in former years, usually in connection with problems of sound ranging, aircraft location, and fixing of enemy gun positions. Notable in this regard is the method of Rothwell, published in 1947, but developed earlier in connection with two series of experiments carried out in 1930 and 1931, on the south coast of England, for the purpose of aircraft location by acoustical means.

Cox, Plagge and Reed (1954) have devised a simple method for estimating the range and intensity of blast waves from surface bursts; their work was recently extended by Gilbert (1961) in the course of investigations carried out at the Suffield Experimental Station.

A widely used approximation is due to Emden (1918) who integrated the expression

$$\cot e' = \frac{dx}{dz} = \frac{C \cos e + u}{C \sin e}, \quad (2-35)$$

derived earlier, in our notation, as equation (2-35). Emden integrated this equation for linear changes of sound and wind speed.

In the outline of Emden's derivation quoted by Mitra (1947) and reproduced in part below, it is specifically assumed that the speed of sound decreases with height while the wind speed increases, a typical situation usually encountered in the troposphere.

Let the speed of sound and wind speed at height z be given by

$$C = C_0(1 - az)$$

and $w = w_0(1 + bz)$,

where a and b are positive constants and the subscript o refers to values at the zero reference level.

Beginning with Rayleigh's equation in the form

$$\cos e = \frac{C}{C_0 / \cos e_0 + w_0 - w}$$

we obtain approximately, for small values of z

$$\cos e \approx \frac{\cos e_0}{1 + Bz} \quad (2-56)$$

where $B = a - \frac{w_0 b}{C_0} \cos e_0$, neglecting terms containing squares and higher powers of z . Substitution of (2-56) into (2-35) leads to

$$\frac{dx}{dz} = \frac{\cos e_0 + (1 + Bz)w/C}{[(1 + Bz)^2 - \cos^2 e_0]^{\frac{1}{2}}} \quad (2-57)$$

Neglecting again higher powers of z , this expression yields, on integration:

$$\begin{aligned}
x = & \frac{\cos e_o}{B} (\sqrt{2Bz + \sin^2 e_o} - \sin e_o) \\
& + \frac{w_o}{C_o} \left\{ 1 - (a + b) (\sin^2 e_o - Bz \cos^2 e_o) / (3B \cos^2 e_o) \right\} . \\
& \left\{ (\sqrt{2Bz \cos^2 e_o + \sin^2 e_o} - \sin e_o) / (B \cos^2 e_o) \right\} \\
& + \frac{w_o}{C_o} (a + b) z \sin e_o / (3B \cos^2 e_o) \quad (2-58)
\end{aligned}$$

This is the expression given by Mitra* for the horizontal distance covered by a ray in a wind field increasing linearly with height. This equation gives good results for values of z up to about 1 kilometer, but its accuracy diminishes rapidly for layers over 1.5 km thick. To be sure, thick layers may be subdivided, as required, but then the work of computation increases in direct proportion to the number of layers used. Obviously, the labour involved in calculating ray paths to heights of 60 to 100 km is considerable, or at least used to be so before the advent of the high speed computer. Even now it is most desirable that the calculations be carried out for as thick a layer as possible, having due regard only for the available aerological data: the ensuing savings in computer time are very substantial, especially so at great heights where lack of detailed information would hardly warrant detailed calculation.

*The expression in Mitra's book is incomplete: the factor w/C_o at the beginning of the fourth line seems to have been lost in print. Mitra denotes the speed of sound by v_o .

2.13. The Complete Integral of the Ray Equation for Propagation Parallel to the Wind.

In addition to the disadvantage noted above, Mitra's form of Emden's equation is prone to fail for certain distributions of wind and temperature, and the case $w_0 = 0$ is obviously ruled out altogether. It is desirable, therefore, to redefine the wind equation, and solve the differential equation (2-35) without recourse to approximations.

Let the speed of sound and the wind speed at height z be given by

$$C = C_0(1 - \alpha z) \quad (2-59a)$$

$$w = w_0 + \beta z$$

where α and β are again appropriate constants.

Beginning with Rayleigh's equation in the form

$$\cos e = \frac{C}{\frac{C_0}{\cos e_0} + w_0 - w}$$

we obtain, after substitution of (2-59) and $h = \cos e_0$,

$\lambda = \beta \cos e_0 / C_0$ for convenience, the exact expression

$$\cos e = \frac{h(1 - \alpha z)}{1 - \lambda z} \quad (2-60)$$

Substituting for $\cos e$ and $\sin e$ in the differential ray equation

$$\frac{dx}{dz} = \frac{\cos e + w/C}{\sin e} \quad (2-35)$$

and introducing the expressions for w and C , we find, after simplification:

$$\frac{dx}{dz} = \frac{h(1 - \alpha z)^2 C_0 + (w_0 + \beta z)(1 - \lambda z)}{C_0(1 - \alpha z) \sqrt{(1 - \lambda z)^2 - h^2(1 - \alpha z)^2}} \quad (2-61)$$

Writing R for the root in the denominator, the root may be expressed more conveniently by

$$R = (a + bz + cz^2)^{\frac{1}{2}} \quad (2-62)$$

$$\text{where } a = \sin^2 e_0 \quad b = 2(\alpha h^2 - \lambda) \quad c = \lambda^2 - \alpha^2 h^2$$

The numerator N of (2-61) is of the form

$$N = A + Bz + Gz^2 \quad (2-63)$$

$$\text{with } A = hC_0 + w_0$$

$$B = \beta - 2\alpha hC_0 - w_0\lambda$$

$$G = C_0\alpha^2 - \beta\lambda$$

With these substitutions, the ray equation takes the simpler form

$$\frac{dx}{dz} = \frac{A + Bz + Gz^2}{C_0(1 - \alpha z)R} \quad (2-64)$$

This expression may be further reduced to

$$\frac{dx}{dz} = \frac{Dz}{R} + \frac{E}{R} + \frac{F}{(1 - \alpha z)R}$$

where D , E , F are constants defined by

$$D = -\frac{G}{\alpha C_0},$$

$$E = -\frac{1}{\alpha C_0} \left(\frac{G}{\alpha} + B \right)$$

$$F = \frac{1}{C_0} \left[A + \frac{1}{\alpha} \left(\frac{G}{\alpha} + B \right) \right]$$

The horizontal distance x covered by the ray while ascending from $z = 0$ to $z = z$ is, therefore,

$$x = D \int_0^z \frac{z dz}{R} + E \int_0^z \frac{dz}{R} + F \int_0^z \frac{dz}{(1 - \alpha z)R} \quad (2-65)$$

or, more compactly,

$$x = EI_0 + DI_1 + FI_2 \quad (2-65')$$

where we have put

$$I_0 = \int_0^z \frac{dz}{R}, \quad I_1 = \int_0^z \frac{z dz}{R}, \quad I_2 = \int_0^z \frac{dz}{(1 - \alpha z)R}$$

These integrals are known and may be evaluated directly.

$$I_0 = \frac{1}{\sqrt{c}} \ln \frac{(R + \sqrt{c} z + b/(2\sqrt{c}))}{(\sin e_0 + b/(2\sqrt{c}))} \quad \text{if } c > 0 \quad (2-66a)$$

$$I_0 = \frac{1}{\sqrt{-c}} \left[\arcsin(b/\sqrt{-q}) - \arcsin((2cz + b)/\sqrt{-q}) \right] \quad \text{if } c < 0 \quad (2-66b)$$

where $q = 4ac - b^2$ when c is negative.

$$I_1 = \frac{1}{c}(R - \sin e_0) - \frac{b}{2c} \cdot I_0 \quad (2-67)$$

In order to write I_2 as compactly as possible we put:

$$k = a\alpha^2 + b\alpha + c,$$

$$\text{and } \epsilon = -(\alpha b + 2c)$$

With these substitutions we obtain 3 expressions for I_2 , according as $k > 0$, $k = 0$, and $k < 0$:

$$I_2 = \frac{1}{\sqrt{k}} \ln \left[\frac{2k + \epsilon(1 - \alpha z) + 2\alpha\sqrt{k} R}{(1 - \alpha z)(2k + \epsilon + 2\alpha\sqrt{k} \sin e_0)} \right] \quad (2-68a)$$

if $k > 0$

$$I_2 = \frac{1}{\sqrt{-k}} \left[\arctan\left(\frac{2k + \epsilon(1 - \alpha z)}{-2\alpha\sqrt{-k} R}\right) - \arctan\left(\frac{2k + \epsilon}{-2\alpha\sqrt{-k} \sin e_0}\right) \right] \quad \text{if } k < 0 \quad (2-68b)$$

$$I_2 = \frac{2\alpha}{\epsilon} \left[\frac{R}{1 - \alpha z} - \sin e_0 \right] \quad \text{if } k = 0 \quad (2-68c)$$

These integrals may all take several forms for certain values of some constants, e.g. $\alpha = 0$; such special cases are summarized in Appendix II.

The travel time is found directly from equation (2-13) with $\cos i = \sin e$:

$$t = \int_0^z \frac{dz}{C \sin e} \quad (2-13)$$

After substitution for C and $\sin e$ the time is

$$t = \int_0^z \frac{(1 - \lambda z) dz}{C_0 (1 - \alpha z) R} \quad (2-69)$$

where the symbols have the same meaning as above; we distinguish 3 main cases:

$$(1) \quad \alpha \neq 0$$

$$\begin{aligned} t &= \frac{\lambda}{\alpha C_0} \int_0^z \frac{dz}{R} + \frac{1}{C_0} \left(1 - \frac{\lambda}{\alpha} \right) \int_0^z \frac{dz}{(1 - \alpha z) R} \\ &= \frac{\lambda}{\alpha C_0} I_0 + \frac{1}{C_0} \left(1 - \frac{\lambda}{\alpha} \right) I_2 \end{aligned} \quad (2-70a)$$

$$(2) \quad \alpha = 0$$

$$t = \frac{1}{C_0} \int_0^z \frac{dz}{R} - \frac{\lambda}{C_0} \int_0^z \frac{z dz}{R} = \frac{1}{C_0} (I_0 - \lambda I_1) \quad (2-70b)$$

$$(3) \quad \lambda = 0$$

$$t = \int_0^z \frac{dz}{C_0 (1 - \alpha z) R} = \frac{I_2}{C_0} \quad (2-70c)$$

2.14 Zones of Anomalous Audibility and the Acoustic Shadow Zone.

If the acoustic records from observing stations surrounding a large explosion are examined for sound arrivals, and the observations plotted on a map as "sound heard", or "sound not heard", the following pattern emerges if the reports are numerous enough: immediately surrounding the site of the explosion is the so-called normal or inner zone of audibility, a zone characterized by high intensity blast waves and shock fronts expanding at supersonic speeds. These blast waves degenerate within seconds to "ordinary" sound waves of normal speed and infinitesimal amplitude. The average radius of the inner zone is about 50 kilometers, but its size and shape depend greatly on meteorological conditions and, to a lesser extent, on the magnitude of the explosion.

Bordering on the inner zone and extending outward for one hundred or more kilometers is the first zone of silence, or acoustic shadow zone. The zone of silence is precisely what the name implies: little or no sound at all is received within this region, and what reception there may be is, at best, only weak and sporadic. The shape of this zone depends again largely on meteorological factors, and its size varies greatly from season to season; on rare occasions the zone may be missing altogether, and direct sound may be received at great distances, e.g. Cox (1947) recorded a direct wave 182 km from a 125 ton blast of TNT. To reach such distances it is

necessary that the sound be reflected and refracted repeatedly between the earth and temperature inversion layers in the lower troposphere. Discontinuities in the low level wind field are another likely cause of normal propagation to great distances.

Adjoining the zone of silence and enclosing it at times completely is the first zone of anomalous audibility. Its inner boundary is, as a rule, clearly marked by a kind of caustic but the focal line of greatest sound intensity is usually found some tens of kilometers beyond the inner edge. Instances are known where the first anomalous zone formed a complete ring around the source but annular sectors are observed much more frequently; they are more likely to occur on account of the prevailing stratospheric wind distributions. The effect of the seasonal wind field in the ozone layer is such that the sound is propagated predominantly toward the east in winter, and toward the west in summer.

Wegener (1925) has shown that the radius of the zone boundary is also subject to wide seasonal variations. The mean radial distance to the inner edge is found to vary from about 105 kilometers in January and February to 200 kilometers in early August.

The outer edge of the first anomalous zone of audibility is generally diffuse and ill-defined. Indeed, the very existence of a true outer limit is still open to discussion and, it seems, largely a question of definition. To be sure, parts of a second and even a third ring of audibility have been found often enough, but whether these segments are truly distinct entities and not just part of a large single zone is another matter altogether. This is not to say, of course, that the sound energy received at twice or three times the radial distance of the first zone may not be due to multiply-reflected rays. On the other hand, it is easily shown that these same distances can be bridged just as readily in a single skip by rays of somewhat shorter

travel time. The situation is complicated further by the infrasonic signals returned from the ionosphere. The travel times of these signals are one or two minutes longer, and their frequencies too low to be perceived by ear, but they may still land close enough to the source to fall within the proper domain of the first zone, loosely ascribed as being due to returns from the ozone layer only. It is thus obvious that the zonal patterns will tend to overlap in space, but not necessarily in time; and we may envisage an hour-long interplay of everchanging sound and shadow, spreading out and ebbing away.

Little work has been done to date on the detailed structure of the zonal pattern, although a good many investigators have concerned themselves, at one time or other, with the practical problem of predicting the blast effects from large surface bursts. Cox (1949), analyzing the records of the Helgoland explosion, seems to have been the first to have noted that the high frequency content of the early part of a signal may increase with distance - an unexpected effect, and contrary to existing theory. Cox attempted to explain this effect by proposing a theory based on Schrödinger's dispersion equation. This theory had to be discarded, however, when it was found to rest on invalid premises.

Before we consider these questions further we shall turn to the important problem of sound absorption in the atmosphere.

III. THE ATTENUATION OF SOUND IN THE ATMOSPHERE

3.1. Introduction

The problem of sound absorption in gases has held the interest of many illustrious investigators for well over a century. Stokes in 1845 developed the first theory of sound attenuation, based exclusively on the damping action of viscosity. In 1868 Kirchhoff provided a second theory which took account of the equally important effects of thermal conductivity. The "classical" phase of development may be said to have ended with Lamb, who considered the effects of both mechanisms and unified the two theories. Lamb's solution resulted in more rapid damping than that predicted by Stokes, but the speed of propagation in the viscous, conducting fluid turned out to be very nearly equal to the speed in a perfect, frictionless fluid.

The combined theory of Lamb is based, of course, on the macroscopic view of heat flux and viscous stress, in full accord with the principles of classical hydrodynamics. Thermal conductivity and viscosity are thus considered as bulk properties of the fluid, and used as such, appropriately, in the Navier-Stokes equations. Theoretical calculation of sound absorption made on the basis of Lamb's theory are usually found to be in excellent agreement with experimental results, and there can be little doubt by now that

the theory is at least correct in essence if not in every detail. To be sure, recent precise measurements seem to indicate that the macroscopic view alone is inadequate, and that such factors as molecular binding should be considered as well. Refined versions of the Kinetic Theory of Gases show, furthermore, that the viscous stresses and the heat flux, as used in the hydrodynamic equations, are correct only to a first approximation. If, for instance, the number of molecules contained in a cube of dimension characteristic to the problem considered is small, the first-order approximation may be not accurate enough, and the Navier-Stokes equations need no longer apply. Two examples come readily to mind: ultrasonic waves of very short wave-length, and propagation in highly rarified gases. The first is of little concern here, but the second is most pertinent in connection with wave propagation at great altitudes, and thus of considerable interest in the present inquiry.

The effect of low density of the medium on the speed and damping of waves was examined by Tsien and Schamberg (1946) who made use of the second order approximations of viscous stress and heat flux given earlier by Chapman and Cowling (1939). The results of this investigation are most assuring, however, for they indicate that the increase of speed will not exceed the normal sea level value by more than 2 per cent, even in the most extreme conditions. If anything, the extra terms of the second order approximations

tend to maintain the speed of sound constant over a wide range of densities.

On the other hand, this investigation showed also that the absorption increases markedly when the mean free path of the gas molecules is no longer small compared with the wave length of sound. This "low density absorption" will thus seriously limit the transmission of sound through the atmosphere above 100 kilometers where the mean free path increases rapidly with height. (See Table IV, p.86). At lower heights, say 30 to 70 kilometers, this kind of absorption is still quite small, however, and may be safely neglected in comparison with damping due to viscosity and heat conduction. If, in addition, consideration is given only to propagation at frequencies below 100 cycles per second, the energy losses due to radiation, diffusion and molecular excitation may be neglected as well, and the attenuation can be accounted for entirely by classical theory.

In the present study the attenuation of sound will be treated largely from this point of view, with particular reference to the classical work of Lamb and Schrödinger, and a more recent paper by Lindsay.

3.2. The Absorption of Sound as a Relaxation Effect

We shall first consider the relaxation dissipation of plane sound waves in one dimension. Following Lindsay (1948) we denote the displacement of the medium in the x-direction by ξ , the excess pressure by p_e and the equilibrium density by ρ .

The equation of motion for small departures from equilibrium is clearly

$$\rho \frac{\partial^2 \xi}{\partial t^2} = - \frac{\partial p_e}{\partial x} \quad (3-1)$$

If we substitute for p_e from Hooke's law in the form

$$p_e = B \frac{\rho_e}{\rho} = - B \frac{\partial \xi}{\partial x}, \quad (3-2)$$

where ρ_e is the excess density and B denotes again the bulk modulus, we obtain the well-known relation

$$\frac{\partial^2 \xi}{\partial t^2} = c^2 \frac{\partial^2 \xi}{\partial x^2}, \quad (3-3)$$

the wave velocity being given by

$$c = (B/\rho)^{\frac{1}{2}} \quad (3-4)$$

This equation is, of course, equal to equation (2-4) derived earlier, and shows only that the speed of sound is constant if Hooke's law is assumed to hold rigorously, and for all time. Evidently, we have come full circle; the simple wave equation is clearly insufficient for our purpose, and the problem must be considered in a more subtle way.

The reason for this 'breakdown' is to be found in equation (3-2). It is a static equilibrium equation which completely ignores the dynamic mechanism by which the density is changed. To take account of this time-dependent process we must replace this static relation by the more appropriate pressure relaxation equation

$$p_e = B\rho_e/\rho + R\dot{p}_e \quad (3-5)$$

where R is called the relaxation constant, and the dot denotes partial differentiation with respect to time.

If we write $p_e = -\rho(\partial\xi/\partial x)$, the pressure relaxation equation takes the form

$$p_e = -B \frac{\partial\xi}{\partial x} - R\rho \frac{\partial\dot{\xi}}{\partial x} \quad (3-6)$$

Substitution of this expression in (3-1) leads readily to the wave equation

$$\frac{\partial^2\xi}{\partial t^2} = \frac{B}{\rho} \frac{\partial^2\xi}{\partial x^2} + R \frac{\partial^2\dot{\xi}}{\partial x^2} \quad (3-7)$$

This relation differs only in detail from Stokes' equation for the transmission of plane body waves through a viscous medium.

We may solve this equation by considering the harmonic trial solution

$$\xi = e^{-\alpha x} e^{i(\omega t - kx)} \quad (3-8)$$

Direct substitution in (3-7) shows that this is indeed a solution of the equation provided that

$$2\alpha k = \frac{\omega^3 \tau \rho}{B(1 + \omega^2 \tau^2)} \quad (3-9)$$

$$\text{and } k^2 - \alpha^2 = \frac{\omega^2 \rho}{B(1 + \omega^2 \tau^2)} \quad (3-10)$$

The symbols used in these expressions have all their conventional meaning:

ω = the angular frequency

k = the wave vector = $2\pi/\lambda$

α = the attenuation coefficient

and τ = the relaxation time, defined by

$$\tau = \rho R/B \quad (3-11)$$

The relaxation time is equal to the time taken by the density to attain the fractional value $1 - 1/e$ of the equilibrium density ρ_0 , for a given applied excess pressure.

Solving (3-9) and (3-10) for α , we obtain the bi-quadratic equation

$$\alpha^4 + \frac{\omega^2 \rho}{B(1 + \omega^2 \tau^2)} \alpha^2 - \frac{\omega^6 \tau^2 \rho^2}{4B^2(1 + \omega^2 \tau^2)^2} = 0 \quad (3-12)$$

whose real solution is

$$\alpha = \left(\frac{1}{2B/\rho} \right)^{\frac{1}{2}} \left[\frac{(1 + \omega^2 \tau^2)^{\frac{1}{2}} - 1}{(1 + \omega^2 \tau^2)} \right]^{\frac{1}{2}} \omega. \quad (3-13)$$

This equation may be put into more convenient form by means of the well-known expression for the phase velocity V_p , i.e.

$$V_p = \frac{\omega}{k} \quad (3-14)$$

If we solve (3-9) for k , we obtain, after substitution for α from (3-13)

$$k = \frac{\omega^2 \tau \{ (1 + \omega^2 \tau^2) [(1 + \omega^2 \tau^2)^{\frac{1}{2}} - 1] \}^{-\frac{1}{2}}}{(2B/\rho)^{\frac{1}{2}}} \quad (3-15)$$

But since $k = \omega/V_p$, we have

$$V_p = \frac{(2B/\rho)^{\frac{1}{2}}}{\omega \tau} \{ (1 + \omega^2 \tau^2) [(1 + \omega^2 \tau^2)^{\frac{1}{2}} - 1] \}^{\frac{1}{2}}, \quad (3-16)$$

Since the phase velocity depends on the frequency, there will be dispersion. We may now express the absorption coefficient more completely with the aid of (3-16). The result is simply

$$\alpha = \frac{\omega^2 \tau V_p}{(2B/\rho)(1 + \omega^2 \tau^2)} \quad (3-17)$$

3.3 The Dissipation of Sound due to Viscosity

The viscosity absorption coefficient may be deduced easily from (3-17) if we make use of the relaxation constant introduced by Stokes. This constant is usually given as

$$R = 4 \mu / 3\rho \quad (3-18)$$

but, when derived directly from the fundamental hydrodynamic equations, it turns out to be

$$R = (\chi + 2\mu)/\rho \quad (3-18a)$$

where χ is the longitudinal coefficient of viscosity, and μ the more familiar shear coefficient of viscosity. Since χ could not be measured by experiment, Stokes made the assumption that fluids show no viscous reaction to uniform bulk compression. Hydrodynamic theory requires, however, that the bulk or volume coefficient of viscosity μ' associated with uniform compression be given by

$$\mu' = \chi + 2\mu/3,$$

and hence, if we assume with Stokes that $\mu' = 0$, it is necessary that $\chi = -2\mu/3$. Substitution of this theoretical value leads then directly to equation (3-18).

We may now express the relaxation time entirely in terms of directly measurable physical quantities for, using (3-11) we obtain

$$\tau = 4\mu/3B \quad (3-19)$$

or, with

$$c^2 = B/\rho$$

$$\tau = \frac{4\mu}{3\rho c^2} \quad (3-19a)$$

The relaxation time is very short in most cases of practical interest. Direct substitution in (3-19) shows, for instance, that $\tau = 1.7 \times 10^{-10}$ sec. for air of density 0.00121 gm/cm^3 at 20°C .

The coefficient of viscosity, given by the familiar expression*

$$\mu = \frac{1}{3} mn\bar{v}\ell$$

is not a function of the pressure. The question naturally arises at this point whether this result of simple kinetic theory is still valid at greatly reduced pressures. There can be little doubt that this expression will hold as long as the gas obeys the postulates of the Kinetic Theory or, in particular, if the collision frequency of the gas molecules is sufficiently high. Experimental investigations on rarified gases confined in vessels of convenient laboratory size clearly show, on the other hand, that the viscosity falls to low values at low pressures. It may be shown, however, that this effect is largely due to the size of the vessel. As long as the size of the vessel is much larger than the mean free path of the molecules, the coefficient of viscosity should remain independent of pressure. Since this condition would seem to be always fulfilled in the earth's free atmosphere, we will accept the pressure independence of μ without further reservation.

*m - mass of the molecule

n - number of molecules in a unit volume of gas

$\bar{v}$ - the root-mean-square molecular velocity

ℓ - mean free path

We now return to consider equations (3-19) and (3-17): since the adiabatic bulk modulus is equal to γp and the viscosity of air is 1.81×10^{-4} dyne sec/cm², it follows that the value of τ in seconds is very small compared to unity for all pressures above 1 dyne/cm², and still quite small for pressures as low as 0.01 dyne/cm². Since a pressure of 1 dyne/cm² is nearly equal to one millionth the normal sea level pressure, the relaxation time will be always small for our purpose, and we may safely assume that $\omega\tau \ll 1$ for all frequencies below the ultrasonic range. Under these conditions we find that (3-17) reduces to the form

$$\alpha_v = \frac{2\mu\omega^2}{3\rho c^3}, \quad (3-20)$$

where we have made use of the subscript v to denote viscosity absorption.

3.4. The Dissipation of Sound due to Heat Conduction

The other important mechanism for producing absorption and dispersion in fluids is heat conduction. In the passage of a sound wave the temperature will be raised as a result of compression, and local temperature gradients will be set up. This may lead to significant flows of heat by conduction before the subsequent rarification can arrest and reverse the trend. Dissipation due to heat conduction is thus also in essence a relaxation effect, and of about the same order of importance

as viscosity dissipation. The derivation of the corresponding absorption coefficient is rather lengthy, however, and will not be given here. We shall, therefore, refer to Rayleigh's treatise* for a detailed derivation, and consider here merely the result.

The theory indicates that the relaxation time for this process given by

$$\tau = \frac{\eta}{\rho c^2 c_p} \quad (3-21)$$

where η is the thermal conductivity of the fluid and c_p the specific heat at constant pressure. For most fluids the relaxation time is so small for all frequencies below the mega-cycle range that the complete theoretical equation for the attenuation coefficient due to heat conduction, α_h , may be reduced to

$$\alpha_h = \frac{\gamma - 1}{2c} \omega^2 \tau \quad (3-22)$$

which becomes, after substitution of τ from (3-21)

$$\alpha_h = \frac{\eta(\gamma - 1)}{2\rho c^3 c_p} \omega^2 \quad (3-23)$$

Substitution of the appropriate numerical values shows that the attenuation resulting from heat conduction in gases is somewhat less than viscosity dissipation but generally of the same order of magnitude. (See Table III, p.81).

* Lord Rayleigh, the Theory of Sound, Vol.II, sec. 349.

3.5 The Classical Attenuation Coefficient

It is customary to assume that viscosity and heat conduction act independently in the damping of acoustic waves. On this basis we may combine equations (3-20) and (3-23), obtaining

$$\alpha = \frac{\omega^2}{2\rho c^3} \left(\frac{4\mu}{3} + \frac{\eta(\gamma - 1)}{c_p} \right) \quad (3-24)$$

Acoustic attenuation of particle displacement, pressure, etc. are frequently expressed in nepers, a natural logarithmic unit corresponding to a reduction in amplitude to $1/e$ of the initial or equilibrium value. Since plane waves are attenuated directly as $e^{-\alpha x}$, the product αx is said to have the "dimensions" of nepers. By the same token the coefficient α has the dimensions of nepers/m.

It may not be amiss to point out that, on the basis of the derivation, α is the amplitude absorption coefficient. Since the intensity is proportional to the square of the amplitude, the intensity absorption coefficient is twice as large, and usually denoted by 2α . The intensity must therefore vary with distance in accordance with the relation

$$I_x = I_0 e^{-2\alpha x} \quad (3-25)$$

The change in intensity level of an attenuated wave, expressed in decibels, is thus given by

$$\Delta IL = 10 \log_{10}(I_x/I_0) = -8.7\alpha x \quad (3-26)$$

and 8.7α may be considered to be a measure of the rate of decrease of intensity level with distance, expressed in db/m.

Since the attenuation coefficient depends on the square of the frequency ν , it is usually preferable to tabulate values of α/ν^2 . Computed and observed values of this ratio for some atmospheric gases are given in the table below:

All data for $T = 293^\circ\text{K}$ and $p = 1 \text{ atm.}$ $\nu = 2\pi\omega$	Classical Calculation of α/ν^2			Observed α/ν^2 $\frac{\text{neper}\cdot\text{sec}^2}{\text{m}}$
	Heat Conduction	Viscosity	Total	
	$\frac{\text{neper}\cdot\text{sec}^2}{\text{m}}$	$\frac{\text{neper}\cdot\text{sec}^2}{\text{m}}$	$\frac{\text{neper}\cdot\text{sec}^2}{\text{m}}$	
Argon	0.77×10^{-11}	1.08×10^{-11}	1.85×10^{-11}	1.87×10^{-11}
Helium	0.22	0.31	0.53	0.54
Oxygen	0.47	1.14	1.61	1.92
Nitrogen	0.39	0.96	1.35	1.64
Dry Air	0.38	0.99	1.37	2.0
CO ₂	0.31	1.09	1.40	-

TABLE III.

Acoustic Attenuation in Atmospheric Gases

(After Kinsler and Frey)

3.6 Schrödinger's Attenuation Coefficient

The attenuation coefficient may be put into a form more suitable to atmospheric acoustics by recourse to the kinetic theory relations for the viscosity and the thermal conductivity:

$$\mu = \frac{1}{3} \rho \bar{v} \ell \quad (3-27)$$

$$\eta = 2c_v \mu = \frac{2}{3} c_v \rho \bar{v} \ell \quad (3-28)$$

where $\bar{v}$ is again the root-mean-square velocity, ℓ the mean free path, and c_v the specific heat at constant volume. Also, since $\bar{v} = (3p/\rho)^{\frac{1}{2}}$ and $C = (\gamma p/\rho)^{\frac{1}{2}}$, we have

$$\frac{\bar{v}}{C} = \left(\frac{3}{\gamma}\right)^{\frac{1}{2}}$$

Substituting these expressions in (3-24) we obtain

$$\alpha = 13.15 \left(\frac{3}{\gamma}\right)^{\frac{1}{2}} \left(\frac{2}{3} + \frac{\gamma-1}{\gamma}\right) \frac{\nu^2 \ell}{c^2} \quad (3-29)$$

with $\frac{4\pi^2}{3} = 13.15$, and $\nu = 2\pi\omega$, the frequency.

If we take $\gamma = 1.41$ as the value for air, the coefficient reduces to

$$\alpha = 18.4 \frac{\nu^2 \ell}{c^2} \text{ cm}^{-1} \quad (\text{or } \dots \times 10^2 \text{ m}^{-1}) \quad (3-30)$$

The attenuation coefficient of energy is thus

$$2\alpha = 36.8 \frac{\nu^2 \ell}{c^2} \text{ cm}^{-1} \quad (\text{or } \dots \times 10^2 \text{ m}^{-1}) \quad (3-31)$$

$$= 36.8 \frac{\ell}{\lambda^2} \text{ cm}^{-1} \quad (\text{or } \dots \times 10^2 \text{ m}^{-1}) \quad (3-31a)$$

where λ is the wave length in cm, or m, as required.

This form of the equation shows clearly the decisive influence of the ratio ℓ/λ^2 on sound propagation in the atmosphere.

The attenuation of high frequency sound is seen to be much greater than that of the bass and infra-sonic frequencies, particularly so in the high atmosphere where ℓ is large.

Equation (3-31a) was first derived by Schrödinger in 1917 in a semi-empirical way. He took account of the height dependence of α by assuming that the thermal conductivity* and viscosity coefficients divided by the density are proportional to the mean free path of the molecules, i.e.

$$\frac{\eta}{\rho} = K\bar{v}\ell \quad \text{and} \quad \frac{\mu}{\rho} = K'\bar{v}\ell$$

Making use of the physical constants of his day, he calculated K and K' , and found the energy attenuation coefficient to be

$$2\alpha = 30.10 \frac{\ell}{\lambda^2} \quad (3-31b)$$

Schrödinger originally denoted the constant by D (Dämpfungsexponent). Kölzer (1925) subsequently computed D values for air, nitrogen, hydrogen and mixtures of air and nitrogen, and hydrogen and nitrogen; for air he found $D = 33.026$.

Schrödinger's absorption curves are based on Equation (3-31b). Neglecting the small effect of atmospheric temperature variations in comparison with the much greater influence of the mean free path, he found ℓ from the simple barometric formula

$$\ell = 10^{-5} e^{\beta z} \text{ cm} \quad (3-32)$$

*Schrödinger actually used the thermal diffusivity, given by $q = \frac{\eta}{c_v \rho}$ in terms of our quantities.

where $\beta = 1.2516 \times 10^{-6} \text{ cm}^{-1}$ for an isothermal atmosphere at 0°C . Putting $x = 10^5 \text{ cm} = 1 \text{ km}$, and

$$\begin{aligned} \kappa &= -2\alpha x = -30.10 \frac{\ell x}{\lambda^2} \\ &= -30.10 \frac{e^{\beta z}}{\lambda^2}, \end{aligned} \quad (3-33)$$

Schrödinger could determine the height z at which a wave of length λ suffered a fractional intensity decrease

$$\begin{aligned} \frac{I}{I_0} &= e^{\kappa}. \\ \text{Since by (3-33) } \ln e^{\kappa} &= -30.10 \frac{e^{\beta z}}{\lambda^2} \end{aligned} \quad (3-34)$$

it is seen that

$$z = \frac{1}{\beta} \left[2 \ln \lambda + \ln \left(-\frac{\ln e^{\kappa}}{30.10} \right) \right] \quad (3-35)$$

This is Schrödinger's absorption equation in its final form.*

3.7 Other Forms of the Attenuation Coefficient

In the present study we shall take some account of atmospheric temperature variations and the mean free path by writing Equation (3-24) in the form

$$2\alpha = \frac{4\pi^2 \nu^2}{c^3} \left[\frac{4}{3} \frac{\mu}{\rho} + \frac{(\gamma-1)}{c_p} \frac{\eta}{\rho} \right] \frac{\ell}{\ell_0} \quad (3-36)$$

where ℓ_0 is the mean free path at ground level. We assume again with Schrödinger that the ratios μ/ρ and η/ρ are proportional to the mean free path at the height z .

*In our notation. Schrödinger denotes the vertical axis by x , the horizontal direction by z , and uses α for β .

Introducing the numerical values

$$\begin{aligned}
 \mu &= 1.71 \text{ gm/cm sec} & (0.171 \text{ kg/ m sec}) \\
 \rho &= 1.29 \times 10^{-3} \text{ gm/cm}^3 & (1.29 \text{ kg/m}^3) \\
 \gamma &= 1.40 & (1.40) \\
 c_p &= 0.24 \text{ cal/gm}^\circ\text{K} & (240 \text{ cal/kg } ^\circ\text{K}) \\
 \eta &= 5.33 \times 10^{-5} \frac{\text{cal}}{^\circ\text{K cm sec}} & (5.33 \times 10^{-3} \frac{\text{cal}}{^\circ\text{K m sec}}) \\
 \ell_0 &= 0.7 \times 10^{-5} \text{ cm} & (0.7 \times 10^{-7} \text{ m})
 \end{aligned}$$

into the last equation, we find

$$2\alpha = 1.4 \times 10^6 \ell v^2 / c^3 \text{ cm}^{-1} \quad (\text{or } \dots 10^8 \text{ m}^{-1}) \quad (3-36a)$$

If we substitute the physical data used by Schrödinger, the coefficient turns out to be

$$2\alpha = 1.0 \times 10^6 \ell v^2 / c^3 \text{ cm}^{-1} \quad (\text{or } \dots 10^8 \text{ m}^{-1}) \quad (3-36b)$$

The kinetic theory expression for the mean free path is

$$\ell = \frac{1}{n d^2 \pi \sqrt{2}} \quad (3-37)$$

where n is the molecular density, i.e. the number of molecules in a unit volume, and d the average diameter of the molecules.

The molecular density may be calculated from the equation of state in the form

$$p = nkT \quad (3-38)$$

For the purpose of this investigation we shall use Equation (3-36a) and assume that the mean free path is adequately represented by

$$\ell = 0.7 \times 10^{-5} \cdot e^{0.136z} \quad (3-39)$$

This expression is derived from Veldkamp's table of ℓ , calculated from NACA upper air data*; it holds with sufficient accuracy to heights up to 70 km, although an index of $0.146z$ would represent the variation of the mean free path rather better at levels near 40 km.

Height (km)	(cm)	Height (km)	(cm)
0	0.7×10^{-5}	120	13
20	1.1×10^{-4}	130	28
40	2.5×10^{-3}	140	56
60	2.4×10^{-2}	150	120
80	1.7×10^{-1}	160	250
100	2.1	170	500

TABLE IV.

Mean Free Path of Air Molecules.
(After Veldkamp and NACA)

Cox (1957) gives a useful relation for the coefficient in terms of the absolute temperature, the density and the frequency:

$$2\alpha = 9.5 \times 10^{-9} \frac{\nu^2}{\rho T} \text{ per km.} \quad (3-40)$$

This expression may be derived easily from (3-31) and the standard Clausius formula for the mean free path,

$$\ell = \frac{3m}{4\pi d^2 \rho} \quad (3-41)$$

*National Advisory Committee for Aeronautics, Note No.1200.

Using $m = 46.5 \times 10^{-24}$ gm as the mass and $d = 3.1 \times 10^{-8}$ cm as the diameter of the nitrogen molecule (for oxygen $d = 3 \times 10^{-8}$ cm), the mean free path is

$$\ell = 1.15 \times 10^{-8} / \rho \quad \text{cm} \quad (3-42)$$

Equation (3-40) follows then directly from (3-31) and Laplace's equation.

Since the density of atmospheric air varies much more rapidly with height than with temperature, we may use the barometric formula

$$\rho = \rho_0 e^{-z/7.7} \quad (3-42)$$

where ρ_0 is the density at sea level and z the height in kilometers. Taking $\rho_0 = 1.29 \times 10^{-3}$ gm/cm³ we find

$$2\alpha = 7.4 \times 10^{-6} \frac{v^2}{T} e^{+z/7.7} \text{ per km.} \quad (3-43)$$

The numerical constant in the exponent, called the "e-height" of the atmosphere, has been determined by averaging density values at 20 kilometer intervals from sea-level to 100 km. We may compare it with the height of the "homogeneous" atmosphere, usually given as 8 kilometers. This is the height of an atmosphere of constant density whose temperature and pressure at sea level are 273°K and 1000 millibars.

3.8 Energy Flux and Energy Density in the First Anomalous Zone of Audibility.

We now proceed to derive an expression due to Gutenberg (1942) for the energy distribution in the first anomalous zone. Gutenberg's equation is strictly valid only in quiet air, but since the wind velocity is usually much smaller than the speed of sound we shall use it also to compute the energy flux in moving air.

We assume that the energy released at 0 by an explosion is initially contained within a small hemisphere* of radius r and transmitted in all directions with equal intensity. (See Fig. 8).

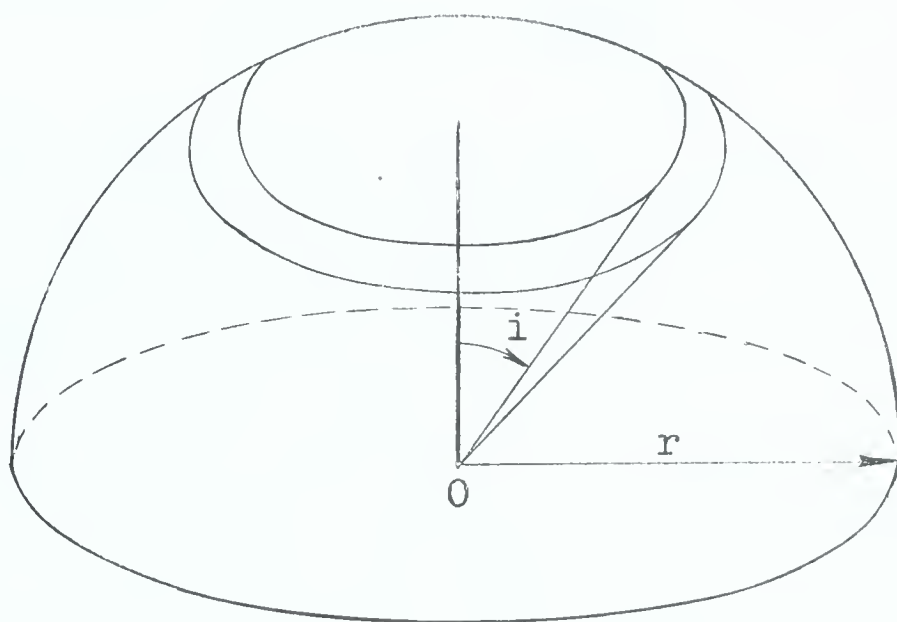


Fig. 8

Initial Energy Distribution at Ground Zero.

*We may remark in this regard that the blast wave of the 100-ton Suffield explosion of Aug. 3, 1961, appears to have had good hemispherical symmetry. See Ditto W.J. and J. Holdsworth, Eng. Journal, July 1962, pp.29-32.

If Q is the total energy released, and none is absorbed by the ground, the energy transmitted through an elemental ring on the hemisphere is

$$\begin{aligned} dQ &= \frac{Q}{2\pi r^2} \cdot 2\pi r \sin i \cdot r \, di \\ &= Q \sin i \, di \end{aligned} \quad (3-44)$$

where i is the initial angle of incidence.

The absorption of energy along the ray path S , (Fig.9) assumed to proceed exponentially, is proportional to e^{-I} ,

where

$$I = \int_0^S (2\alpha) ds = 2 \int_0^{\Delta/2} (2\alpha) \frac{d\Delta}{\sin i} \quad (3-45a)$$

or, equivalently

$$I = 2 \int_0^{z_m} (2\alpha) \cdot \frac{dz}{\cos i} \quad (3-45b)$$

Here Δ denotes the horizontal distance covered by a ray landing in the first anomalous zone, and z_m is the height of the ray apex.

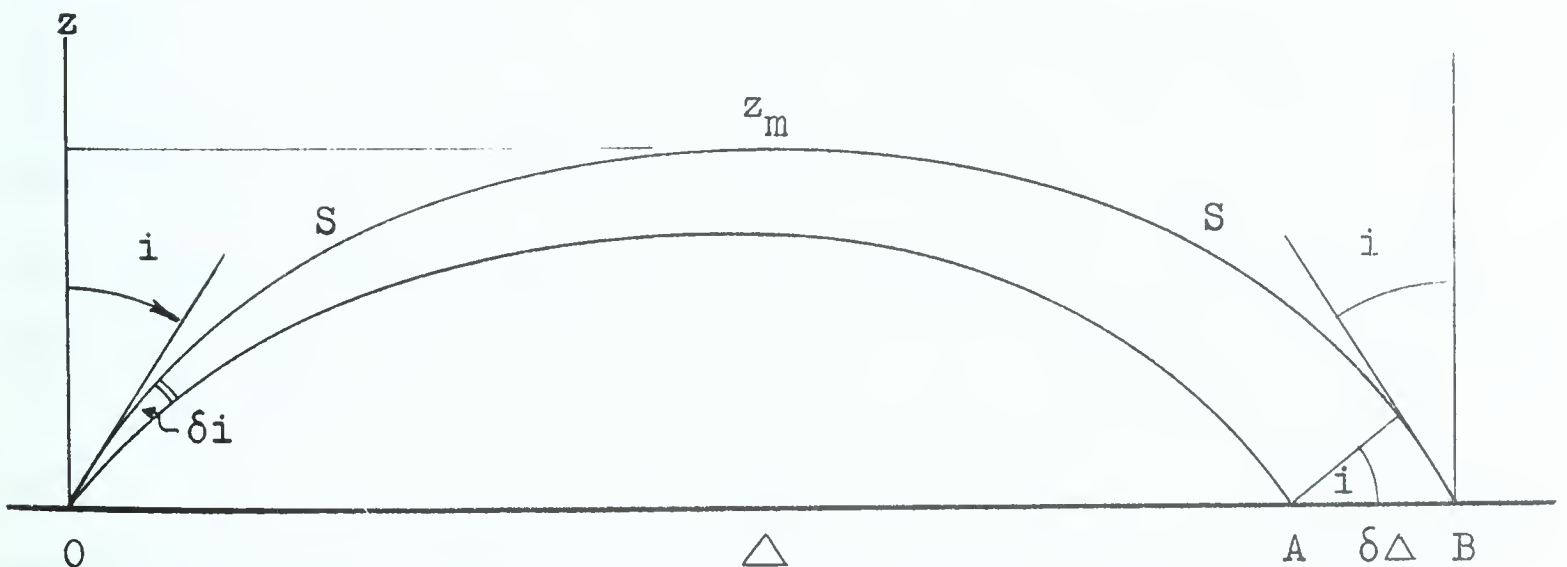


Fig. 9

Geometry of Sound Rays refracted in the Atmosphere.

The transmitted energy returning to ground arrives on a zone of area $2\pi\Delta d\Delta$. The surface energy density at Δ is thus given by

$$E(\Delta) = \frac{Q}{2\pi} e^{-I} \frac{\sin i}{\Delta} \cdot \frac{di}{d\Delta} \quad (3-46)$$

Fig. 10 illustrates the dominant effect of the term $di/d\Delta$ on the energy distribution in the first zone.

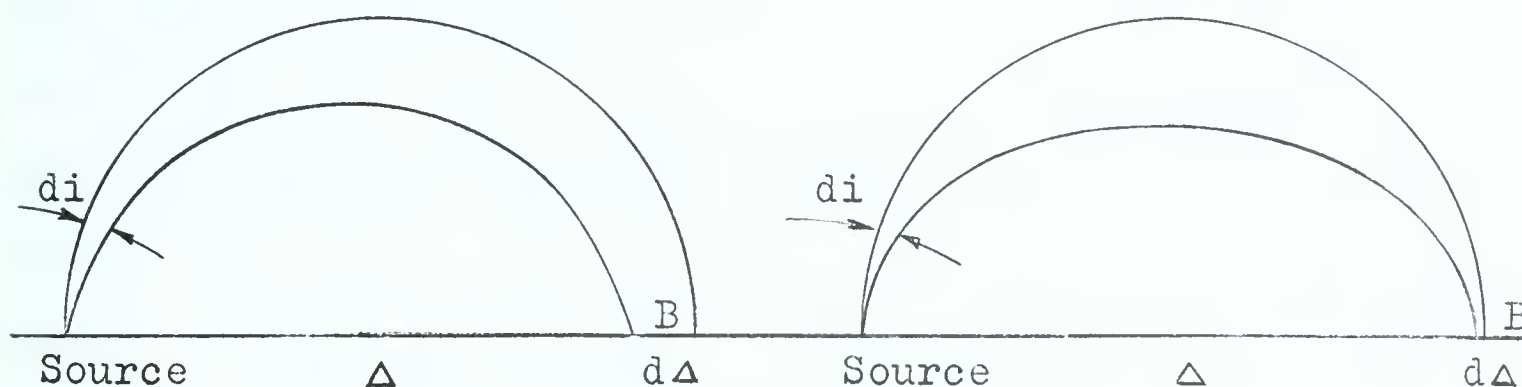


Fig. 10

Effect of the $di/d\Delta$ term on the Sound Intensity.

If i changes rapidly with distance, the energy will be concentrated near B , but if it changes slowly, the energy density will be small. If two rays of small angular difference arrive at the same point B , $di/d\Delta$ becomes infinite and the energy will be focussed into a caustic. By contrast, the energy density near the outer boundary of the zone will be very small, for $di/d\Delta \rightarrow 0$ for a critical angle i_c such that

$$\sin i_c = (T_o/T_m)^{\frac{1}{2}}$$

where T_o is the temperature at the surface and T_m is the maximum temperature in the refracting layer; rays with angles

of incidence smaller than i_c will not be returned to ground at all. A little energy will be returned by rays leaving the source with angles slightly greater than i_c , but this energy will be diffused over a large area, and difficult to detect.

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At this point one might wish to insert a numerical value for Q , but this presents several difficulties. The energy released at the "epicentre" is proportional to the weight of the explosive and amounts to 4.2×10^9 joules per ton of TNT. It is known that only a small fraction of the energy is put into the ground as shock and cratering, but the amount of energy radiated as light and heat to "infinity" may be quite large. A nuclear air-burst is said to radiate as much as one half of the total energy in the form of heat and light. (Hunt et alii, 1960). A large chemical explosion on the ground will presumably radiate less than a nuclear explosion of comparable size, but reliable estimates of the energy lost by radiation are not available at present. It may be instructive, however, to quote Hunt's estimates derived from data released by the U.S. Atomic Energy Commission.

When the blast from a nuclear explosion on the ground has reached a radius at which the shock over-pressure is one pound per square inch, the kinetic energy in the positive phase is approximately 8% of the total energy released by the explosion, and the kinetic energy in the negative phase is

of the order of $\frac{1}{2}$ %. Hence the energy at this stage has been partitioned approximately in the following way: kinetic energy in the blast 10%, radiated away from the region of the explosion as electromagnetic radiation 20 to 30%, cratering and ground shock 1 to 5%, and the remaining 55 to 77% as heat and compressional energy in the air.

In view of these uncertainties we will remove the factor $Q/2\pi$ by dividing Equation (3-46) by

$$E'(\Delta) = \frac{Q}{2\pi \Delta^2} \quad (3-47)$$

where $E'(\Delta)$ is the energy flux of "normal" sound propagating radially through an isotropic atmosphere. We shall neglect the absorption of normal sound travelling along the surface of the earth; since the mean free path is small at ground level, this absorption should be also small, even for large values of Δ .

The ratio $E(\Delta)/E'(\Delta)$ will be called the fractional energy flux F ; it is evidently given by

$$F = e^{-I} \Delta \sin i \frac{di}{d\Delta} \quad (3-48)$$

We shall use this expression later to compute the energy distribution of the sound received in the first anomalous zone of audibility.

3.9 Wave Dispersion

Most forms of wave motion in the atmosphere are dispersive, i.e. the speed of propagation (phase velocity) of individual waves is dependent on their frequency. The energy of such wave systems is propagated at a speed characteristic for the group of waves comprising the system, the so-called group velocity, which is generally different from the phase velocity. If the group velocity exceeds the phase velocity, new waves will emerge ahead of the original wave train; if it is less, the slower components of the initial train will lag behind and a long trailing wave may develop.

The nature of atmospheric dispersion is largely determined by the thermal and hence velocity structure of the atmospheric sound channels, but dispersion can also occur in the isosonal* medium of an isothermal atmosphere. In most media waves of low frequency travel faster than high frequency waves, (normal dispersion) so that a pulse of initially very short duration may be spread out into a long train of oscillations if the travel time is long. This is always the case with waves from large nuclear explosions and natural aeroclysms such as the eruption of Krakatoa and the Great Siberian Meteorite. Such waves will circle the globe several times and be dispersed into wave trains lasting many minutes.

The longest waves travel about 1.5% faster than the shortest. The duration of the signal is thus in essence proportional to the distance from the source to the observer, and about equal to 1.5% of the travel time.

The dispersion of waves from the much smaller chemical explosions considered here is not only of a different order of magnitude, but inherently different in that the lengthening of the wave train would seem to be largely due

* Constant speed of sound.

to a superposition of pulses which reach the observer by several distinct paths of slightly differing travel time. This is not to say that normal dispersion is not observed,* but only that it is of a lesser order and hence more difficult to identify than multiple path scattering. By the same token it is also true that normal dispersion is always small for waves from small explosions, and appreciable only for the long infrasonic modes excited by the largest explosions.

Schrödinger (1917) has shown that the phase velocity of plane sound waves propagating vertically in an isothermal atmosphere is

$$v_p = \frac{\omega}{k} = c \left[1 + \frac{\lambda^2}{16 \pi^2 H^2} \right]^{\frac{1}{2}} \quad (3-49)$$

where H is the height of the homogenous atmosphere, equal to about 8 km.

This expression shows clearly that the dispersion is very small for all but the longest waves. Indeed, the phase velocity increases by only 0.3% for wavelengths of 8 km (period 27 sec) and 1.5% for wavelengths of 18 km (period 53 sec.) It is evident, therefore, that normal dispersion is negligibly small for all sound waves in the audible and ultrasonic range, and of little consequence for a wide band of infrasonic frequencies as well.

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*See for instance Gutenberg's article in the Compendium of Meteorology, page 373.

IV. FIELD WORK, EQUIPMENT AND OPERATIONAL PROCEDURES.

4.1. Introduction

Some five years ago the Defence Research Board of Canada started a research program for the study of blast waves at the Suffield Experimental Station, a 1000 square mile field test area in Southeastern Alberta. Small charges of a few pounds of TNT were used in the initial experiments, but by August 1959 the charge-weight had been increased to 20 tons. These experiments soon aroused the interest of several geophysical groups in Western Canada, and when it became apparent that the test program would likely be continued for a number of years, suggestions were put forward to the effect that some effort be made to monitor at least the larger explosions. It was generally felt that valuable seismic and acoustic information could be obtained from these tests, at comparatively little expense, and with but minor modifications to standard equipment already on hand.

The Research Committee of the Canadian Society of Exploration Geophysicists (C.S.E.G.) at Calgary, promised to give support to such projects, and suggested that an attempt be made to record the anomalous air waves from some future explosion. Dr. J. A. Mair (1960) and other members of the Imperial Oil Geophysical Research group at Calgary undertook the construction of suitable equipment

for this purpose, and on August 18, 1960, they succeeded in recording anomalous sound waves from a 20 ton explosion, at a distance of some 176 miles from the Suffield "shot point."

This first Canadian venture into aerological seismology deserves to be remembered also for its unorthodox instrumentation. Its main armament consisted of a five-piece battery of 50 gallon oaken whiskey barrels, ingeniously fashioned into microbarographs; these colorful assemblies occasioned, as Dr. Mair readily admits, all sorts of comment. However, Mair pioneered also the use of the EVP-5 Electrotech hydrophone as an aerial detector, with unexpectedly good results. These detectors were later used extensively in connection with the present work, largely on the basis of experience gained from this first experiment, and on Dr. Mair's recommendation.

4.2. The 100 ton Explosion of August 3, 1961

The largest Suffield explosion to date, 100 tons of TNT, was initially scheduled to be set off on August 1, 1961. Preparations to record this explosion began in March of 1961, after some earlier correspondence and discussion between Dr. F. T. Davies, Chief Superintendent of the Defence Research Telecommunications Establishment at Shirley Bay, and Mr. P. J. Beaulieu, Secretary of the Associate Committee on Geodesy and Geophysics, N.R.C., Ottawa. Dr. G. D. Garland assumed most of the responsibilities of general planning and

co-ordination. Messrs. A. Junger and R. H. Carlyle, and Dr. Mair of the C.S.E.G. Research Committee of Calgary contacted all oil and geophysical exploration companies active in the Calgary area, and obtained the co-operation and support of some 25 of them. Mr. R. Keller of the General Geophysical Company co-ordinated the work of the C.S.E.G. group at Edmonton.

By June 15 it appeared that about 30 commercial exploration parties would be available. Most of these would be located within 100 miles of Calgary, but some were expected to work in Saskatchewan and Southwestern Manitoba. In addition, a University of Saskatchewan crew under Dr. Hall was to operate near Flin Flon, and a Geological Survey of Canada party under G. Hobson (H. A. MacAuley, party chief) was to record near Watrous, Sask. A seismological party from the University of Michigan was to observe near Hays, Alberta, almost exactly 100 km southwest of Watching Hill, the site of the explosion. The University of Alberta planned to operate six stations of its own, located at Edmonton, Suffield, Regina, Moosomin, Brandon and Winnipeg. These stations were to be equipped with microbarographs capable of recording long-period air-waves returned from the ionosphere. The station at Regina was to be equipped also with magneto-telluric instrumentation, and monitor the explosion for possible electro-magnetic effects.

But the best laid plans ... often go awry, and this venture was to be no exception. Six transducers for the microbarographs, ordered months in advance, failed to arrive at

Edmonton, in spite of repeated promises by the manufacturer, and the concerted efforts of the Purchasing Department and members of the Faculty. As a result, most of the University stations could not be equipped to record the long-period waves from the ionosphere, and had to improvise with single EVP-5 pressure detectors. To make matters worse, heavy rains fell in the Suffield test area at the end of July. Equipment was damaged, the explosion had to be postponed, and the initial plans had to be modified accordingly. The explosion was set off two days later, on August 3rd, but not all field parties could be notified in time, or be kept up to date on last minute changes. The number of stations which recorded on August 3rd was considerably below the expected total for August 1st, and much valuable information was undoubtedly lost by the delay.

The final count showed that 27 stations* had monitored the explosion at the appropriate time. Several stations failed to obtain proper records for a variety of reasons, but all contributed in some measure to the over-all success of the operation. The locations and distances from the explosion of the 25 groups who turned in records are listed in Table V, and the geographical distribution of the stations is shown in Fig.11. Each station is also identified by a number, but the numbering is completely arbitrary.

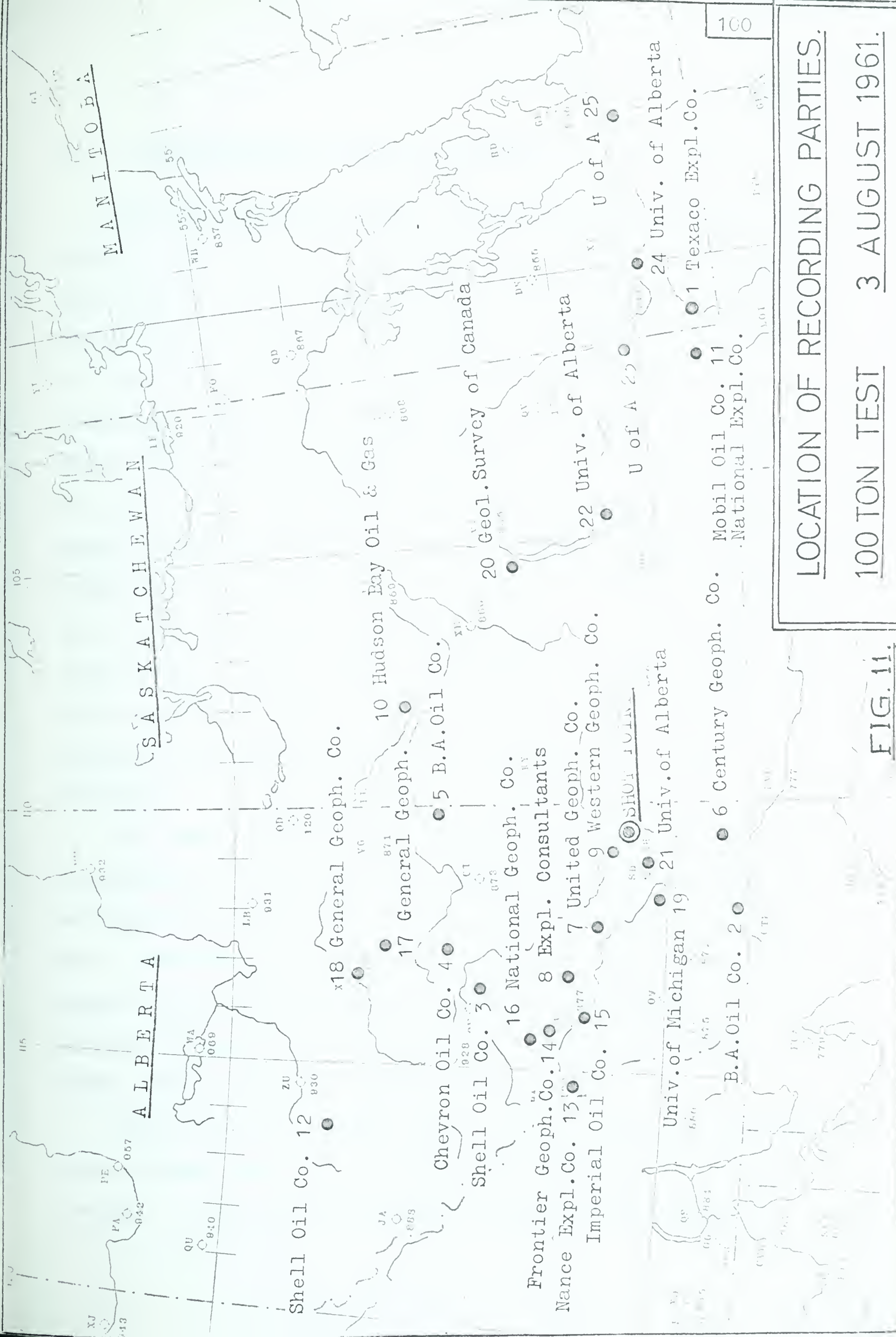
*This count includes the two hot-wire microphones operated by the University of Saskatchewan.

TABLE V.

SUFFIELD EXPERIMENTAL STATION 100 TON TEST AUGUST 3RD, 1961.
LOCATIONS OF RECORDING SITES AND DISTANCES FROM SHOT POINT.

	RECORDED BY	* LOCATION				LATITUDE			LONGITUDE			DISTANCE	
		S	T	R	M	DEG	MIN	SEC	DEG	MIN	SEC	MILES	KM
1	TEXACO EXPL.	6	4	26	W1	49	16	0.0	101	0	0.0	438.2	705.2
2	BA OIL CO.	5	3	15	W4	49	11	10.1	111	58	23.7	102.3	174.3
		5	3	15	W4	49	10	41.4	111	58	53.7	109.0	175.4
3	SHELL OIL CO	11	36	26	W4	52	5	3.9	113	37	6.9	170.2	274.0
4	CHEVRON OIL	17	41	20	W4	52	32	29.7	112	49	59.4	170.9	275.0
5	BA OIL CO.	13	43	2	W4	52	42	16.5	110	8	53.9	154.6	248.7
		13	43	2	W4	52	42	51.2	110	8	53.9	155.2	249.8
6	CENTURY GEOP	21	4	5	W4	49	19	9.1	110	36	38.3	80.8	130.1
7	UNITED GEOPH	27	21	18	W4	50	49	14.4	112	24	16.4	81.4	131.0
8	FXP. CONSULT.	17	37	23	W4	52	10	40.6	113	14	39.3	162.7	261.9
9	WESTERN GEOP	36	19	15	W4	50	39	32.2	110	56	55.1	18.4	29.6
10	HUDSON BAY O	19	46	15	W3	52	59	14.2	108	6	28.7	203.8	328.0
11	MOBIL OIL CO	5	5	33	W1	49	22	6.8	101	52	47.7	397.9	640.3
	NAT. GEOPH. CO	5	5	33	W1	49	22	6.8	101	52	30.8	398.1	640.7
12	SHELL OIL CO	20	56	16	W5	53	51	33.2	116	18	44.5	335.7	540.3
13	NANCE EXP. CO	27	24	10	W5	51	4	14.3	115	17	46.9	208.5	335.6
		27	24	10	W5	51	4	15.6	115	18	.9	208.7	335.9
14	FRONTIER GEO	18	27	2	W5	51	18	46.8	114	15	30.2	168.6	271.4
15	IMPERIAL OIL	9	23	29	W4	50	56	10.9	114	4	24.2	154.4	248.5
		9	23	29	W4	50	56	10.9	114	4	14.8	154.3	248.3
16	NAT. GEOPH. CO	12	30	4	W5	51	33	32.9	114	25	21.5	181.3	291.7
17	GEN. GEOPH. CO	19	49	19	W4	53	15	1.4	112	45	23.6	211.6	340.5
18	GEN. GEOPH. CO	4	52	23	W4	53	27	16.3	113	10	20.1	232.1	373.5
19	UNIV OF MICH	3	13	14	W4	50	3	43.9	111	50	47.0	61.5	99.0
20	GEOL. SURVEY	9	31	25	W2	51	38	59.2	105	27	38.3	238.9	384.5
	OF CANADA	9	31	25	W2	51	38	59.2	105	27	57.0	238.7	384.2
21	UNIV ALBERTA	12	15	9	W4	50	14	38.2	111	6	49.1	27.4	44.0
22	UNIV ALBERTA	33	18	19	W2	50	34	17.1	104	32	54.8	267.9	431.1
23	UNIV ALBERTA	33	13	31	W1	50	8	59.3	101	39	5.9	397.9	640.4
24	UNIV ALBERTA	22	10	19	W1	49	51	18.0	99	57	59.6	475.3	764.9
25	UNIV ALBERTA	0	11	3	W1	49	53	4.1	97	24	4.5	588.4	947.0
0	SHOT POINT	4	18	5	W4	50	29	20.0	110	37	38.0		

* S T R M DENOTE SECTION, TOWNSHIP, RANGE, AND MERIDIAN.



LOCATION OF RECORDING PARTIES.

100 TON TEST 3 AUGUST 1961.

FIG. 11.

4.3 Amplifiers and Recording Equipment

Since the air-waves from a large explosion are predominantly in the infrasonic range, it is important to use equipment with as low a frequency response as possible. It is no easy matter to obtain such equipment in sufficient quantity to record what may well be only a "one-shot" affair. Most commercial exploration parties would be expected to use only their regular field recording systems, i.e. standard refraction amplifiers, and high-speed recording cameras. Low frequency refraction amplifiers would be best, of course, but the higher frequency reflection amplifiers can be made to perform quite well with the low cut filters out. At any rate, it seems likely, as Mr. Junger has pointed out, that this was the first occasion anywhere when so much high frequency seismic equipment was used to record the air-waves from a large explosion.

The commercial exploration companies could obviously not be expected to devote excessive amounts of time and money to an operation for which they would not be reimbursed in any way. Nevertheless, all recording parties took considerable pains to get the best possible results under somewhat unusual circumstances, and some went to great lengths to modify their equipment, particularly cameras.

Since the field parties used a great variety of equipment it was rather difficult to compare and correlate the various records. A few parties provided relative calibration data

for their recording systems, but it was not possible to reduce this information to a common standard. The instrumentation of the University of Alberta stations suffered by necessity from similar handicaps. The equipment used by the five University field parties and the Edmonton monitor station is listed in Appendix III .

4.4 The EVP-5 Detector

The standard pressure detector used by nearly all parties was the EVP-5 Electro-Tech pressure phone. This detector was readily available from the Calgary Branch of the Electro-Technical Labs., at a monthly rental fee of ten dollars. The EVP-5 is a diaphragm-actuated crystal pressure detector with an integral impedance matching transformer and damping resistor (see Fig. 12). The detector has an output impedance of 500 ohms, and its sensitivity is high enough to eliminate the need for pre-amplification ahead of standard seismic amplifiers. The response curve of the EVP-5 is shown in Fig. 13. It has a resonant frequency of about 8 cycles per second; its response is essentially flat for frequencies above 15 cycles per second, restricting its use to the detection of air-waves returned from the meso-incline. Such limitations detract but little from the over-all merits and utility of the EVP-5; it is also readily available, and obviously designed for work in the field, of rugged construction and yet light in weight.

S P E C I F I C A T I O N S

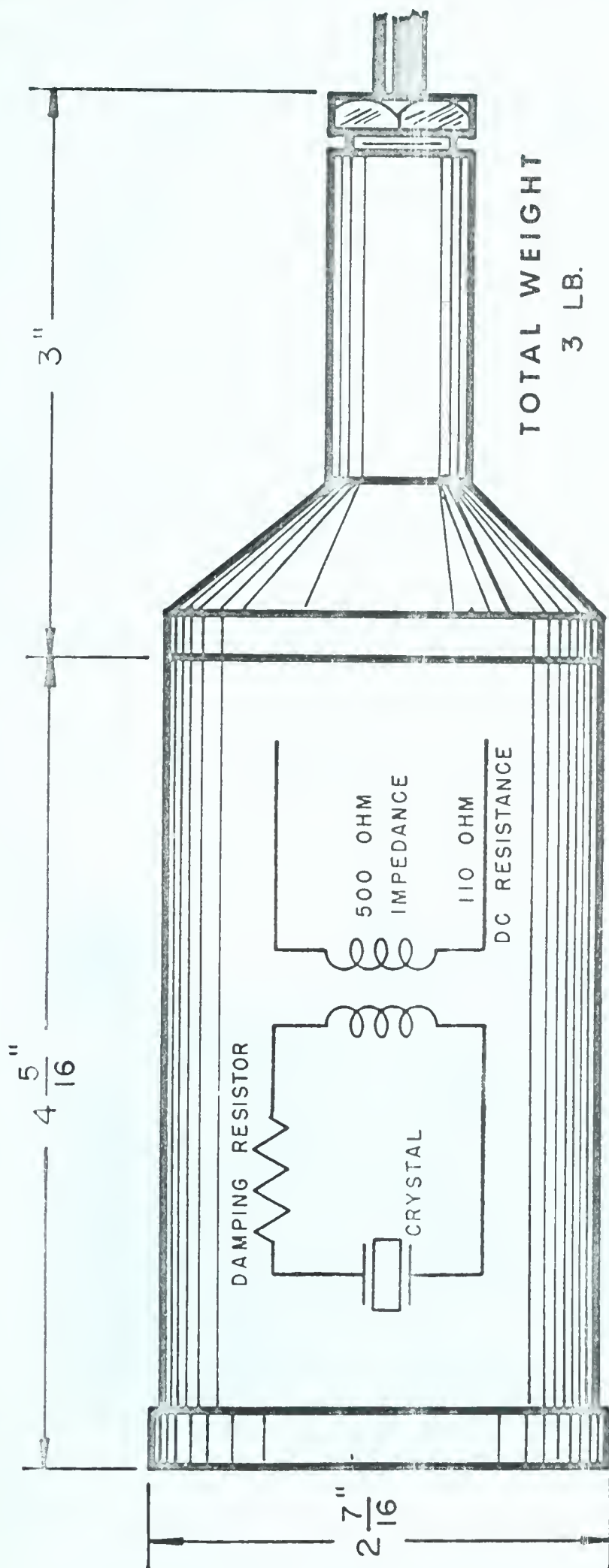


Fig. 12. The EVP-5 ELECTROTECH Pressure Sensitive Detector.

(Actual Size)

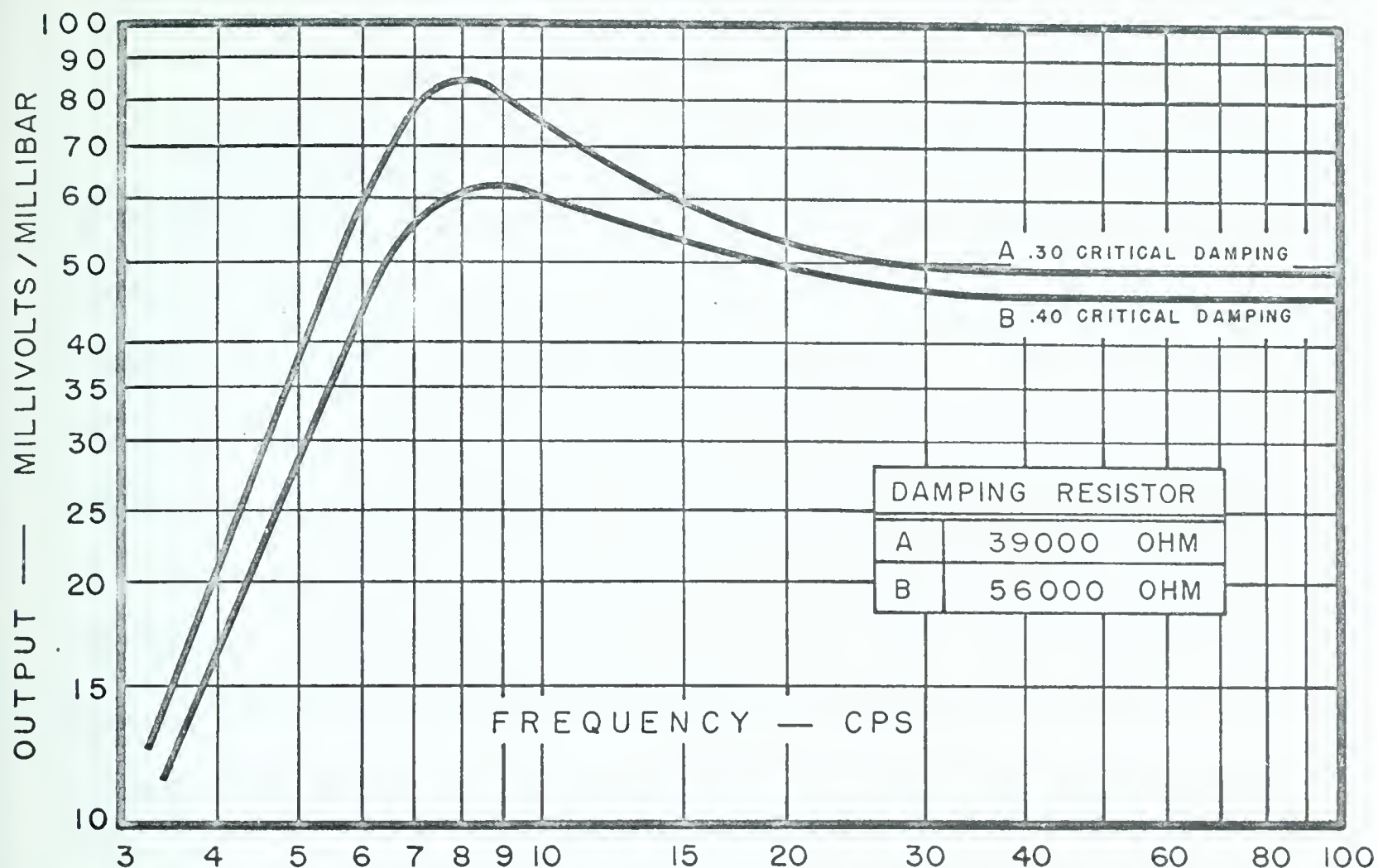


Fig. 13 : Frequency Response Curve of EVP-5.



Fig. 14 : Field Testing of Equipment.

EVP-5 pressure phones in foreground. An S-36 geophone stands in the test rack. A micro-barograph is cleverly hidden behind a big back.

Individual pressure detectors may be tested for operation by blowing into the vents at the base of the shell. A spread of detectors may be tested most readily by means of a shot gun blast or other small explosion. The speed of sound in air at the recording site may be conveniently obtained in the same manner. The EVP-5 will detect the blast of a 12 gauge shot gun over distances of up to one mile, depending on prevailing wind and turbulence conditions. Fig. 15 shows a portion of a record of a shot gun test firing made under conditions of light wind and little turbulence; the shot was fired halfway between the third and fourth detectors of a linear array one mile in length. Fig. 16 shows a test record from a group of detectors placed side by side, some 60 yards from the "shot point"; the experimental arrangements are shown in Fig. 14.

Since the EVP-5 is a non-directional pressure device, its exposure is not too critical. It is, however, quite sensitive to wind fluctuations, and should be afforded some protection from excessive gustiness. The wind noise may be reduced substantially by placing the detector in a shallow trench or other shelter, preferably with the diaphragm end facing downwind.

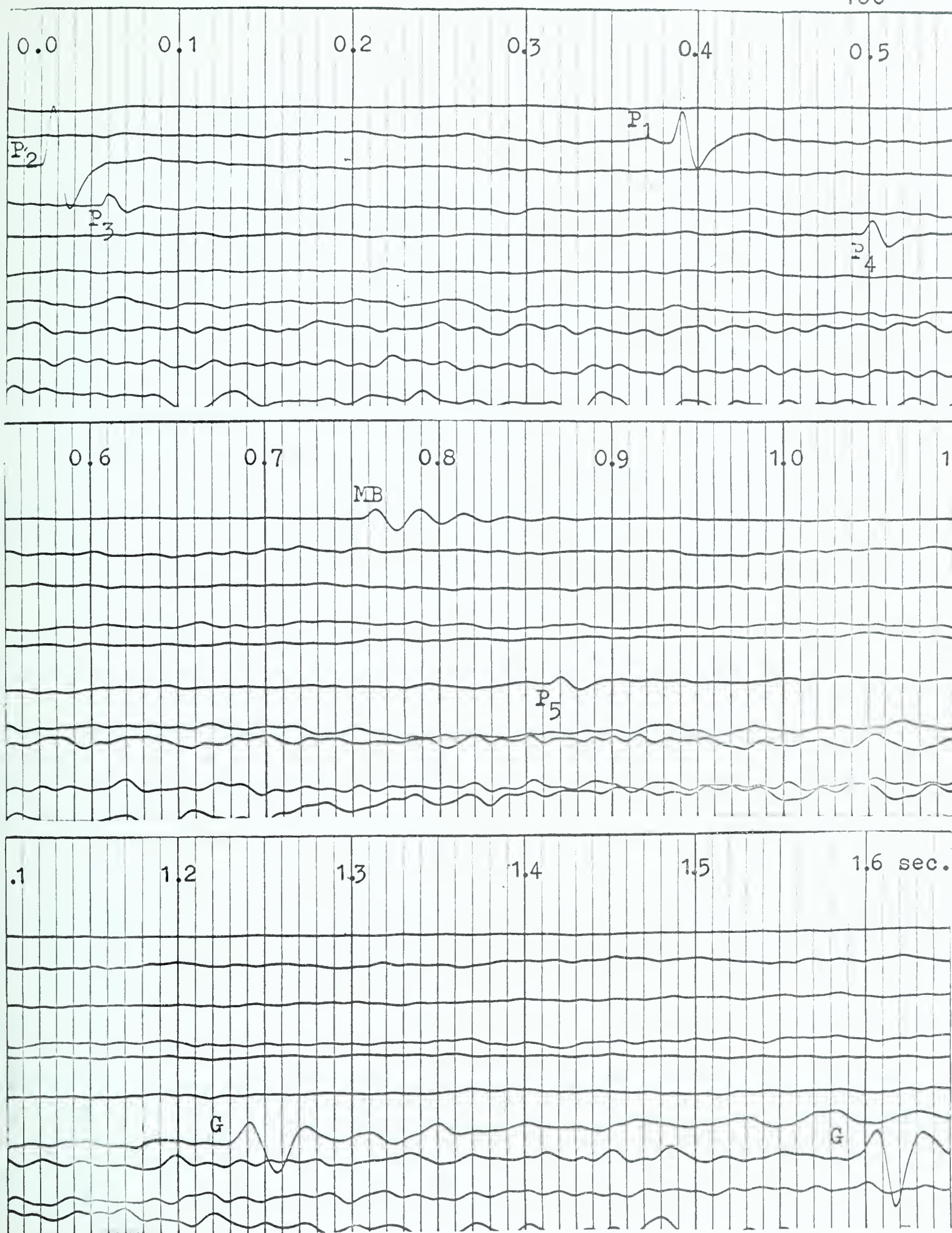


Fig. 15 : Record of a Spread of Detectors.

The test shot was fired between EVP-5 pressure phones P₂ and P₃. MB denotes the microbarograph trace, and the G's denote S-36 TI geophones.

Detector spacing: MB to P₁ @ 125 m; P₁ to P₂ @ 125 m; others at 137.5, 137.5, 125, 125, and 122.5 meters.

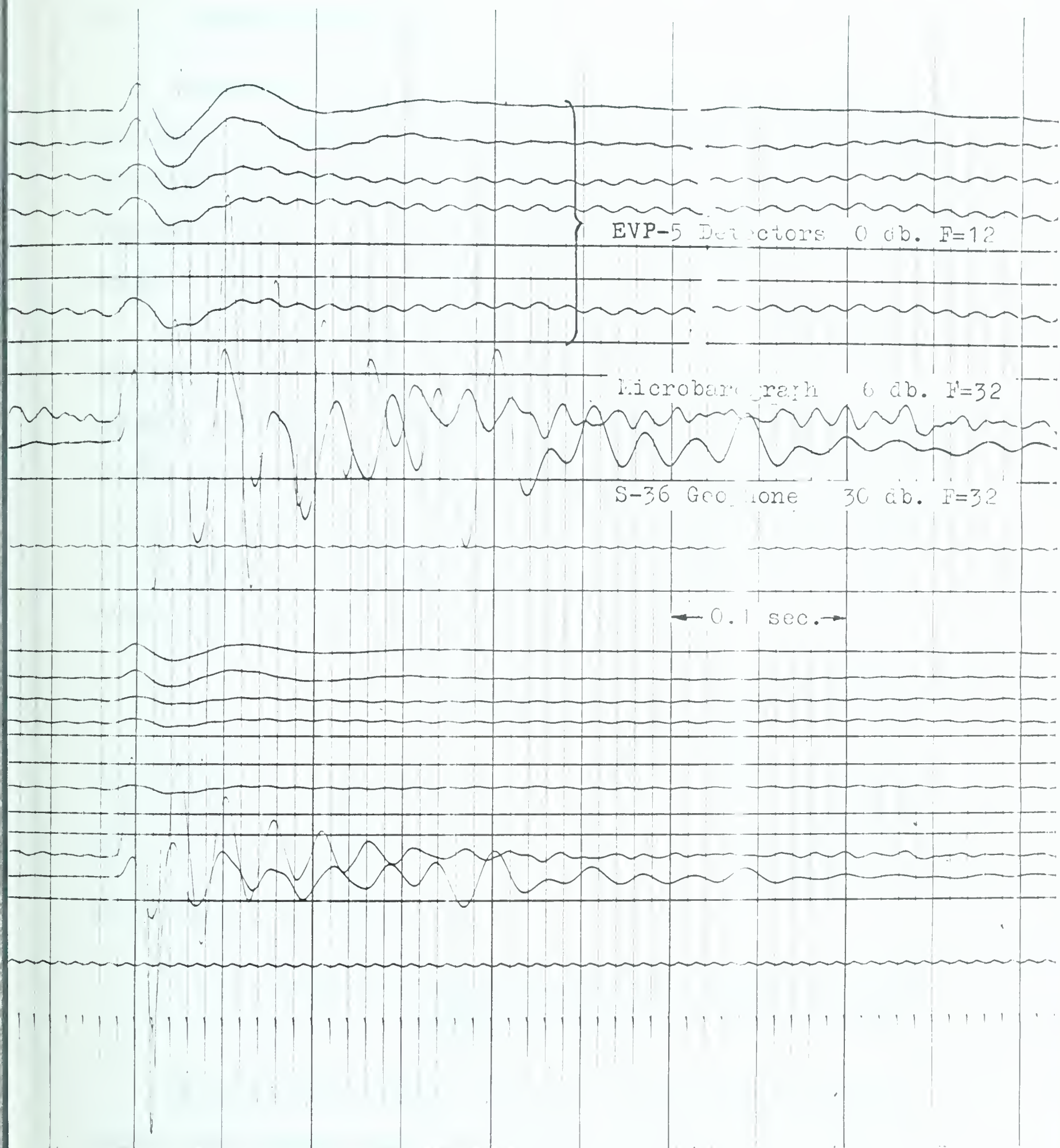


Fig. 16. Test Record of EVP-5 Detectors, Microbarograph, and S-36 TI Geophone. (Shot Gun Blast at 60 yards)

4.5 The Microbarograph

Eight microbarographs were to be built and distributed to the most distant stations, located mainly between Regina and Winnipeg, but delays in shipment of the transducers forced drastic changes in the initial plans, and only two instruments could be made ready in time for the explosion.

The microbarographs were designed by Dr. Garland for operation with Statham transducers PM5 TC and PM197 as the sensing elements. The design is based on R-C filter network theory applied to a mechanical analogue*: an enclosed volume of air v communicating with the free atmosphere (at pressure p) by means of a capillary leak of 'resistance' R . The electro-mechanical analogue is shown below.

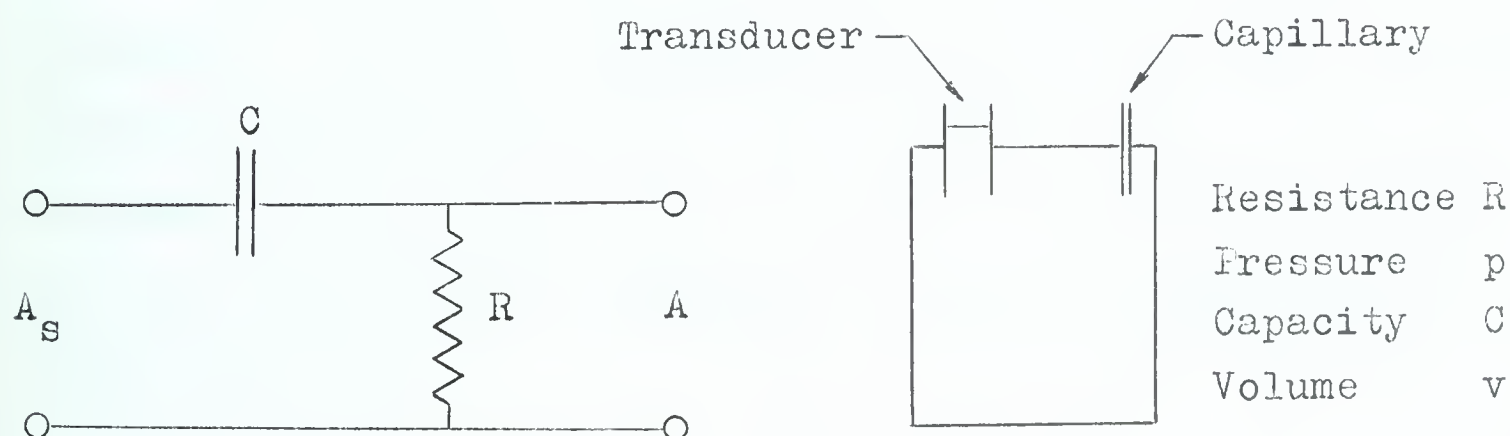


Fig. 17: Electro-mechanical analogue of high pass R-C filter and microbarograph pressure chamber.

If A_s is the voltage input and A the output voltage across the R-C filter, the transfer function is evidently

$$\frac{A}{A_s} = \frac{iR}{i\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}, \quad (4-1)$$

*See W. G. Van Dorn, A Low Frequency Microbarograph, J. Geoph. Res., Vol. 65, 3693-98 (1960)

or, in terms of the period T

$$\frac{A}{A_s} = \left[1 + \frac{T^2}{4\pi^2 R^2 C^2} \right]^{-\frac{1}{2}} \quad (4-2)$$

Designing the filter so that one-half the power is transferred at $T = 20$ sec (3db down) requires that

$$\frac{T^2}{4\pi^2 R^2 C^2} = 1 \quad (4-3)$$

or $(RC)^2 \approx 10 \text{ sec}^2$. The time constant of the system is thus about 3.2 sec.

The long-period natural barometric changes are generally of much greater amplitude than sound waves. If a microbarograph is to be made sensitive and stable it is necessary to minimize the effects of the long-period components as much as possible. The microbarograph must thus be designed to operate as a high pass filter, and provisions must be made to equalize natural barometric differences between the volume of air contained in the chamber of the instrument, and the free atmosphere.

The following one-to-one correspondence may be readily established between electrical and mechanical quantities:

electric charge	q	:	volume of gas	v
potential difference	V	:	pressure of gas	p
electric current	i	:	rate of flow	Q

The analogy is easily extended to capacity and resistance: since $q = VC$, and $V = Ri$ it follows that $v = pC$, and $p = RQ$. The capacity and resistance of the mechanical system are thus seen to be

$$C = v/p \quad (4-4)$$

$$\text{and} \quad R = p/Q \quad (4-5)$$

The long period cut-off of a microbarograph is determined by the resistance of the air leak and the capacity of the pressure chamber. The microbarographs constructed by the University use 10 gallon oil drums as pressure chambers. Since 10 gallons $\approx 45 \times 10^3 \text{ cm}^3$, and the atmospheric pressure is of the order of 10^6 dynes/cm^2 , the capacity of such a chamber is $45 \times 10^{-3} \text{ cgs}$. The resistance and dimensions of the capillary leak are obtained from the time constant and Poiseuille's Law in the form

$$R = \frac{Q}{p} = \frac{8\mu\ell}{\pi r^4} \quad (4-6)$$

where μ is the viscosity of air*, r the radius and ℓ the length of the capillary.

With $RC = 3.2 \text{ seconds}$, the resistance of the system is 70 cgs; the radius of the capillary is 0.08 cm, and its length 5.6 cm. The transfer function of this system is plotted in Fig. 18. on the next page.

*about $2 \times 10^{-4} \text{ poise}$.

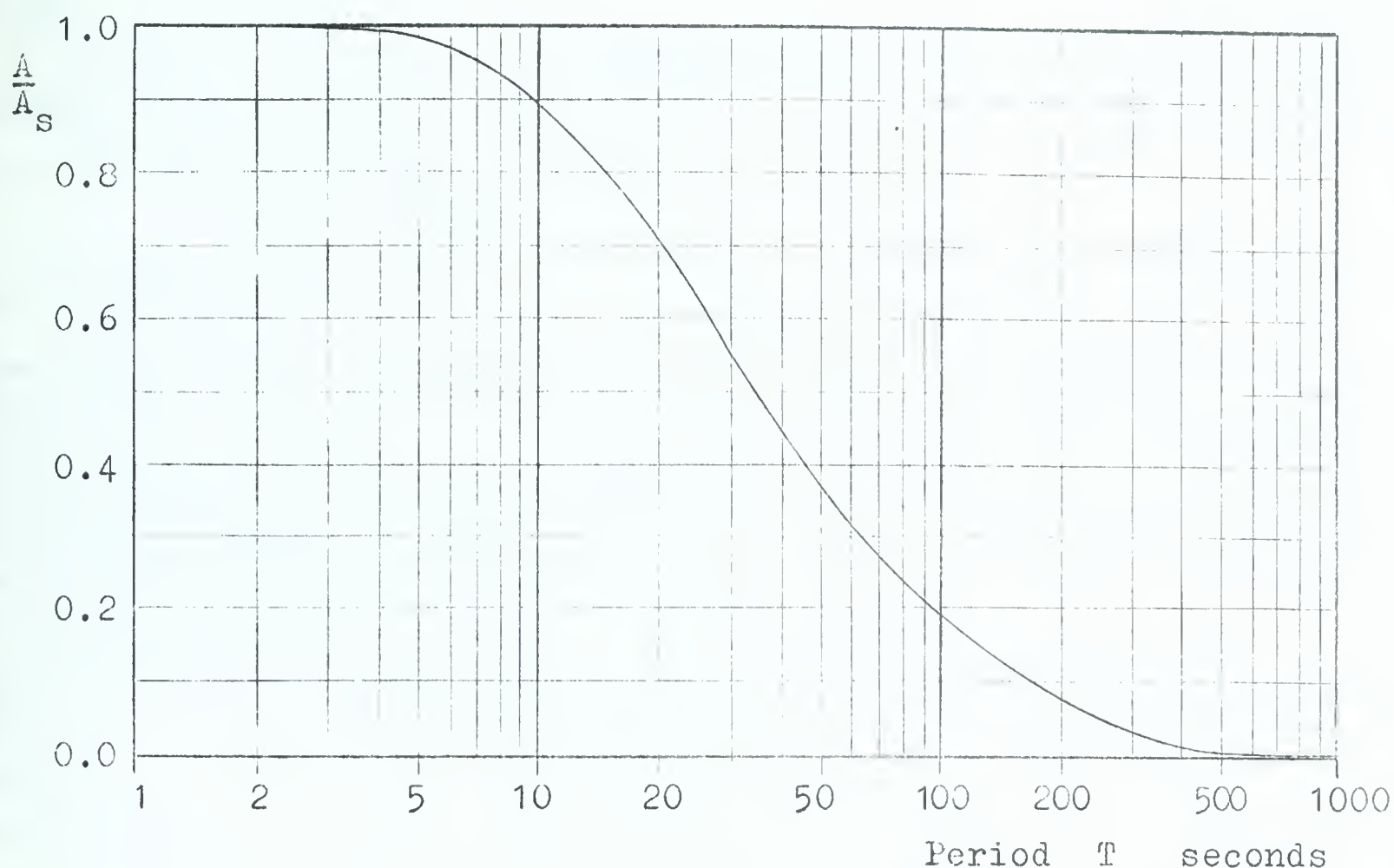
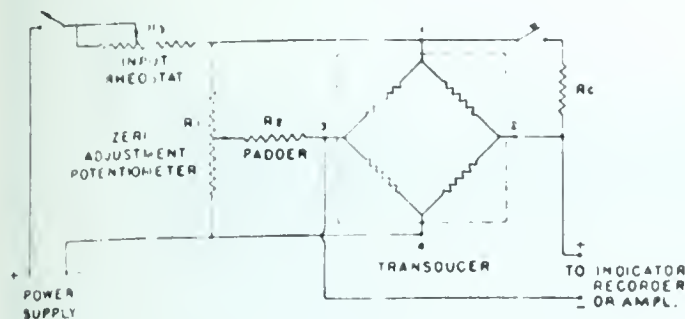


Fig. 18 : Transfer Function of Microbarograph.

A circuit diagram, dimensions and other specifications of the Statham PM5TC differential pressure transducer are shown in Fig. 19. The PM197 pressure transducer has similar characteristics, but it is more sensitive, and has a pressure range of 0.01 pounds per square inch. Photos of two microbarograph assemblies using the PMC197 and PM5 TC are shown on Page 113. Two records of the PMC197 microbarograph are included in the field test records reproduced above (Figs. 14 and 15).

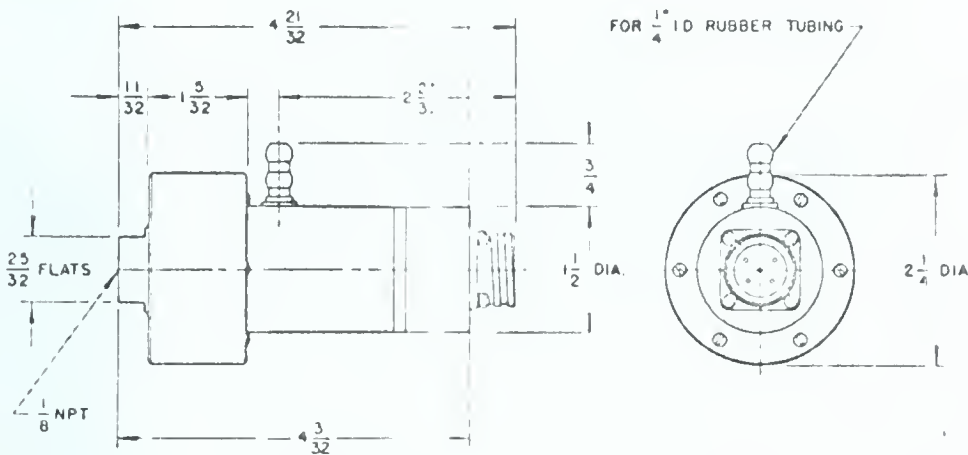
Model PM5TC

Differential Pressure Transducer



TYPICAL TRANSDUCER CIRCUIT SCHEMATIC

- Transducer terminals 1 and 4 == power input • Transducer terminals 2 and 3 == output to receiving instrument
- = energizing voltage (AC or DC), as rated for transducer
 - = calibrating resistor
 - = transducer bridge resistance in ohms measured between terminals 2 and 3
 - = transducer bridge resistance in ohms measured between terminals 1 and 4
 - = wire-wound potentiometer with a value between 20,000 and 30,000 ohms
 - = wire-wound resistor with a value of about 100R for full scale zero adjustment range
 - = wire-wound rheostat for transducer input voltage adjustment (sensitivity)



RANGES	±0.15 through ±0.7 psid.
PRESSURE LIMITS	±3.5 psid both positive and negative directions
POSITIVE PRESSURE MEDIA	Non-corrosive gases and/or fluids.
REFERENCE PRESSURE MEDIA	Dry, non-corrosive gases.
MAXIMUM INTERNAL PRESSURE	15 psig.
TRANSDUCTION	Resistive, balanced, complete unbonded strain gage bridge.
NOMINAL BRIDGE RESISTANCE	350 ohms.
EXCITATION	From 11 to 14 volts. See Selection Table.
FULL SCALE OUTPUT	±20 to ±32 millivolts.* See Selection Table.
RESOLUTION	Infinite.
NON-LINEARITY & HYSTERESIS	Less than ±0.5% of full scale output.
AMBIENT TEMPERATURE LIMITS	—100° to +275°F.
THERMAL SENSITIVITY SHIFT	Less than 0.01%/°F. from —65° to +250°F.
THERMAL ZERO SHIFT	Less than 0.01%FS/°F. from —65° to +250°F.
POSITIVE PRESSURE CONNECTION	1/8-27 NPT (internal).
REFERENCE PRESSURE CONNECTION	Fitting for 1/4" ID hose.
ELECTRICAL CONNECTION	Case mounted electrical connector to mate with Cannon WK-4.
IDENTIFICATION	The model designation, serial number and name of manufacturer are permanently marked on each transducer.
WEIGHT	Approximately 11 ounces.

*Open circuit. Unless impedance of receiving instrument is specified, unit is temperature compensated for open circuit operation.

Fig.19. Specifications of STATHAM PM5TC Pressure Transducer.



Fig. 20.

Microbarograph made with
Statham PM5TC Transducer.
Note the capillary leak
on the right.



Fig. 21.

Microbarograph assembly
using FM197 Transducer.
The capillary leak is
again on the right.

4.6 Communications and Timing

Maintenance of adequate communications between dozens of temporary observation stations scattered over several provinces is never an easy matter even under the best circumstances. Since the recorders should be set up in quiet locations, it is usually necessary to move them out into the open country, and away from power- and telephone lines. Once a site has been chosen and the equipment set up, the station is in essence isolated, if not temporarily marooned. This would not be true, of course, for two-way radio links between field stations and a central control, but the costs of a communication network of such proportion would be, in most circumstances, prohibitively high.

The network of stations recording the anomalous airwaves from the 100 ton explosion depended largely on commercial telephone and telegraph communications, and pre-arranged timing procedures. The field parties were simply advised to record at the times appropriate to their locations, and for as long as possible. Simple procedures of this kind work well and present little difficulty if the blast can be set off at the first scheduled time, but any last minute postponements or change in plans will likely be costly and confusing. The first nominal shot time for the 100 ton blast was set for 10:30:00 MST., Aug. 1st, 1961. Since it was possible, however, that the detonation might have to be vetoed on technical grounds in the final seconds of the count-down, it was

necessary to schedule alternate zero times at half hour intervals up to, and including 11:30:00 MST.; any further delays would necessitate postponement of the blast to the following day. To take care of such exigencies it was necessary that the field parties be prepared to record repeatedly, at half hour intervals, until such time as they could be notified that the blast had actually gone off. Every effort had to be made, therefore, to contact the parties as soon as possible after the blast, if waste in time and materials was to be held to a minimum. This problem was all but solved completely through the co-operation of Station CKUA, Radio Alberta. Mr. J. Pinko, the Station Manager, agreed to broadcast a short message immediately after receipt of notification from Suffield that the explosion had been set off. This arrangement worked very well in practice. On the day of the blast, most recording parties operating in Alberta had received confirmation of the shot time within 10 minutes after the initiation of the blast - in many cases well before the arrival of the acoustic wave front. Field parties not within range of CKUA had to be contacted by telephone, however, and not all could be reached in time to prevent wasted effort.

The timing and correlation of field records presented on the whole but few problems. Five of the University of Alberta stations were equipped with Times chronometers, and operated on absolute time. The chronometers were set by tuning in to the 10 AM. MST Dominion Observatory time signal; the sixth

station, operating near Suffield, recorded the timing signals broadcast by WWV. Four of the chronometers had been obtained on loan from the Dominion Observatory at Ottawa, through the courtesy of Mr. Malcolm Bancroft.

Three other field parties were also equipped with chronometers, but only one (at Watrous) had absolute time. All parties not operating on absolute time had made provisions to record regular program material broadcast by any one of three radio stations: CKUA (Alberta), CBX (Alberta) or CBK (Sask.). This procedure consisted simply in recording the audio output of an ordinary broadcast receiver. The output was taken directly from the leads of the speaker voice coil, rectified by a crystal diode, and applied to one of the galvanometers of the recording camera. The recorded trace was found to be of sufficient character to serve as an excellent time marker when compared with audio traces recorded simultaneously at stations operating on absolute time. The Edmonton monitor recorded CKUA, CBX, and the minute and second marks of a Times chronometer. The University station near Regina similarly monitored CBK.

Recorded speech produces generally a more distinctive trace than a musical program, but this difference in character proved to be of little importance in the processing of the records. In the case under study, the broadcast signals were recorded with a variety of equipment of widely differing speed and sensitivity. The traces were thus often super-

ficially very different from each other, but closer inspection led readily to positive correlation and timing of all but one of the records. This single exception only proved the rule, for failure came by accident: the recording crew had tuned their receiver to some "stray" radio station - unmonitored and not identified.

The accuracy attainable with such a simple timing network is surprisingly high. Acoustic arrivals and other events can be readily timed to within ± 5 milliseconds in most cases, and to ± 2 milliseconds or better in high speed records.

The idea of using standard broadcast signals for timing purposes seems to have originated with Dr. Garland. A timing network of this kind suffers, of course, from the disadvantage of operating in an entirely passive manner, and is, therefore, hardly a substitute for a full-fledged two-way radio system. But even so, the passive network has much to recommend it, for it is easy to set up, costs next to nothing, and has the great advantage of working well in actual practice.

4.7 The Determination of Geodetic Distance

Distance between the Suffield test site and the points of the observation network was determined by means of a method due to Richter (1958) who used it to calculate the epicentral distances of earthquakes in the California area. The method takes some account of the sphericity of the Earth, but in practice involves only the computation of the hypotenuse of a

plane right-angled triangle. The sides of this triangle are obtained from the latitude and longitude positions of the points in question, and from two sets of coefficients derived from Clarke's spheroid.

The geodetic distance in kilometers between two points is then simply

$$\Delta = \sqrt{\Delta x^2 + \Delta y^2}$$

where $\Delta x = A \Delta \lambda$

and $\Delta y = B \Delta \varphi$

The coefficients A and B are the lengths in kilometers of one minute of arc of the parallel and meridian, respectively, centered on the mean latitude between the two given points. $\Delta \lambda$ is the difference in longitude, and $\Delta \varphi$ in latitude, of the two points, expressed in minutes of arc. Richter tabulated values of A and B at one minute intervals for the latitude range of California. For the present investigation it was necessary to extend Richter's tables to higher latitudes. A set of tables for use between 46 and 65 degrees is given in Appendix IV. A sample computation follows:

Location of observer	52° 32' 29.7" N	112° 49' 59.4" W
Location of test site	50° 29' 20.0" N	110° 37' 38.0" W
Mean Latitude	51° 30' 54.8" N = 51° 30.91' N	

$$\Delta \varphi = 2^{\circ} 3' 9.7" = 123.16'$$

$$\Delta \lambda = 2^{\circ} 12' 21.4" = 132.36'$$

From Tables (Appendix IV)

$$A = 1.157013 \text{ km/min.} \quad B = 1.854331 \text{ km/min.} \quad ;$$

$$\text{hence } \Delta x = A \Delta \lambda = 153.18 \text{ km}$$

$$\Delta y = B \Delta \varphi = 228.45 \text{ km}$$

$$\text{and } \underline{\Delta = 275.0 \text{ km}} .$$

Richter's method is strictly applicable only to geodetic distances of the order of a few hundred kilometers, but the error is only 0.1 km in 500 km, and a good deal less for two points close to the same meridian or parallel. Such accuracy is quite sufficient for most purposes, and often the best that can be reasonably expected in work of this kind. At any rate, the accuracy of any such computation must finally rest on the precision with which the geodetic co-ordinates of the end points are known. A knowledge of the co-ordinates to within say 1 second of arc would be obviously desirable, but uncertainties of 5 to 6 seconds may be acceptable at times. In the present work the locations of most recording sites were known to better than two seconds. The Technical Office, Department of Lands and Forests, the Province of Alberta, was most helpful in providing the exact positions of the majority of sites in Alberta; these positions, determined by the Geodetic Survey of Canada, were usually known with an accuracy of ± 0.01 seconds. The position of sites in other provinces, or of sites not surveyed with high precision were calculated to better than 1 second from data published by the Dominion of Canada Land Survey.

4.8 Weather Conditions

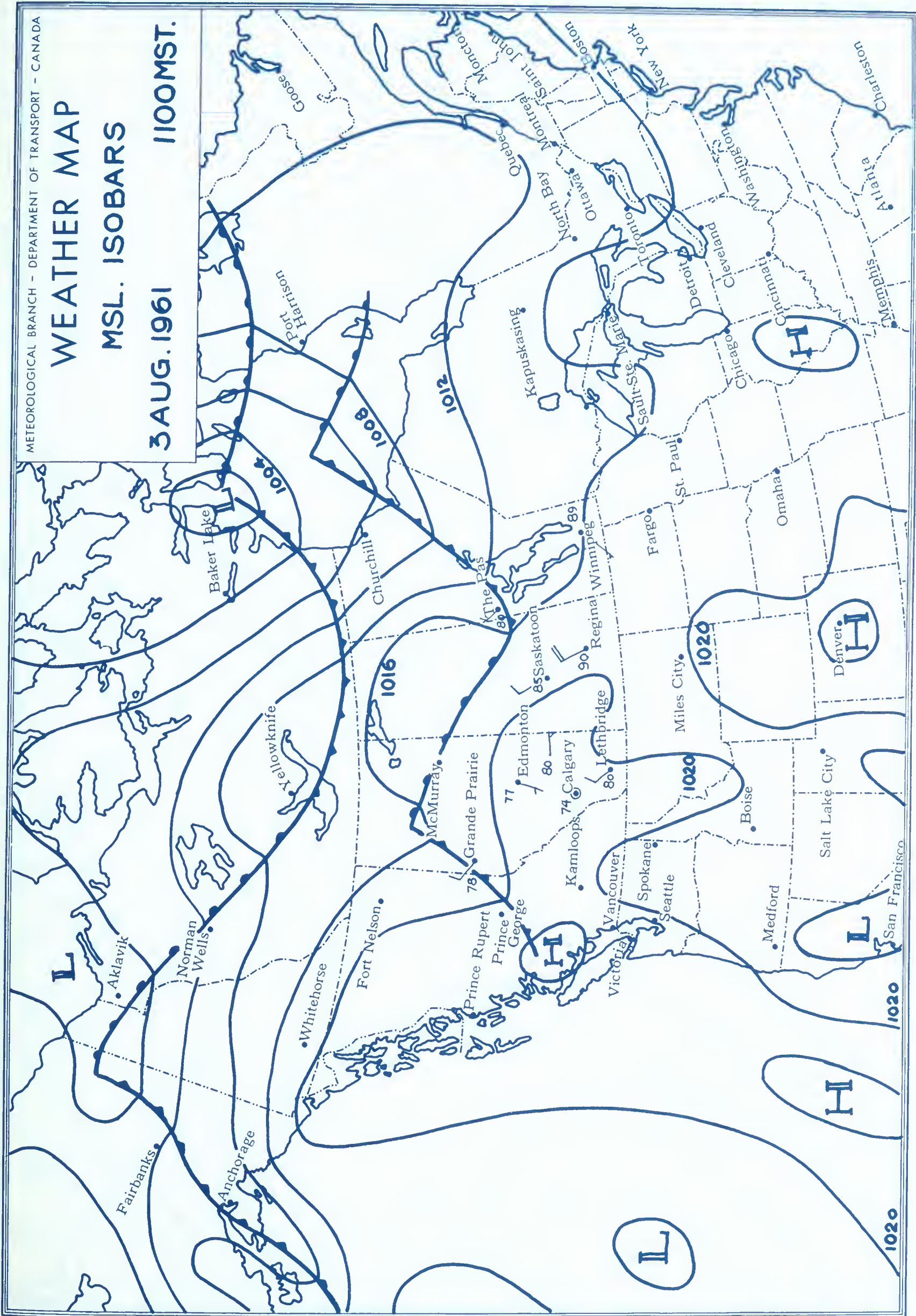
The atmospheric conditions prevailing at the time of a large experimental explosion are of prime concern for a number of reasons. Discounting the nuisance of inclement weather, and possible damage to instruments by rain and wind, it is evidently desirable that temperature and wind distribution be such as to minimize the effects of low level blast propagation. In settled areas it may be necessary to restrict test explosions on this account to times when the atmosphere is free of low level temperature inversions and strong vertical wind shears.

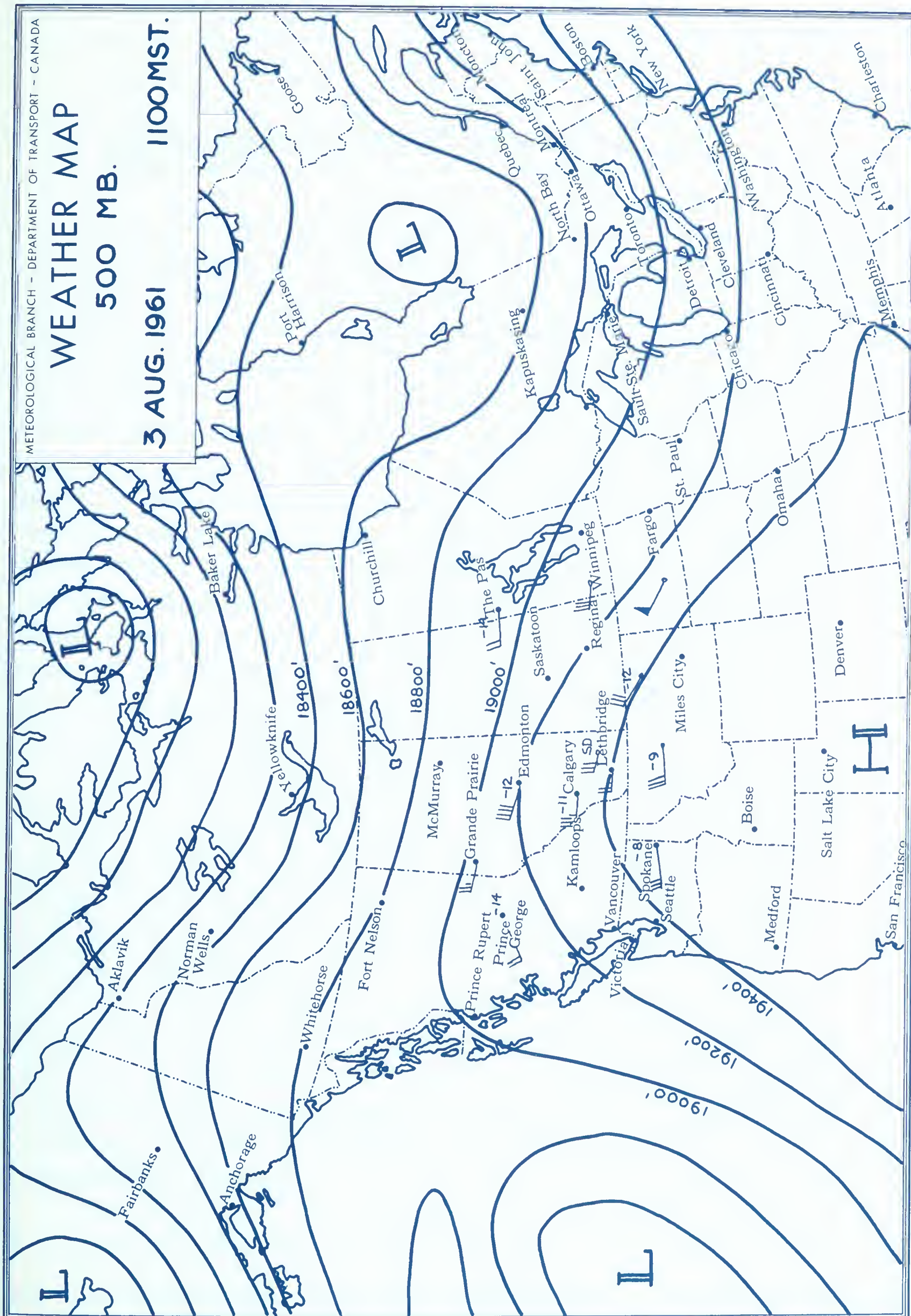
Temperature inversions are usually most pronounced in winter, and the early hours of the morning. The diurnal wind changes are also such that the minimum surface wind occurs normally in the early morning, but since the upper winds do not weaken to the same degree, the prevalently positive wind shear may be enhanced considerably. The combined effects of temperature and wind refraction will tend to concentrate large amounts of energy into relatively narrow zones downwind from the explosion. If serious blast damage is to be avoided it is necessary, therefore, to schedule large explosions in the summer months, or at least at times when temperature and wind conditions are likely to be favorable. The safest time in this respect is evidently mid-afternoon, when surface wind and temperature are at their maximum. But maximum wind and tempera-

ture also mean maximum turbulence and, from the point of view of seismic recording, a high level of background noise. If the noise is to be held to tolerable levels it may be necessary to compromise and choose an optimum time some one or two hours before noon.

The 100 ton blast was set off at 10:30:01 AM. under almost ideal conditions. Skies were clear, and temperatures had climbed above 80°F. in most parts of western Canada. Fairly brisk (20-25 mph) northerly winds had developed in Saskatchewan, but the circulation was generally weak and disordered in Alberta. Simplified charts of the prevailing weather are given on the next two pages. The first chart, Fig. 22, shows the surface pressure pattern and the observed winds and temperatures at a few weather stations in the test area.

The next chart, Fig. 23, is a contour map of the 500 millibar surface, showing in essence the circulation pattern at a height of about 18,000 feet. This chart serves to show that, for the purpose of ray computations, the atmosphere may be considered to be a horizontally stratified, uniform medium. This assumption is well justified in this particular case: it is seen that the northwesterly wind field over the test area is remarkably uniform, and quite devoid of abrupt changes in flow, or other disturbing features. This is borne out also by the radio-sonde balloon observations to be given later, in Chapter V.





4.9 A Brief Résumé of the Anomalous Sound Studies made in 1962.

The sound propagation studies of 1962 were all carried out in connection with one of the major University of Alberta projects of the 1962 summer season, the seismological determination of the crustal thickness in Southern Alberta. Since this investigation has been described in great detail by Weaver (1962), it will be necessary here to recount only briefly those phases of the work concerned directly with anomalous sound propagation. These studies centered specifically on three surface-based explosions set off at the Suffield test range in August and September. The largest of these tests, on August 15th, involved the detonation of 20 tons of TNT. This test yielded excellent records of anomalous sound waves and induced ground waves. It was monitored not only by the two University of Alberta recording trucks, but also by recording parties from B.A. Oil Co. of Calgary, and the General Geophysical Co. at Edmonton.

The two other explosions, of 5 tons each, were set off on August 1st and September 10th. Good records were obtained on August 1st, but the September test yielded little useful information. It produced, at best, a negative result of sorts, for it turned out that the recording trucks were too close to the site of the explosion, and just within the acoustic shadow zone.

The locations and distances of the recording sites are listed below in Table VI, and shown also on a map of Alberta, Fig. 24, on page 126. Four charts of the prevailing weather conditions have been relegated to Appendix V; these will be referred to later, as required, in discussion of the experimental results.

LOCATIONS OF RECORDING SITES AND DISTANCES FROM SHOT POINT.

SUFFIELD EXPERIMENTAL STATION 5 TON TEST AUGUST 1, 1962

RECORDED BY	LOCATION				LATITUDE			LONGITUDE			DISTANCE	
	S	T	R	M	DEG	MIN	SEC	DEG	MIN	SEC	MILES	KM
U OF A TR1	35	17	7	W4	50	29	6.3	110	50	11.2	11.2	18.0
	35	17	7	W4	50	29	6.3	110	51	23.4	12.1	19.4
U OF A TR2	30	18	24	W4	50	33	28.7	113	16	57.0	119.1	191.7
	30	18	24	W4	50	33	28.7	113	18	9.3	120.0	193.1
SHOT POINT	29	17	5	W4	50	28	26.0	110	35	0.0		

SUFFIELD EXPERIMENTAL STATION 20 TON TEST AUGUST 15, 1962

RECORDED BY	LOCATION				LATITUDE			LONGITUDE			DISTANCE	
	S	T	R	M	DEG	MIN	SEC	DEG	MIN	SEC	MILES	KM
U OF A TR1	36	17	28	W4	50	29	6.3	113	42	58.7	138.2	222.4
	36	17	28	W4	50	29	6.3	113	41	46.5	137.3	221.0
U OF A TR2	5	18	2	W5	50	29	51.0	114	13	48.9	160.8	258.8
	5	18	2	W5	50	29	59.0	114	13	48.9	160.8	258.8
	5	18	2	W5	50	29	59.0	114	14	48.4	161.6	260.0
BA OIL CO.	8	18	29	W4	50	31	11.9	113	56	48.2	148.3	238.7
	8	18	29	W4	50	30	51.1	113	56	48.2	148.3	238.7
	8	18	29	W4	50	30	51.1	113	56	21.3	148.0	238.2
GEN. GEOPH. CO	4	52	23	W4	53	27	16.3	113	9	58.3	233.7	376.2
	4	52	23	W4	53	27	16.3	113	10	20.1	233.9	376.4
SHOT POINT	29	17	5	W4	50	28	26.0	110	35	0.0		

SUFFIELD EXPERIMENTAL STATION 5 TON TEST SEPTEMBER 10, 1962

RECORDED BY	LOCATION				LATITUDE			LONGITUDE			DISTANCE	
	S	T	R	M	DEG	MIN	SEC	DEG	MIN	SEC	MILES	KM
U OF A TR1	7	18	22	W4	50	30	51.1	113	0	30.7	90.6	145.8
	7	18	22	W4	50	30	51.1	113	1	43.0	91.4	147.2
U OF A TR2	33	17	26	W4	50	29	7.0	113	30	39.1	112.2	180.6
	33	17	26	W4	50	29	6.3	113	29	27.3	111.3	179.2
SHOT POINT	23	15	8	W4	50	16	10.0	110	59	42.0		

* S T R M DENOTE SECTION TOWNSHIP, RANGE, AND MERIDIAN.

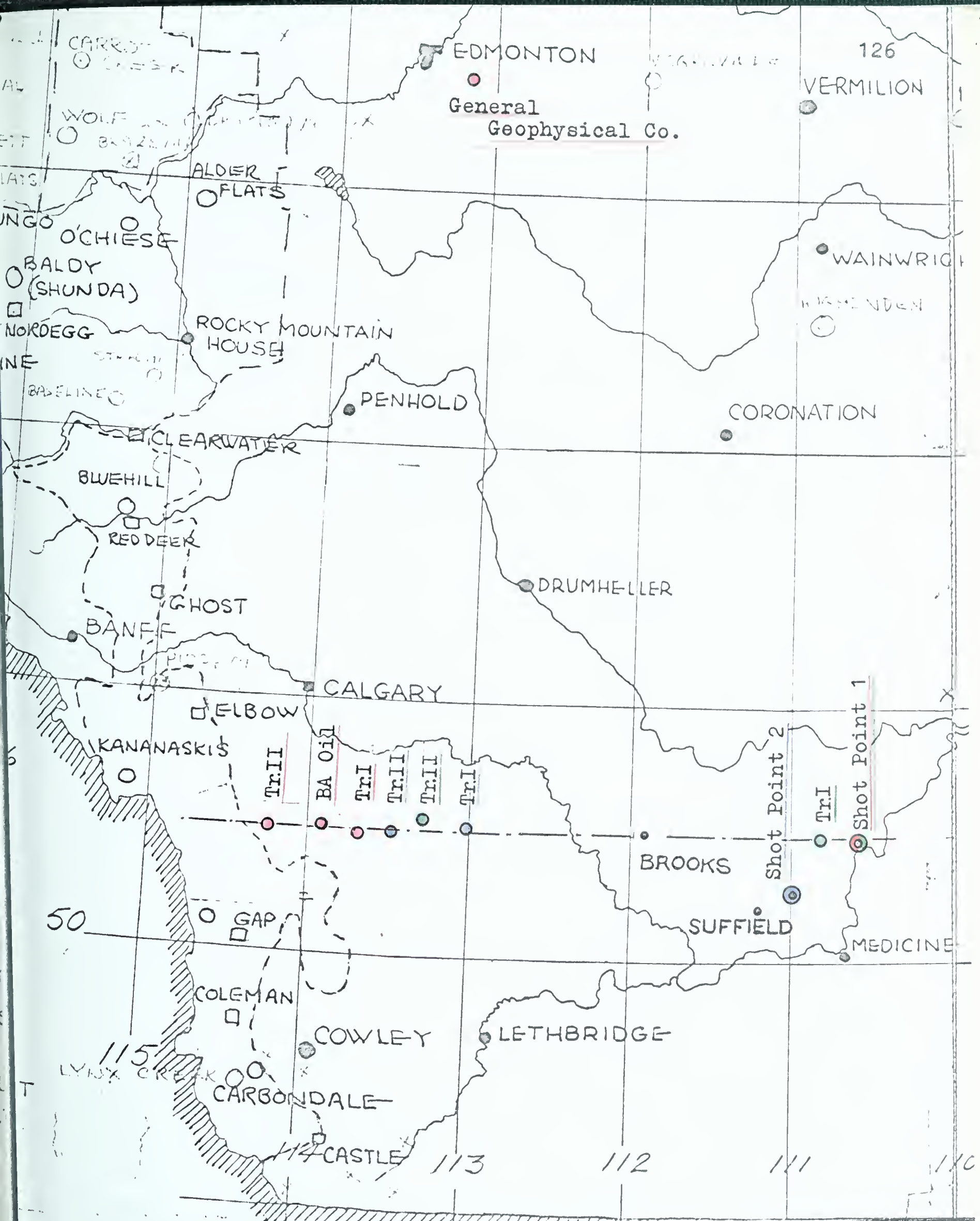


Fig.24

LOCATION OF RECORDING PARTIES.

- 5 Ton Test 1 August 1962.
- 20 Ton Test 15 August 1962.
- 5 Ton Test 10 September 1962.

V. THE INTERPRETATION OF THE RECORDS

5-1. Introduction

The seismic records and other experimental data were interpreted largely from two points of view: geometric acoustics, and spectrum analysis of anomalous waves. Since the 100 ton explosion was the first important test of this kind in Canada, it was only natural that attention should have been given, at least initially, to propagation studies carried out in what might well be called by now the conventional or even classical manner. It is this approach which leads most directly to the determination of wind- and temperature distribution in the upper atmosphere beyond the reach of sounding balloons.

As a first approximation, the effect of the wind was neglected. Ray paths and travel times were computed for various assumed temperature distributions, and the resulting plot of travel time curves was compared with the observed curves. When it had become apparent that such models were inadequate to account for the observed pattern of seismic arrivals, these simple efforts at curve matching were abandoned, and more realistic models were developed. Nevertheless, these preliminary computations served later as valuable checks in the programming and testing of more complicated models. Simple models were used, furthermore, to compute a set of rays for the temperature distribution of the proposed standard atmosphere (extended to a height of 700 km) and for

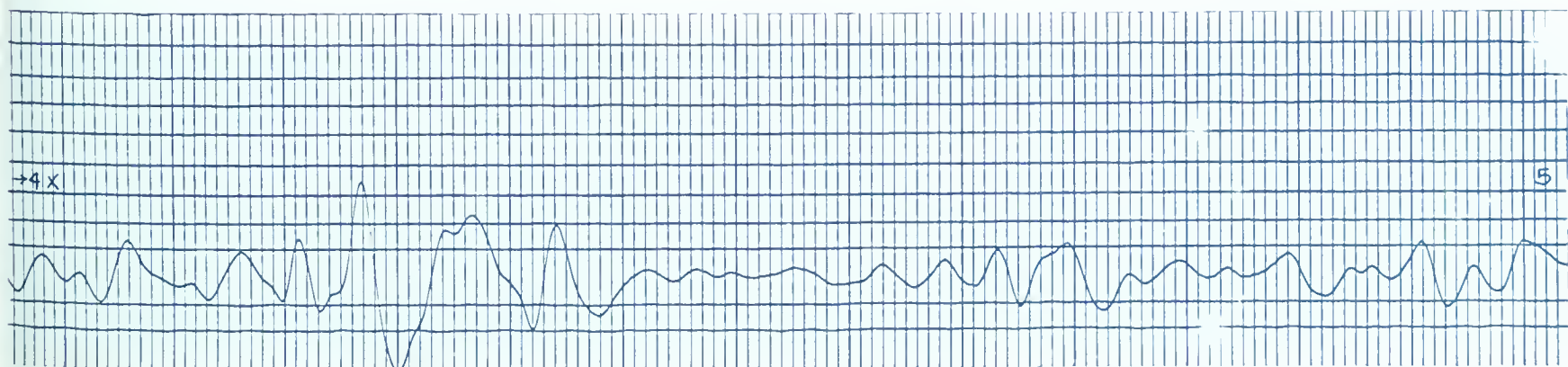
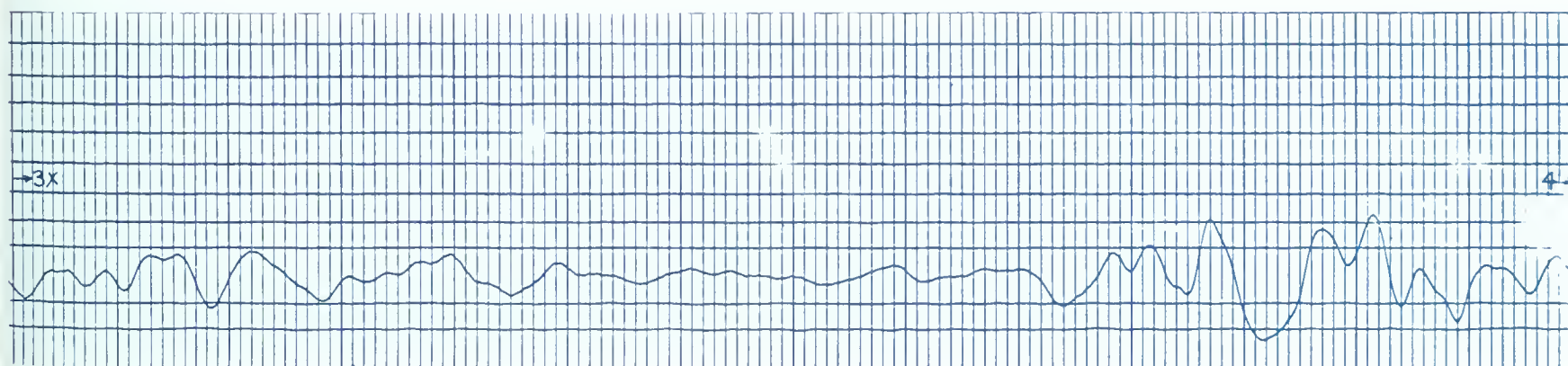
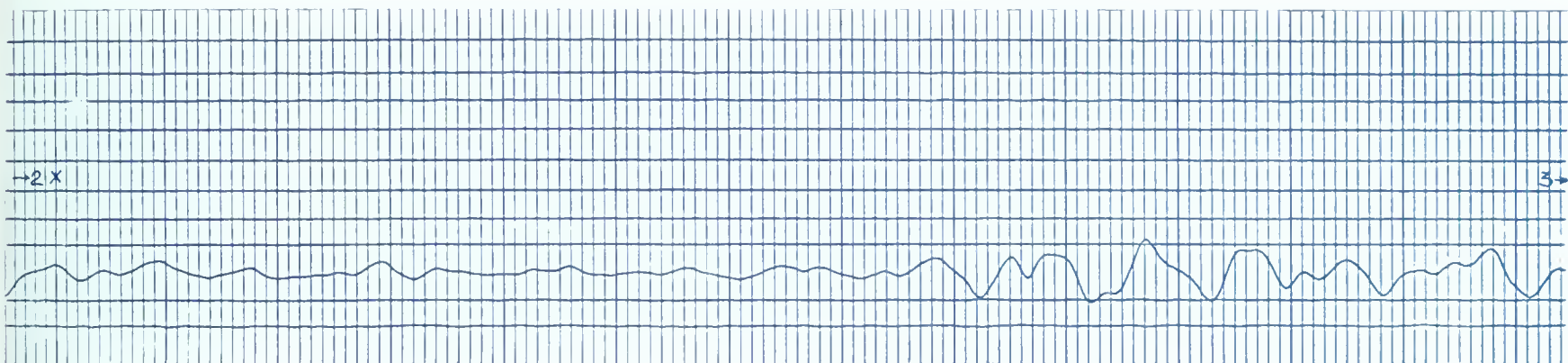
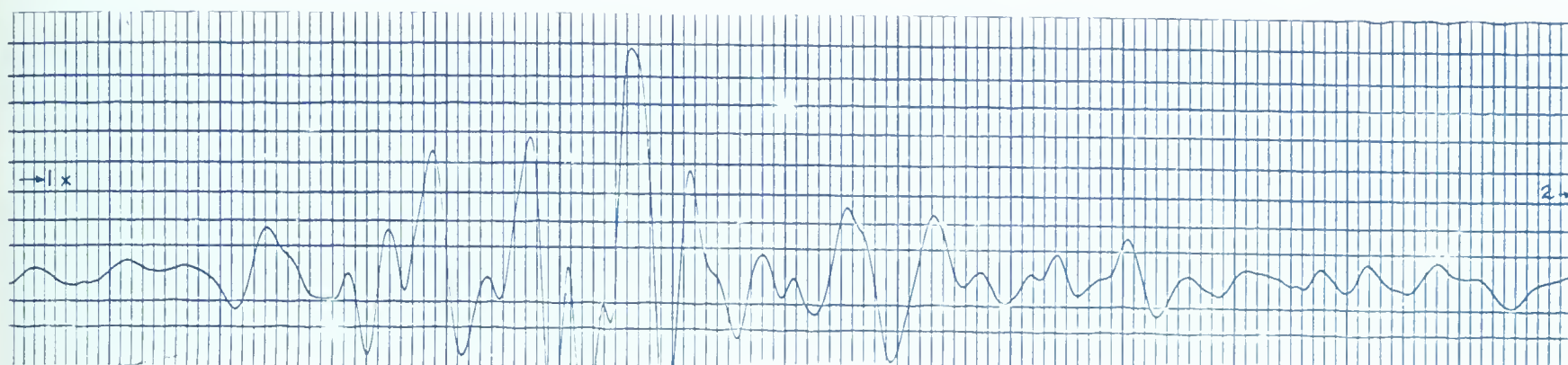
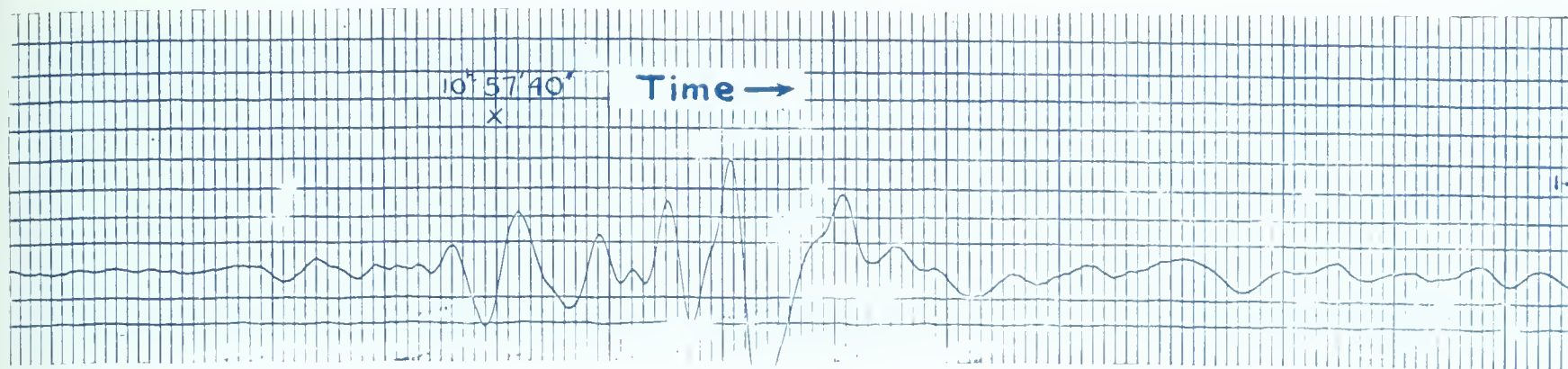
ray paths in an atmosphere with sloping layers above the tropopause.

The second main interest centered on the analysis of the frequency spectrum of the anomalous sound waves. All records containing signals suitable for spectrum analysis were examined in this manner. Most of the records were digitized "by hand", by measuring the amplitude of the recorded signal to the nearest one hundredth of an inch, at intervals of five or ten milliseconds. More recently, several records were processed by means of a newly developed and much faster electro-mechanical digitizing unit. The actual analysis of the digital data was carried out on the IBM 1620 at the University of Alberta Computing Centre.

5-2 The Records

Shown on the next few pages are facsimiles of the more important seismic records obtained from the 100 ton blast of 1961, and the pertinent tests of 1962. Only those records are presented which contain significant and positively identified arrivals, correlated with absolute time. For convenience of presentation, most of the facsimiles are reproduced at about one half the size of the original records. Economy of space has demanded also, in some cases, that a record be trimmed of broadcast signal correlations, unused galvanometer traces, and other irrelevant matter.

The distance between two adjacent vertical timing lines represents, in most cases, a time interval of 10 milliseconds; where the unit of time is different from this, it is so indicated on the appropriate record. The time noted on the face of a record is Mountain Standard Time, with the time increasing from left to right. These timing marks do not necessarily denote the beginning of a pulse or other prominent feature, but serve merely to indicate the time correlation with respect to some convenient timing line.



First Skip Arrivals.



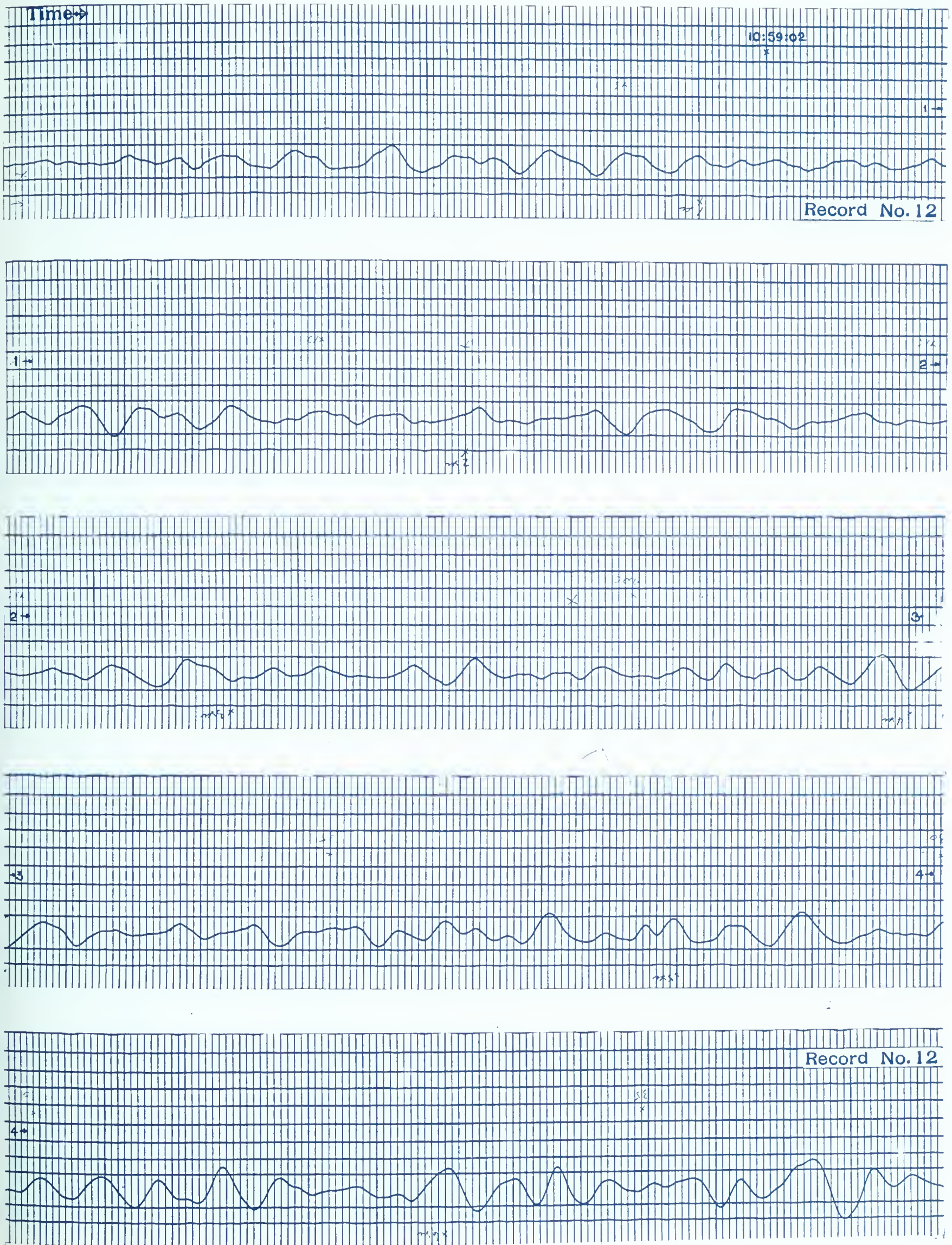


FIG. 26 : Records of Anomalous Sound Waves.

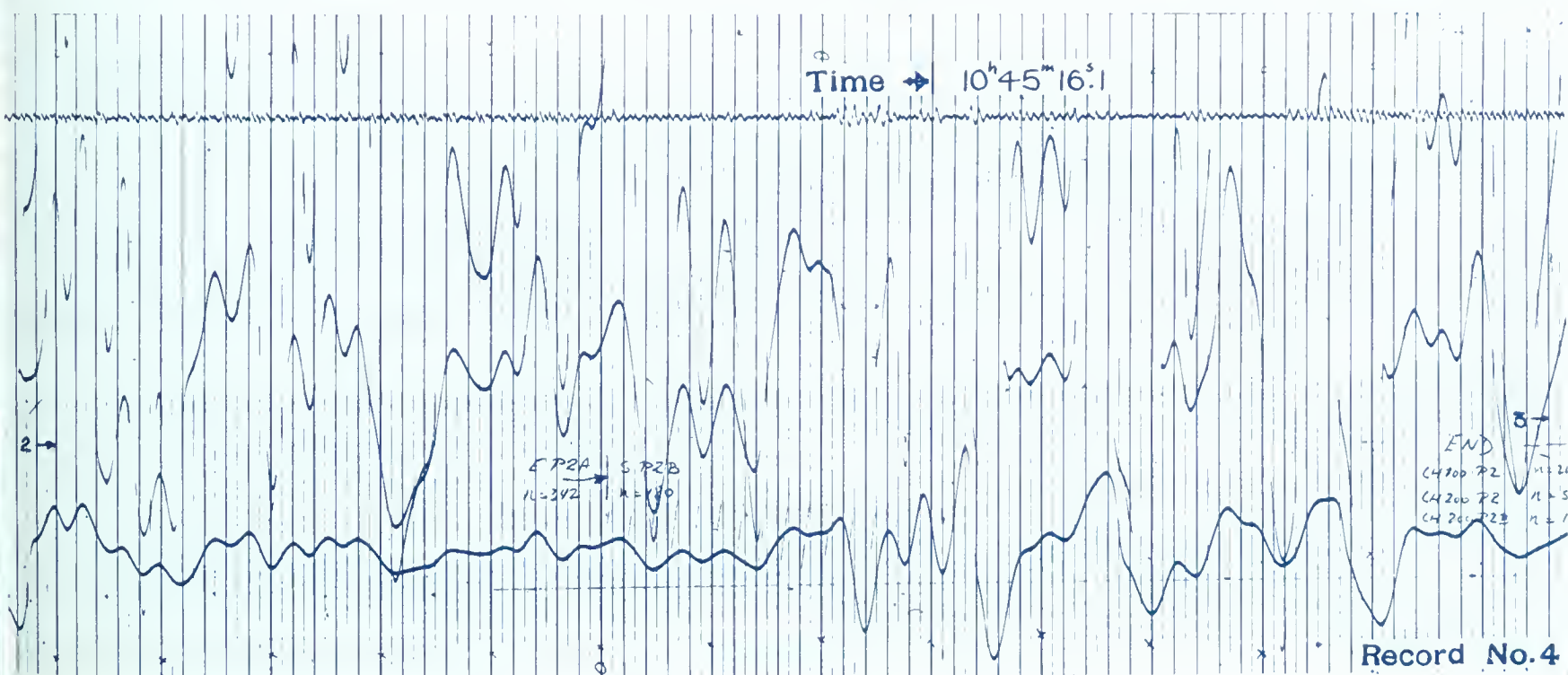
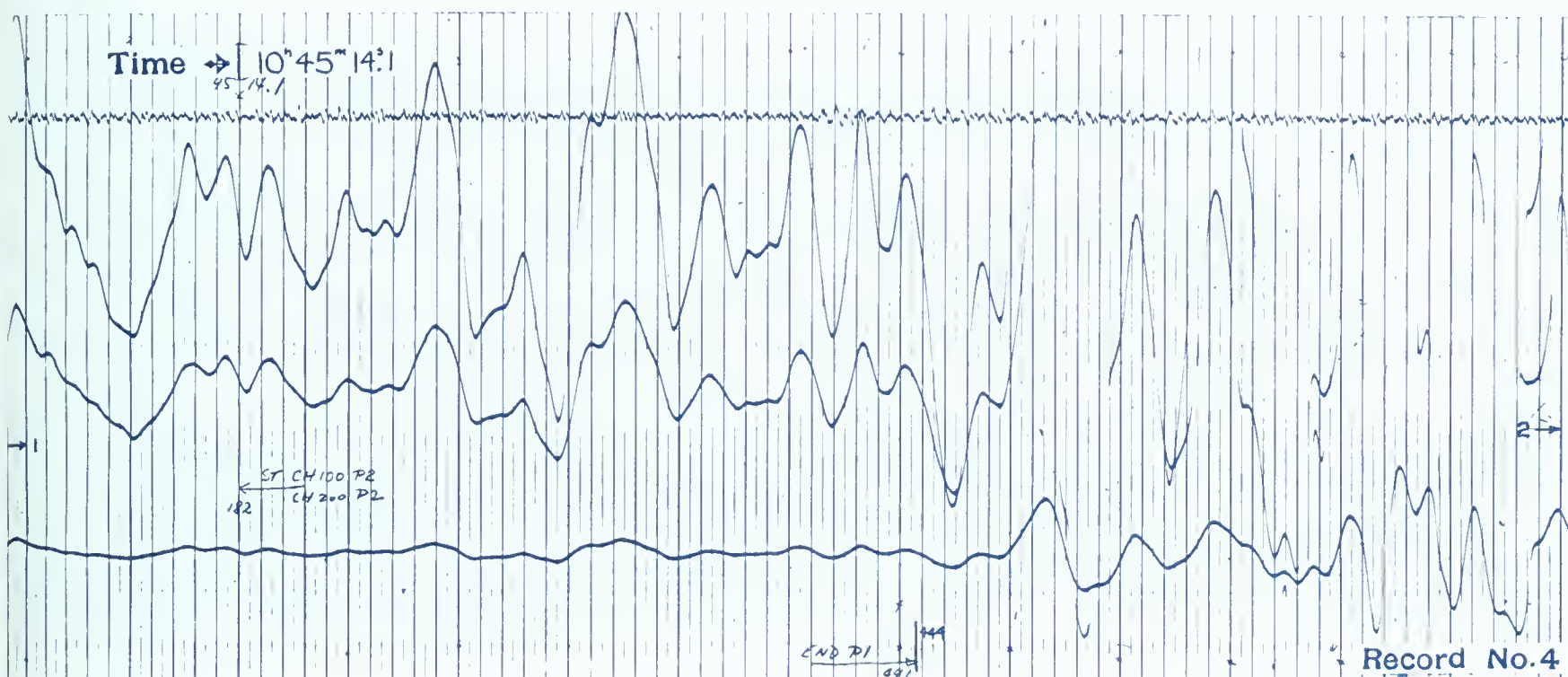
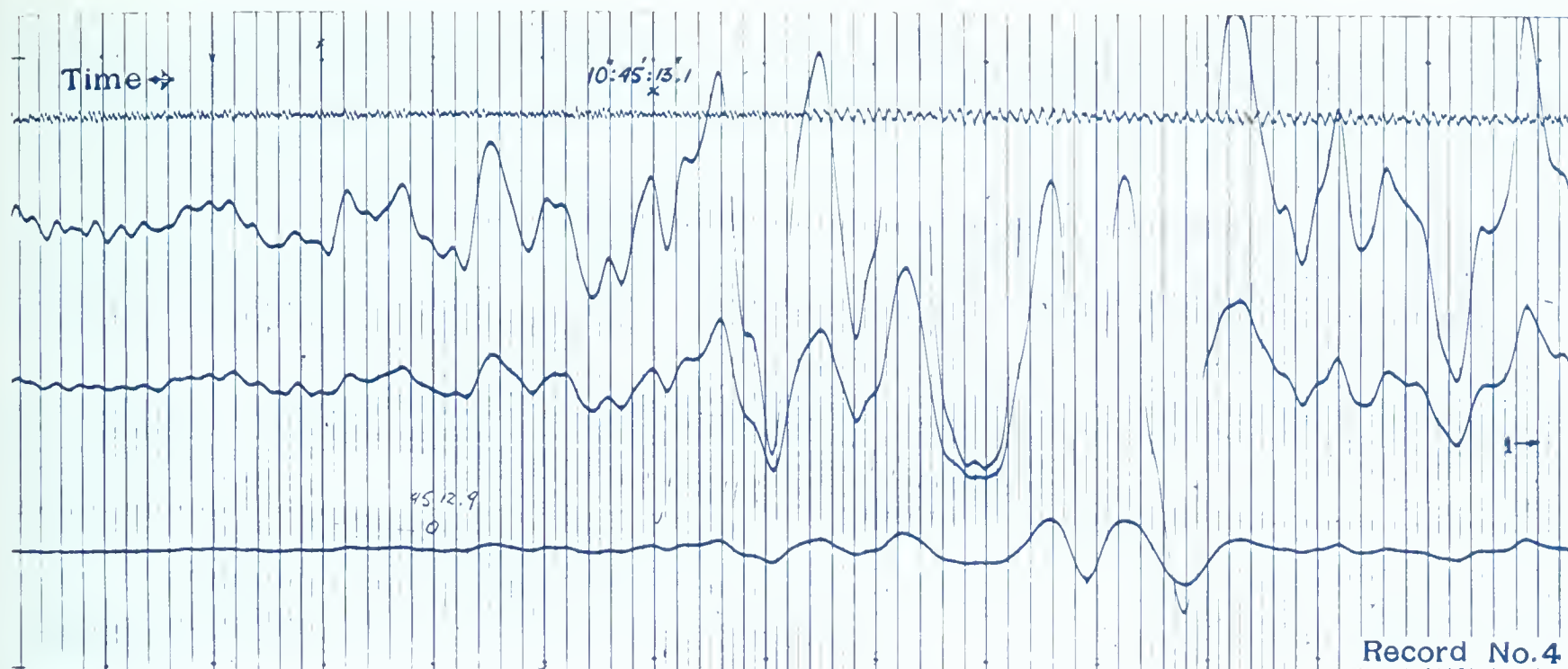


Fig. 27 : Records of Anomalous Sound Waves.

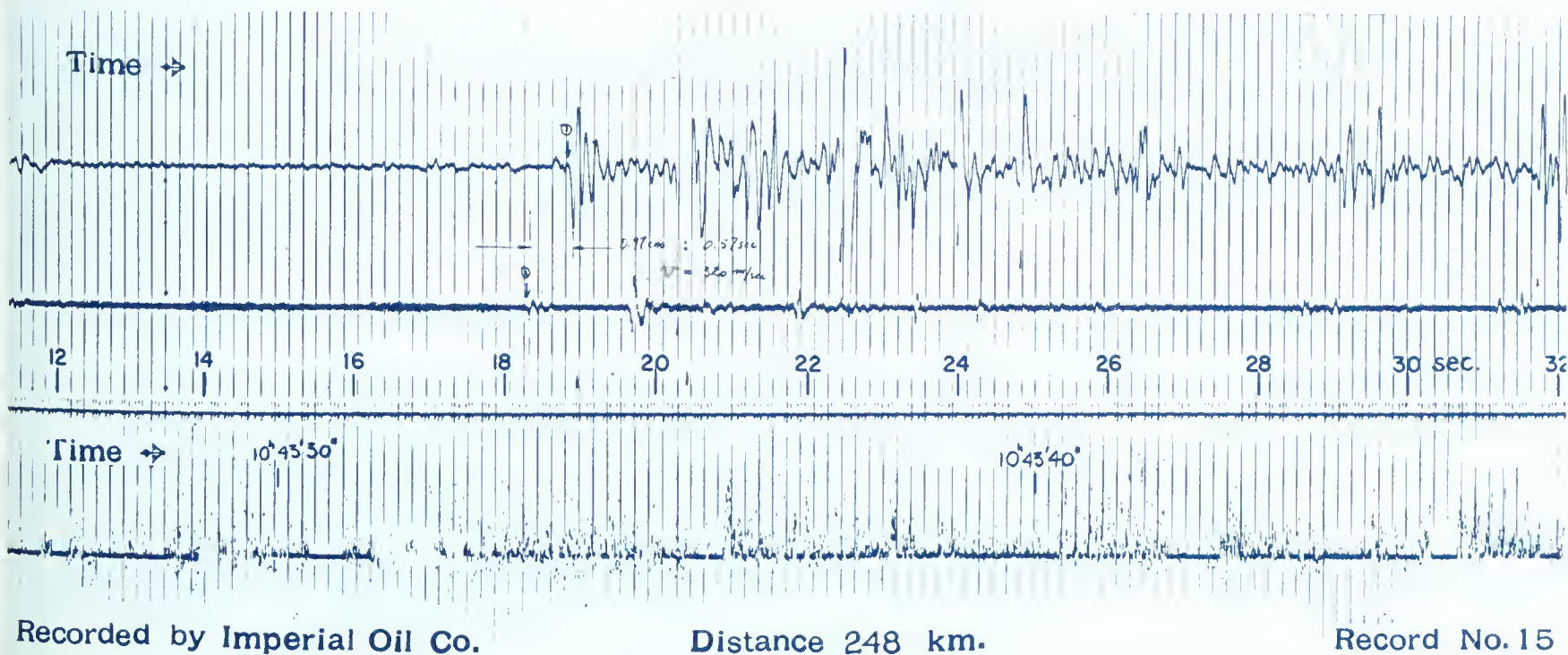
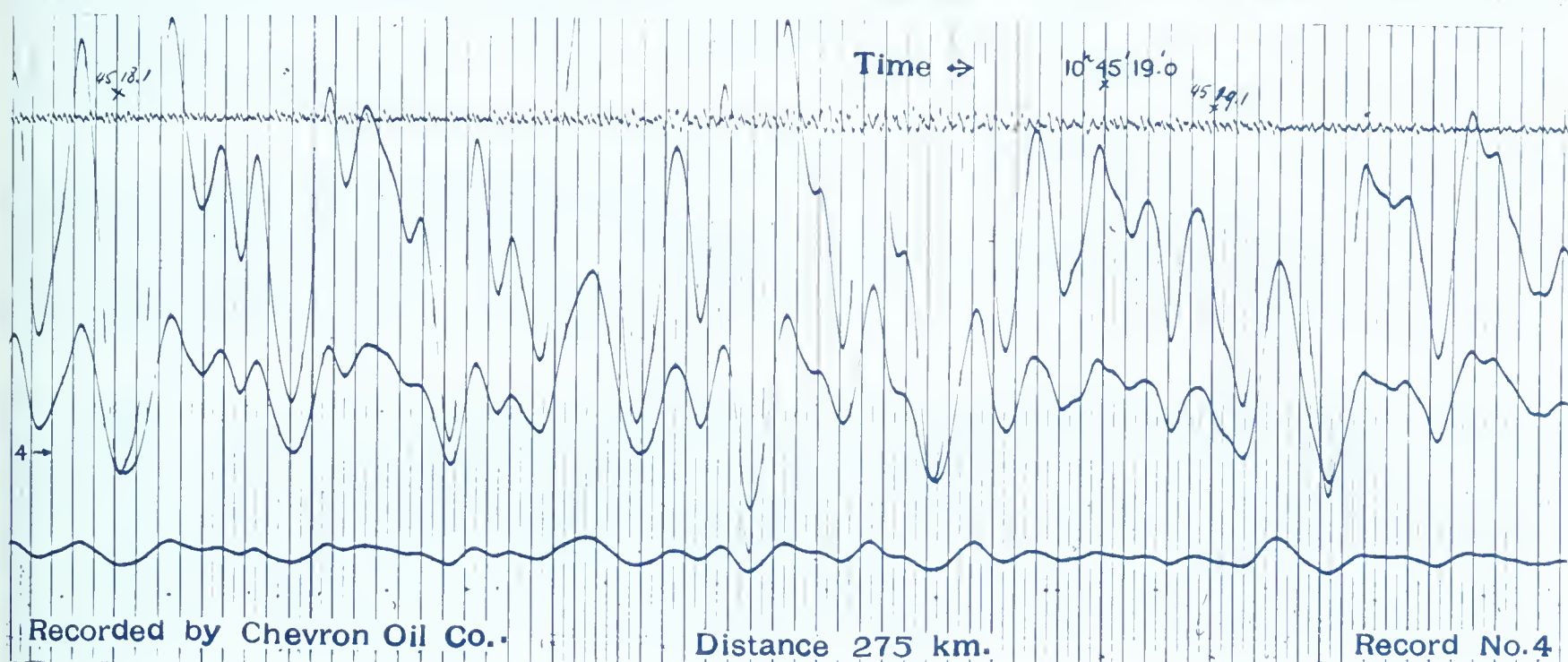


Fig. 28 : Records of Anomalous Sound Waves.

3 August 1961

Single Skip Arrivals

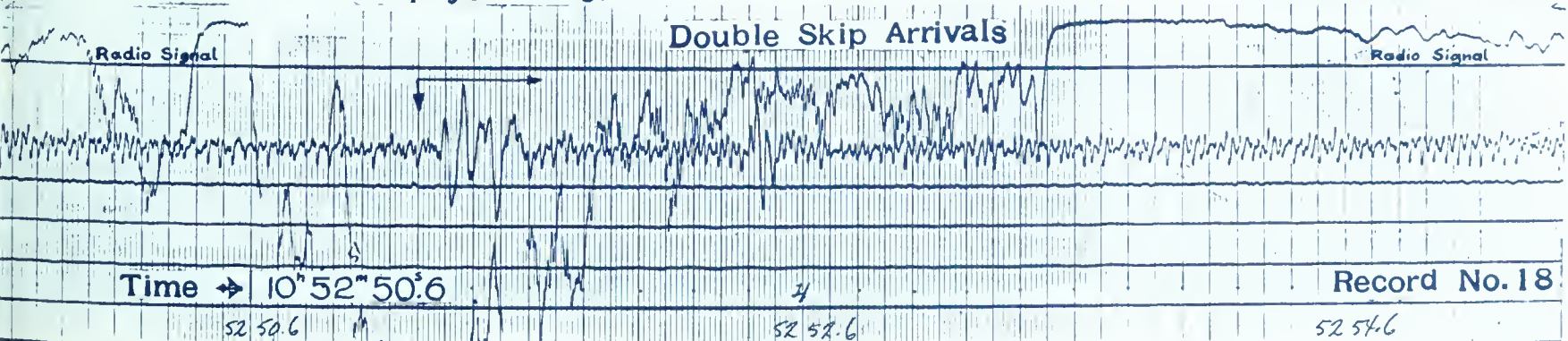
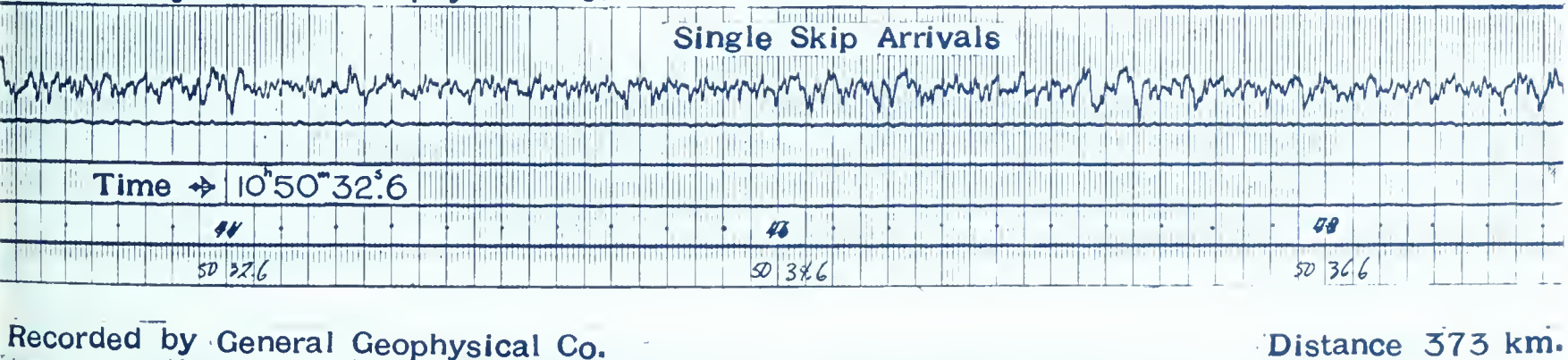
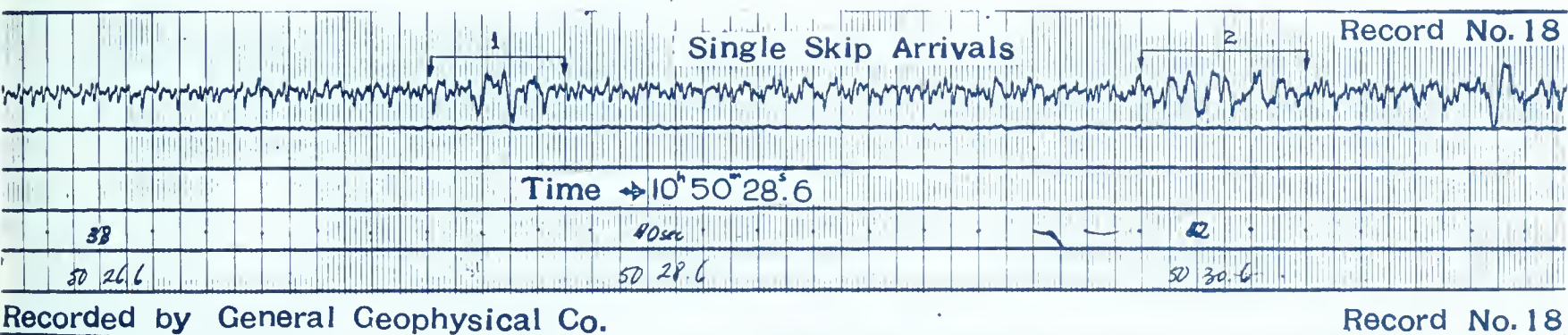
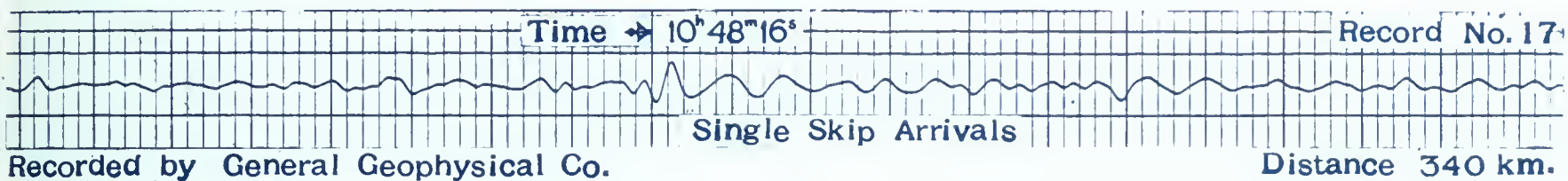
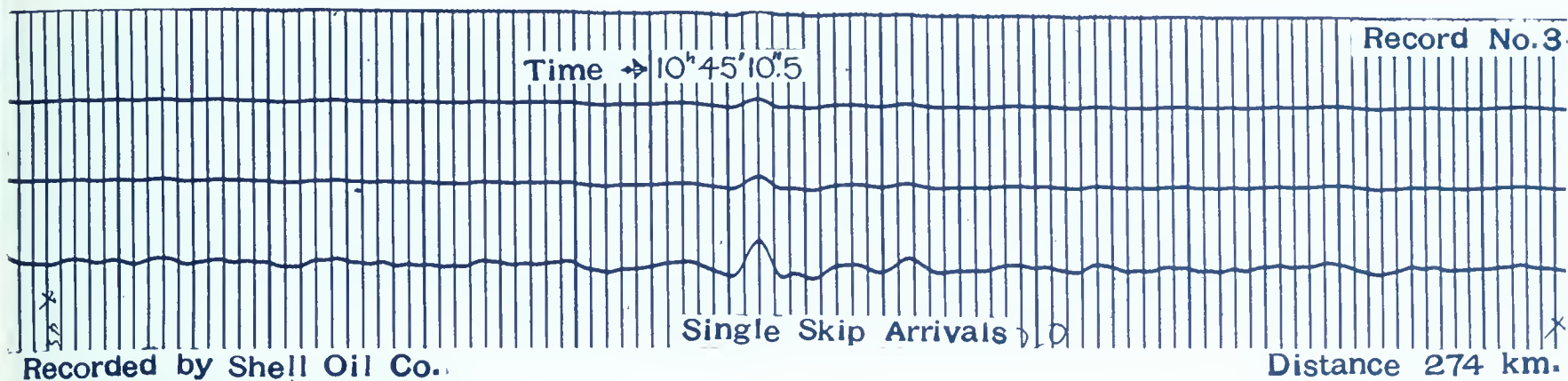
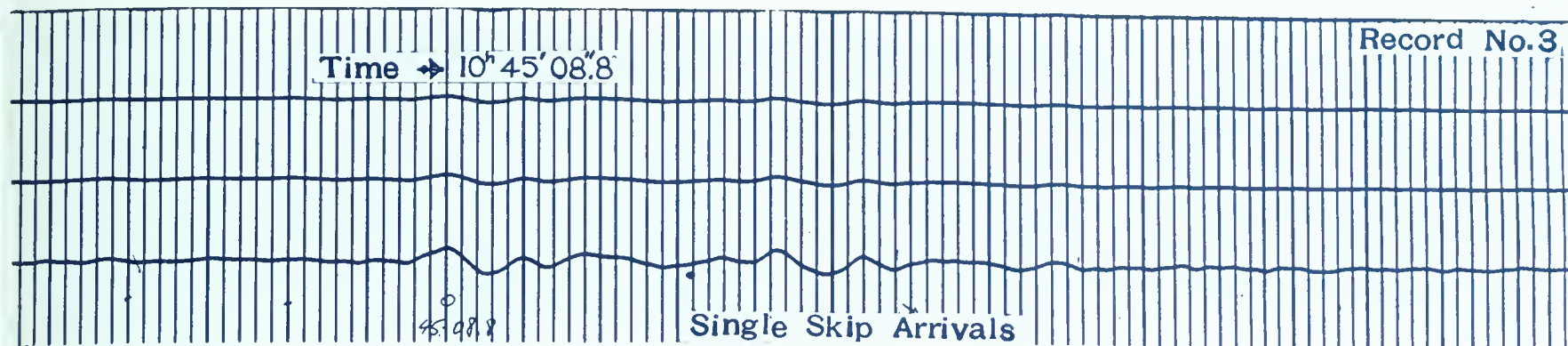


Fig. 29 : Records of Anomalous Sound Waves.

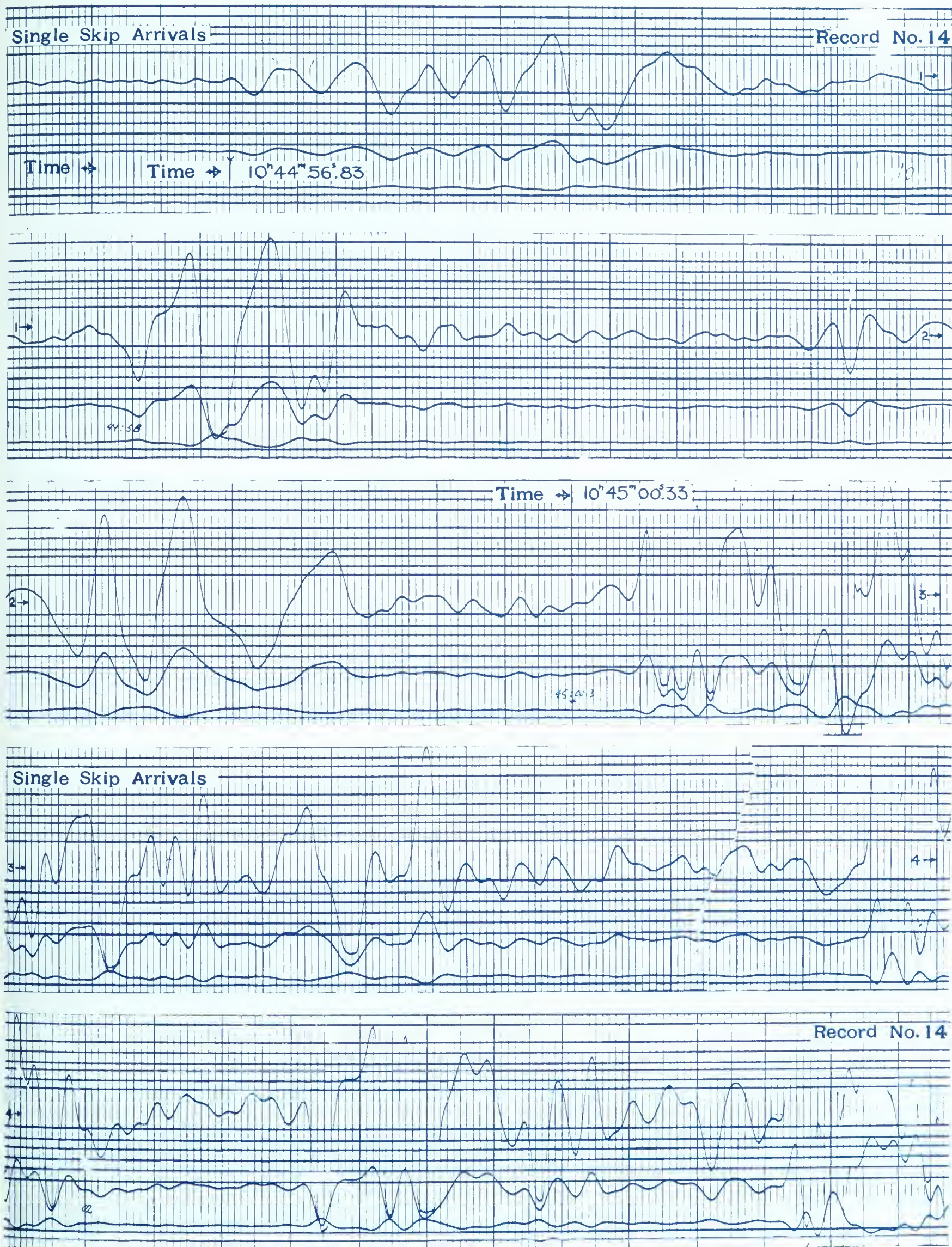
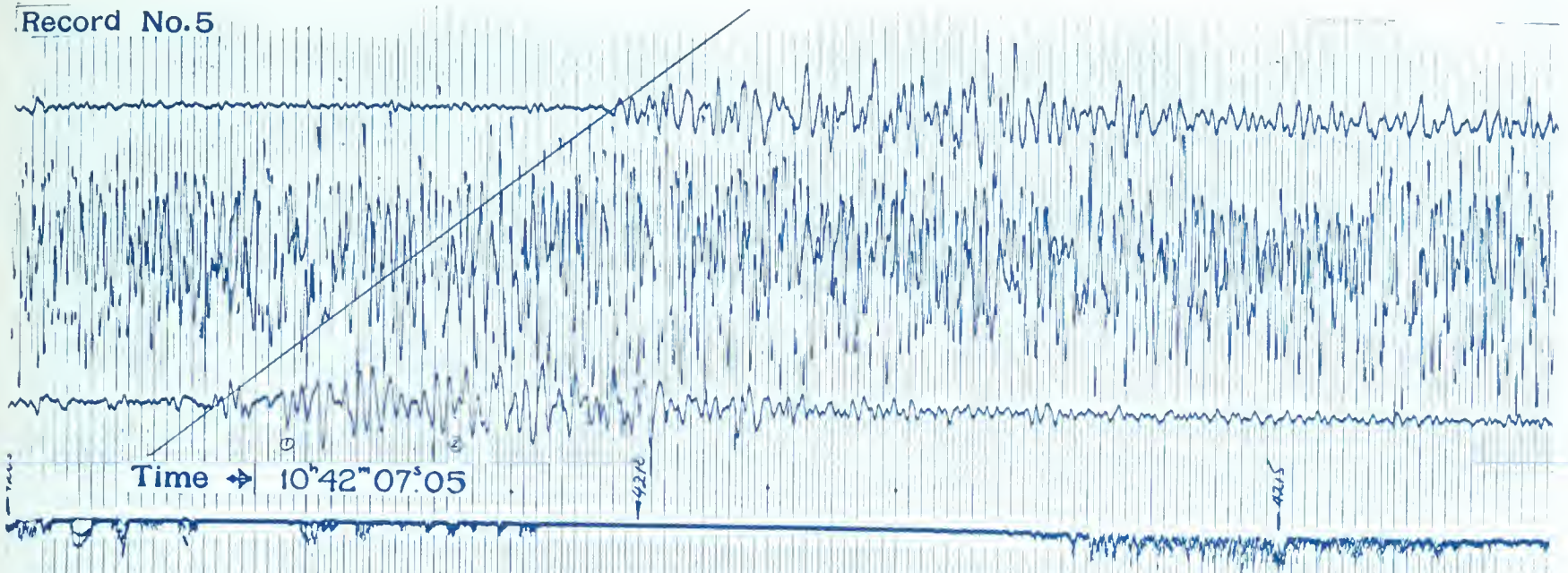


Fig. 30 : Records of Anomalous Sound Waves.

Record No.5

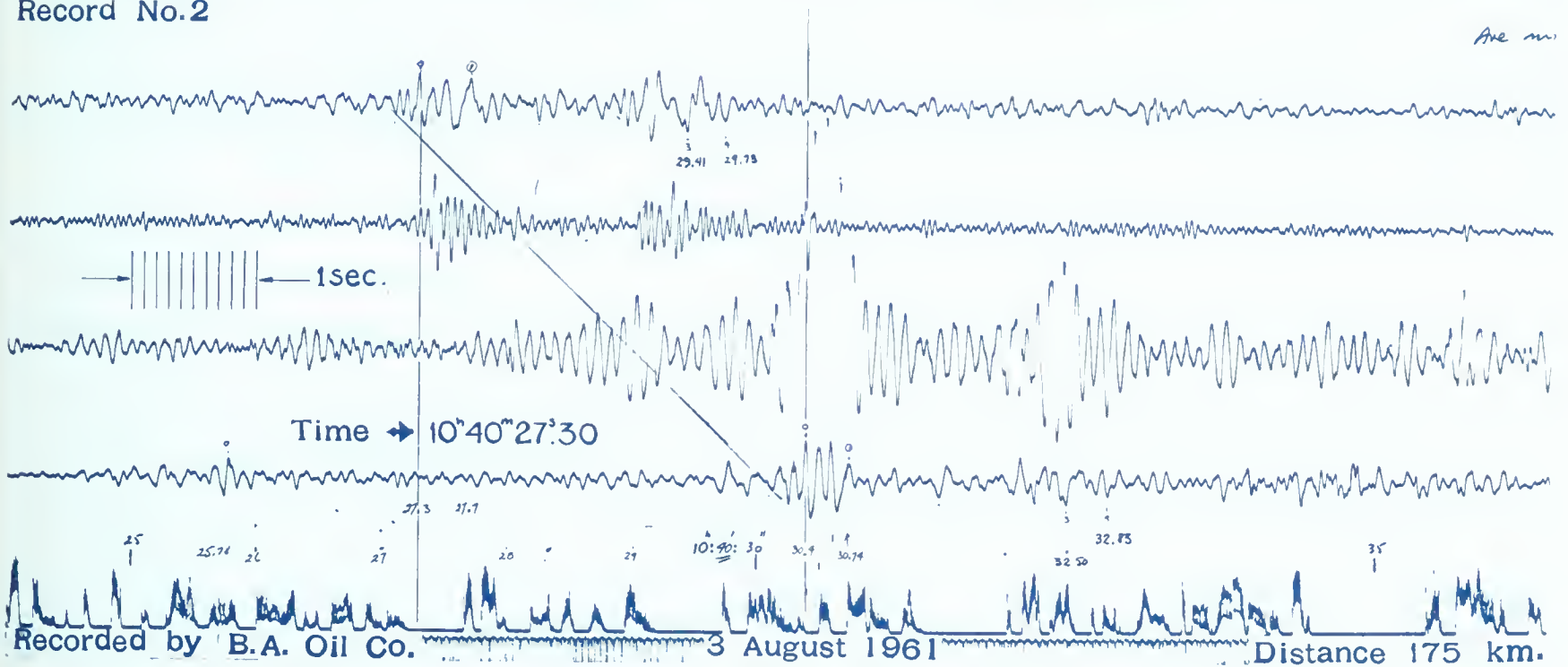


Recorded by B.A. Oil Co.

3 August 1961

Distance 249 km.

Record No.2

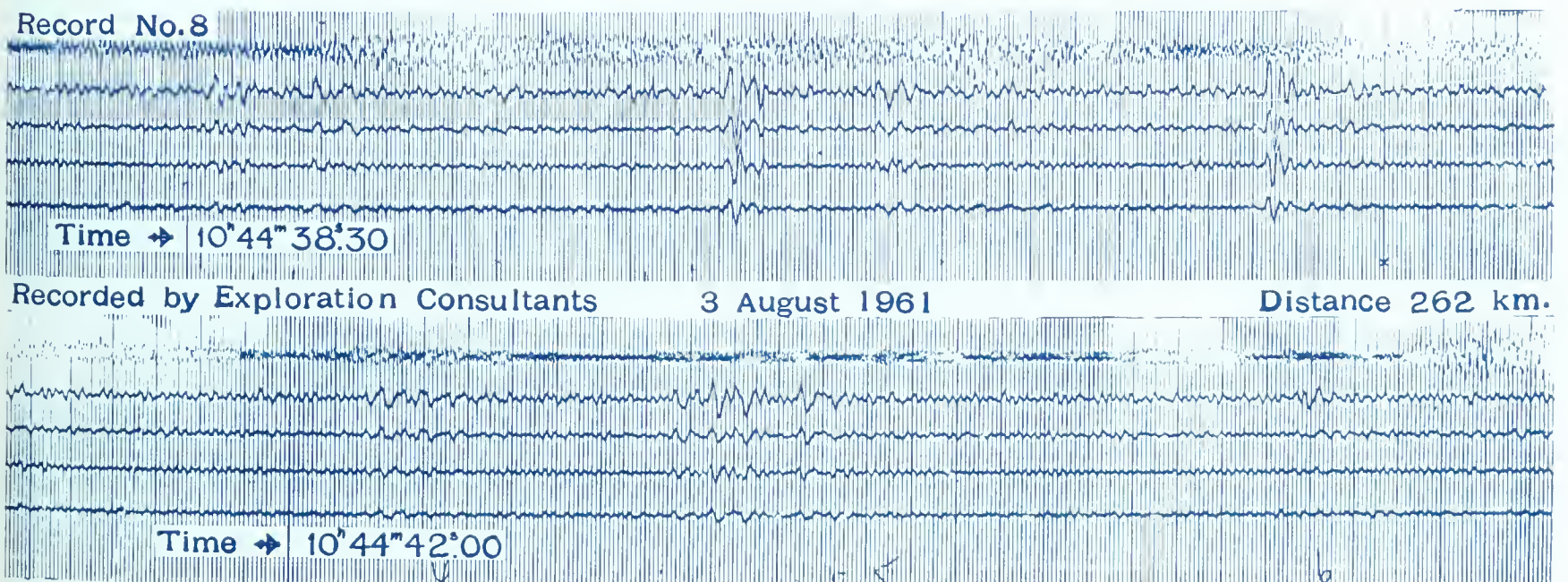


Recorded by B.A. Oil Co.

3 August 1961

Distance 175 km.

Record No.8



Recorded by Exploration Consultants

3 August 1961

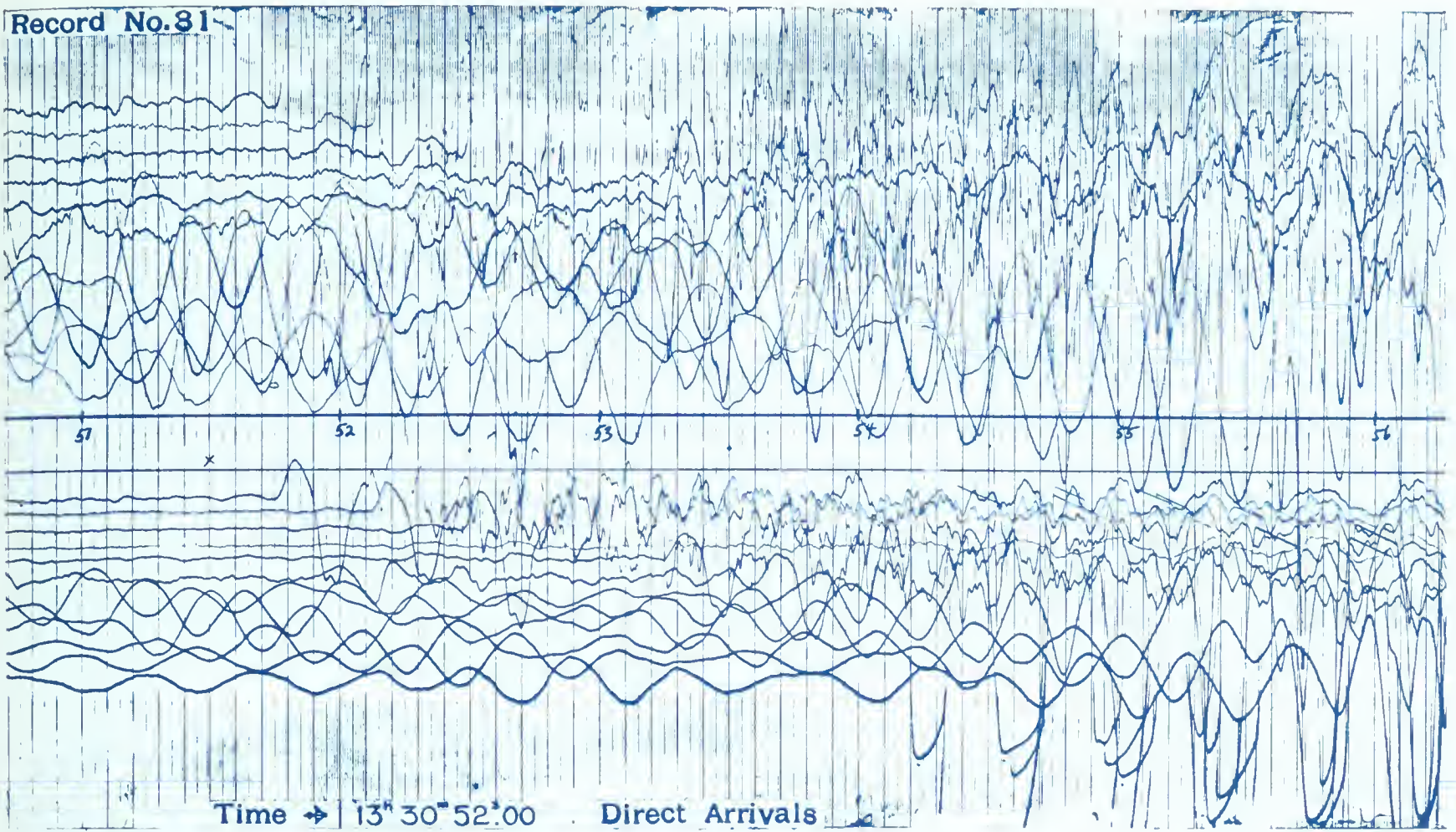
Distance 262 km.

Time $\rightarrow$ 10:44:42:00

Fig. 31 : Records of Anomalous Sound Waves.

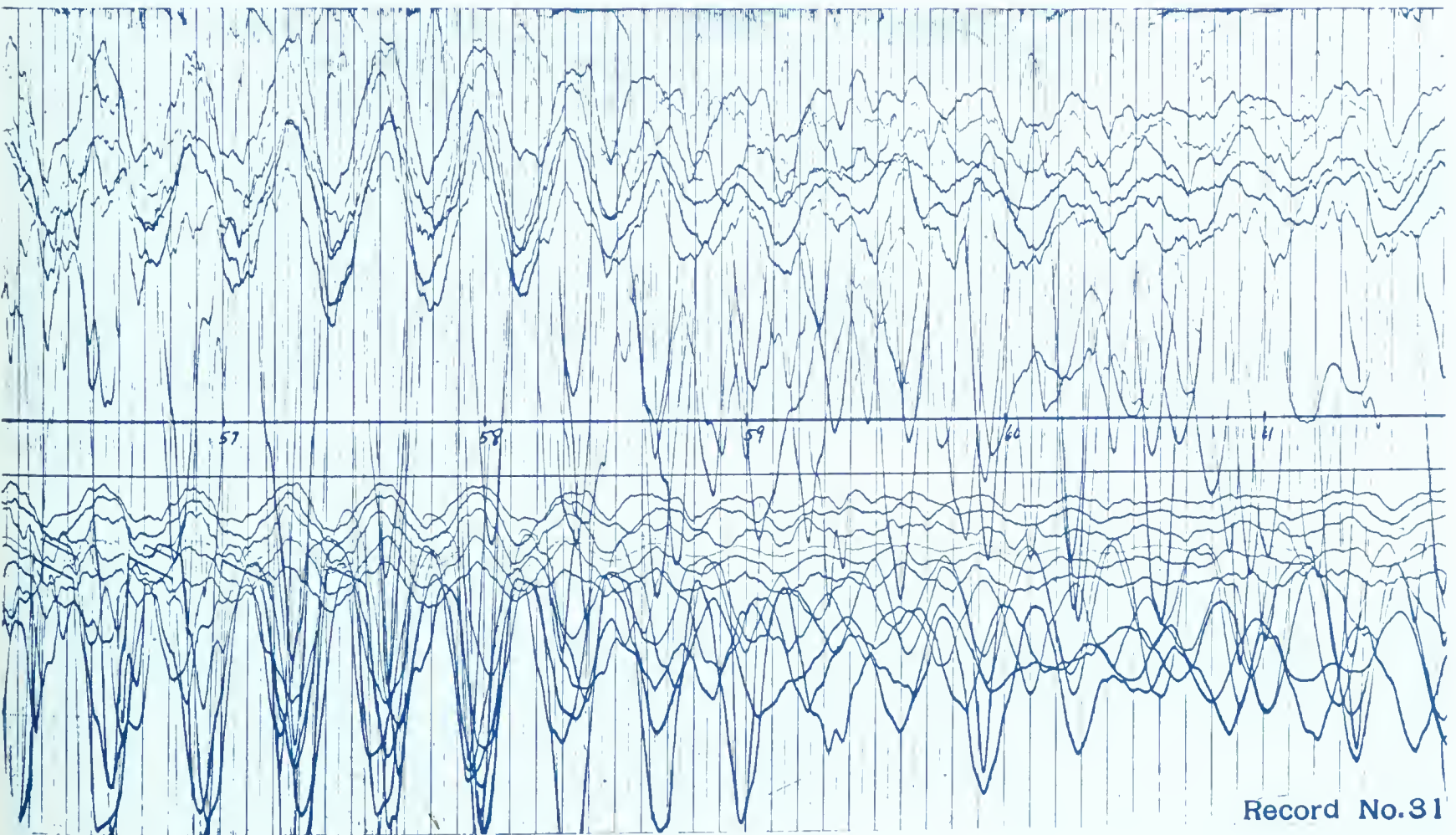
Single Skip Arrivals

Record No. 31



Traces 1-6 Pressure Phones

Traces 7-12 Geophones



Record No. 31

Fig. 32 : Records of Anomalous Sound Waves.

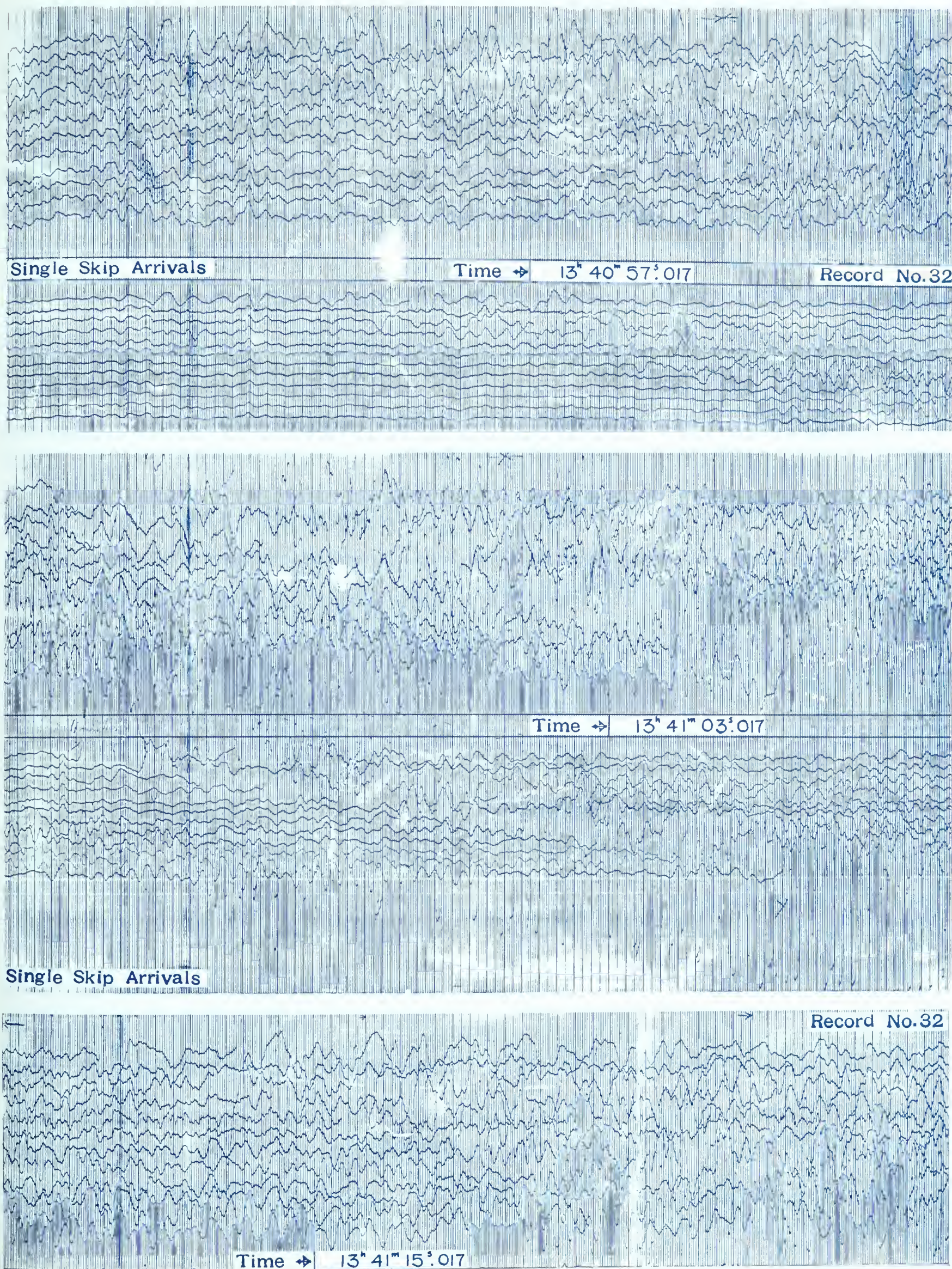


Fig. 33 : Records of Anomalous Sound Waves.

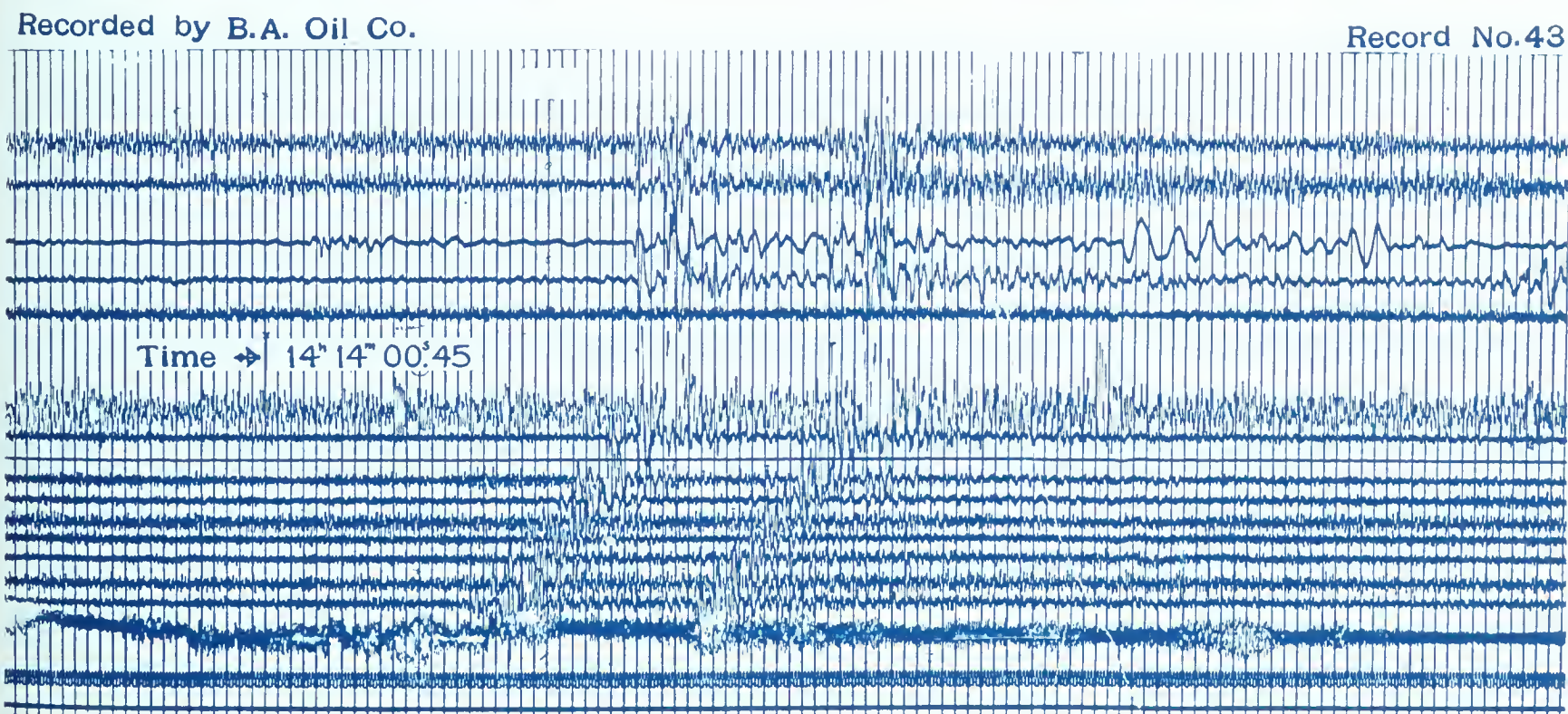
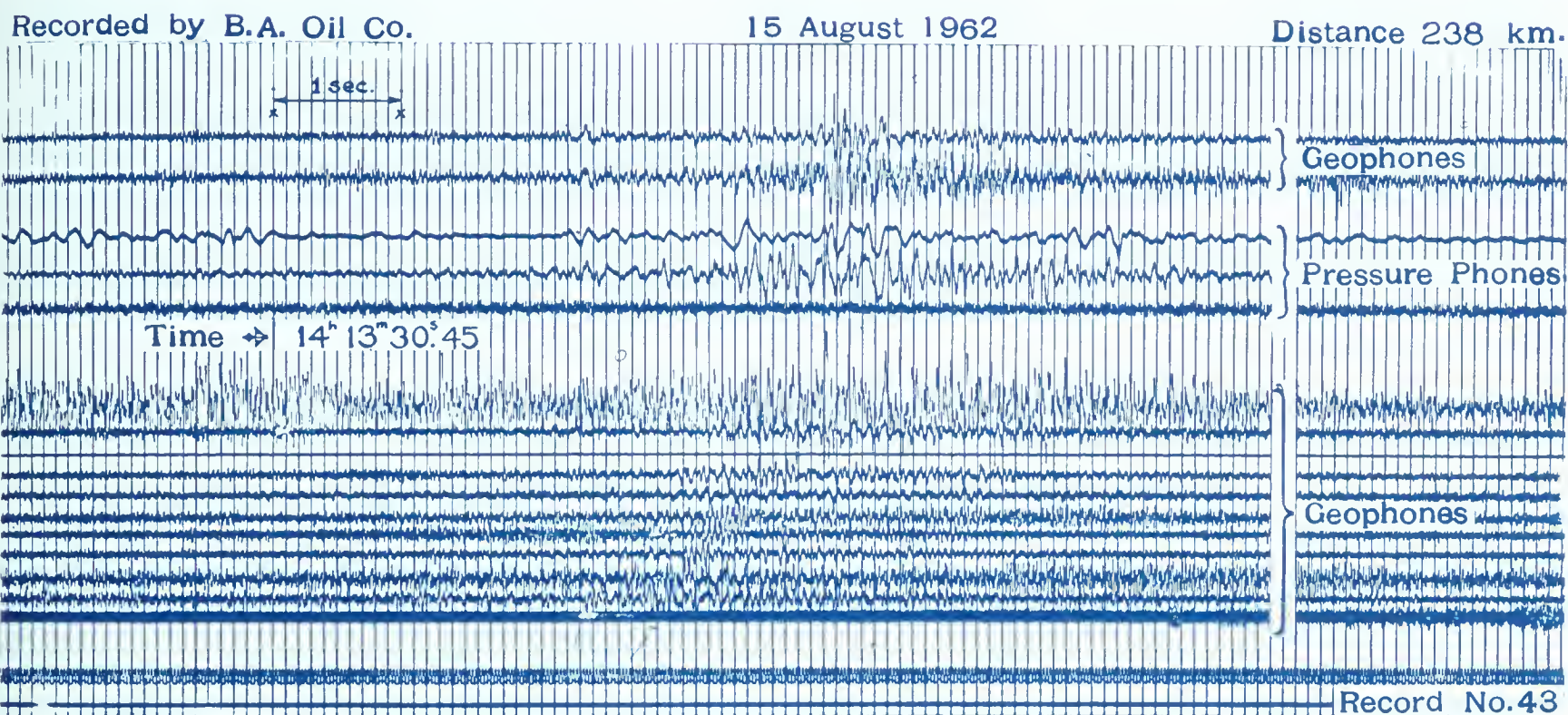
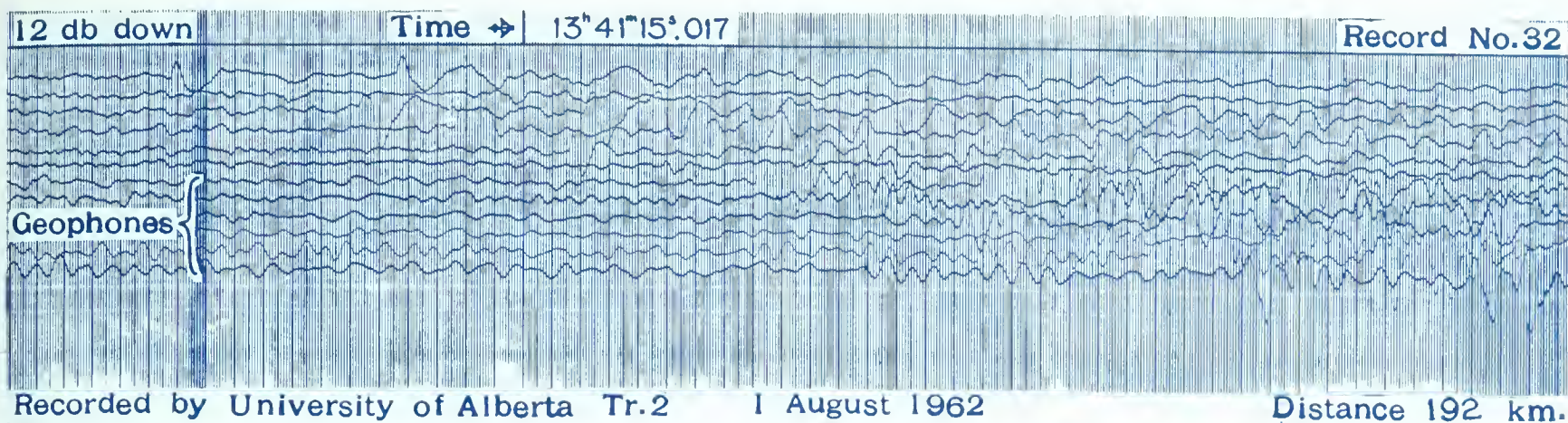


Fig.34 : Records of Anomalous Sound Waves.

Single Skip Arrivals

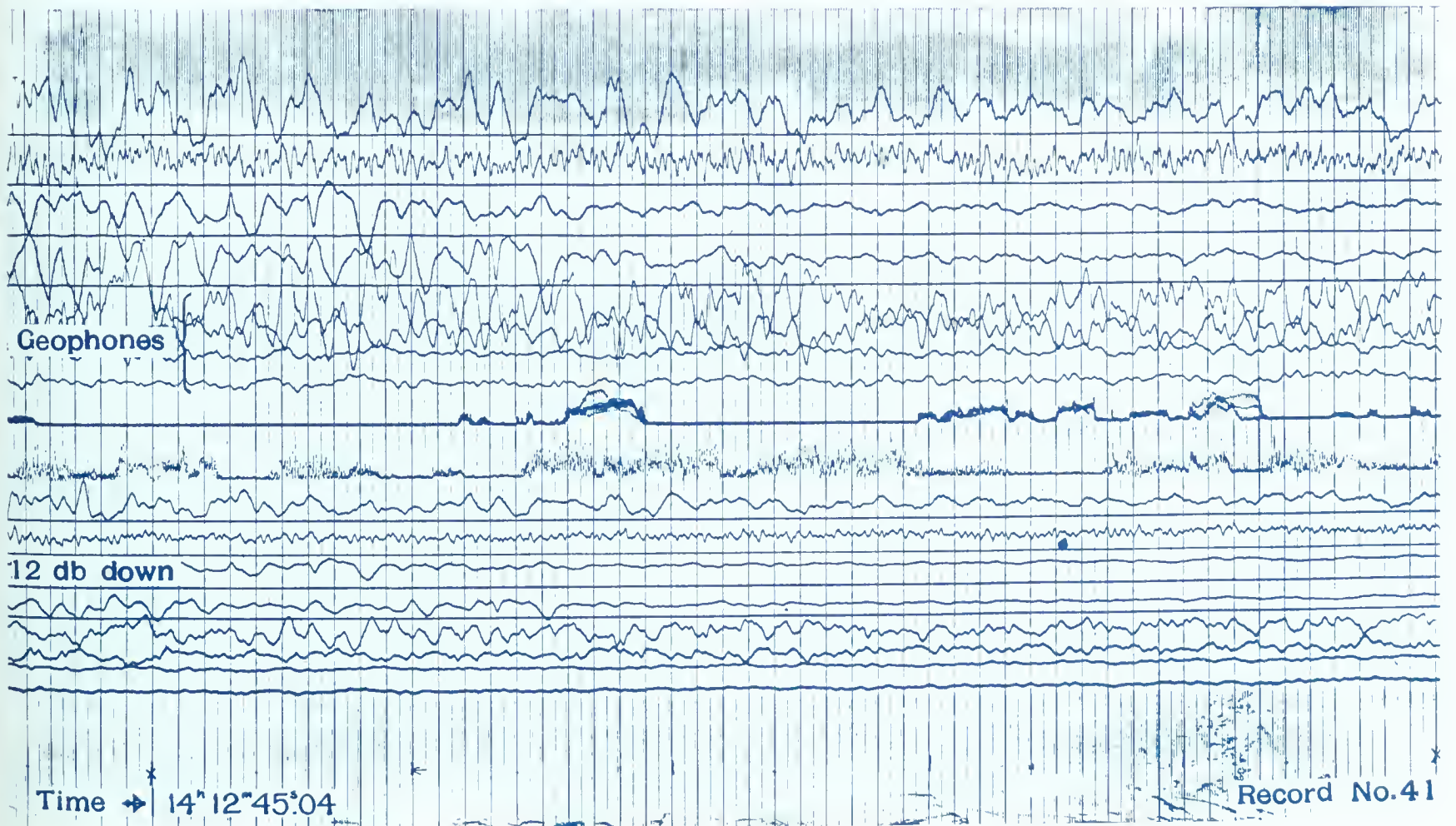
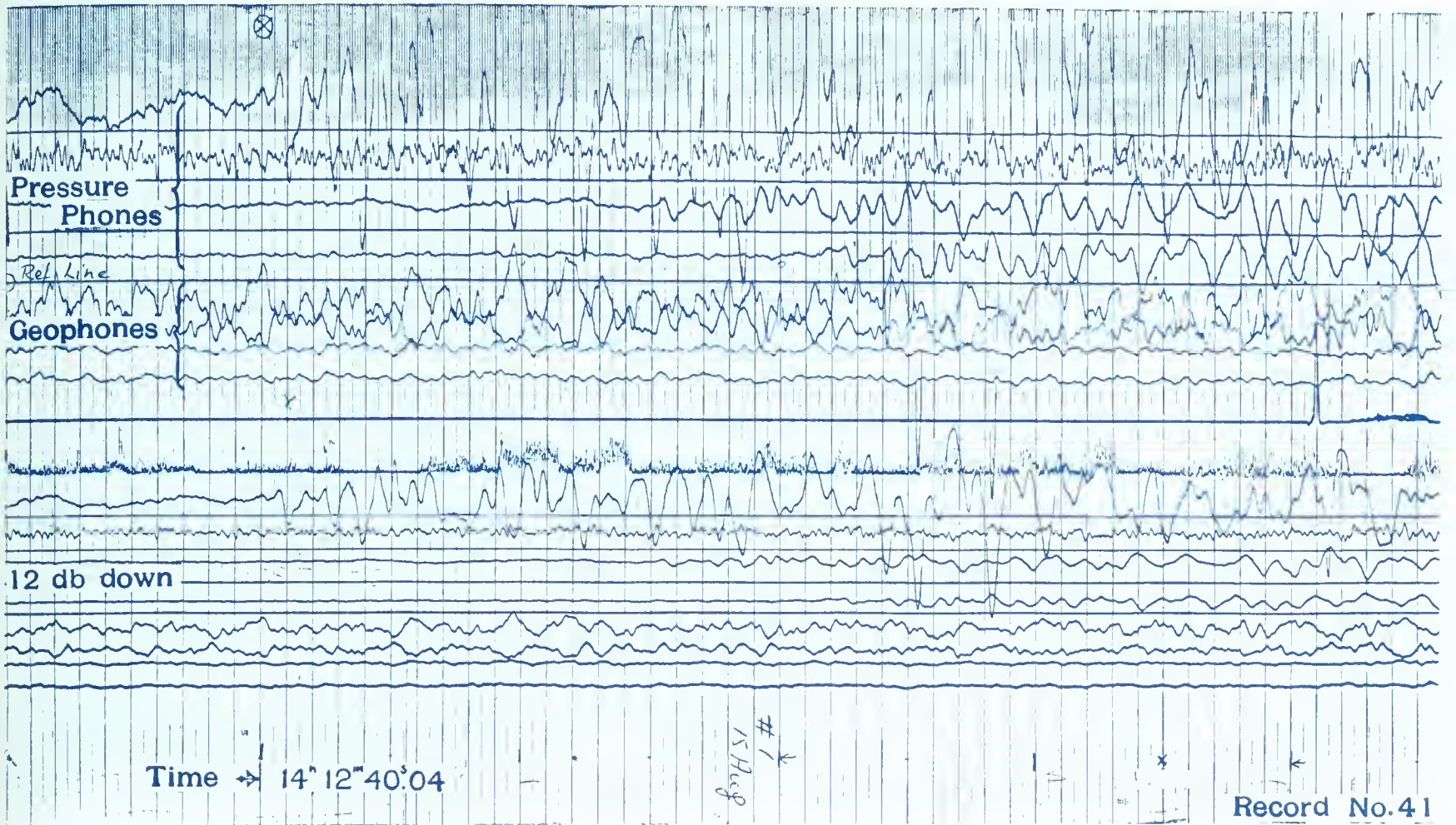


Fig. 35 : Records of Anomalous Sound Waves.

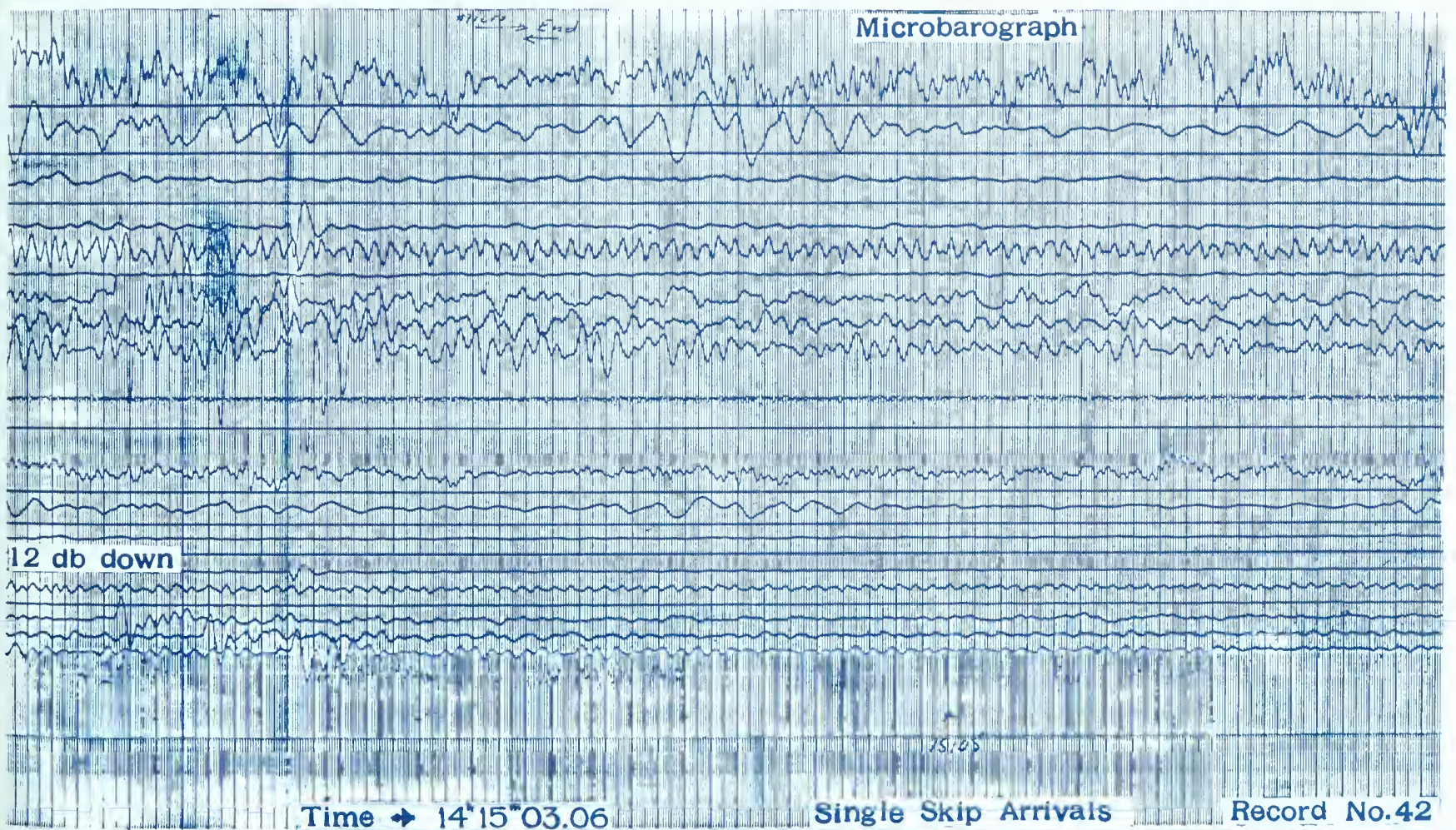
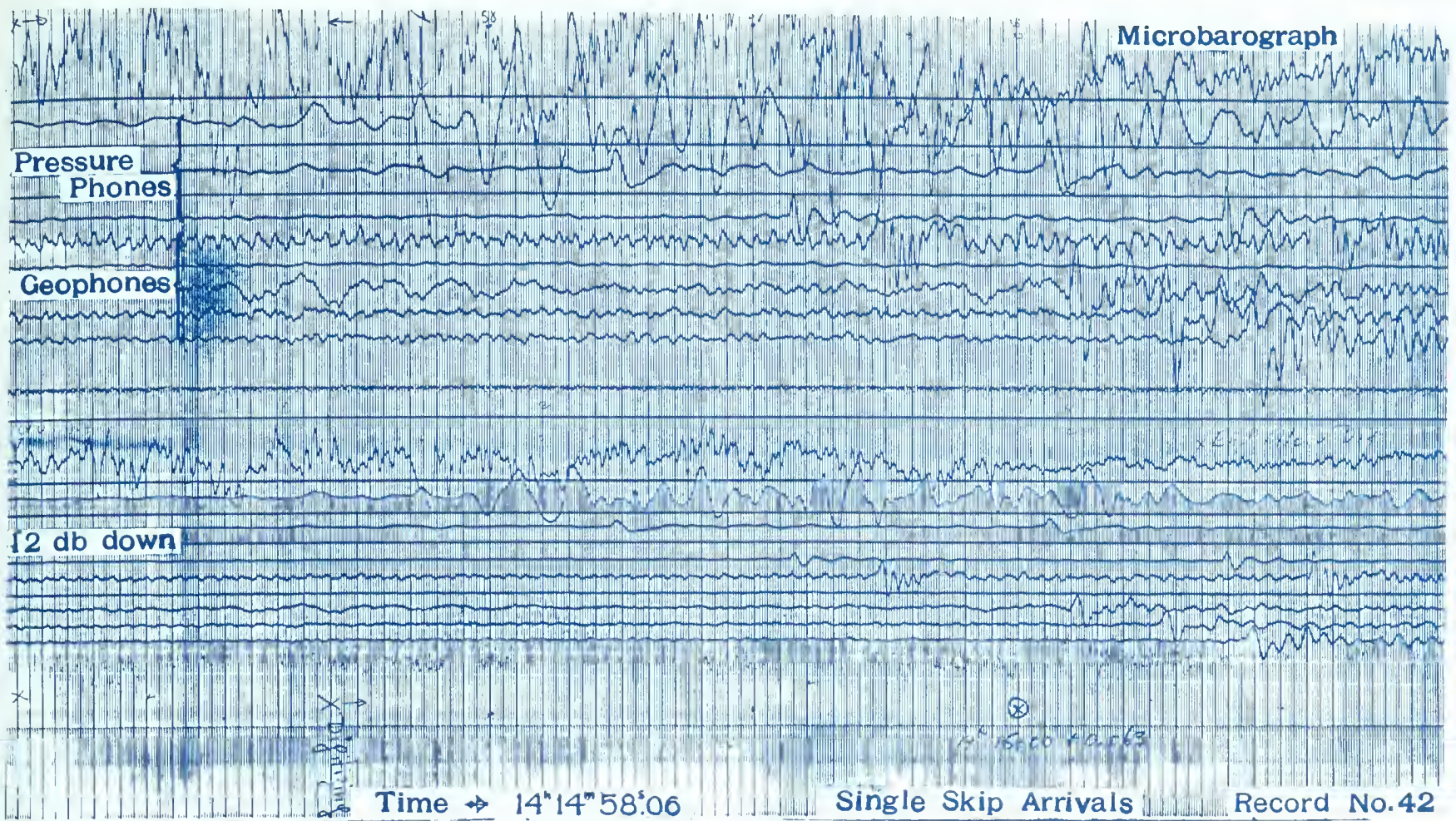


Fig. 36 : Records of Anomalous Sound Waves.

5-3. Travel Time Curves (100-ton Explosion)

The travel time curve observed for the 100-ton explosion is shown in Fig. 37 on the next page. The curves, straight lines to the first approximation, were plotted from the time - distance data given below (Table VII).

Record No.	Time of Observation MST	Travel Time sec.	Distance from Explosion, km.
2	10:40:26.59	625.59	174.3
	10:40:29.70	628.70	175.4
3	10:45:08.50	907.50	274.0
4	10:45:12.80	911.80	275.0
	10:43:17.02	796.02	275.0
5	10:42:06.66	725.66	248.7
	10:42:09.55	728.55	249.8
8	10:44:38.34	877.34	261.9
12	10:57:39.73	1658.73	540.3
	10:59:01.03	1740.03	540.3
14	10:44:56.83	895.83	271.4
15	10:43:31.63	810.65	248.3
	10:43:32.20	811.20	248.5
17	10:48:16	1095	340.5
18	10:49:29.10	1168.10	373.5
	10:52:51.24	1371.24	373.5
19	10:35:44	343	99.0

Time of explosion 10:30:01.00 MST.

Table VII

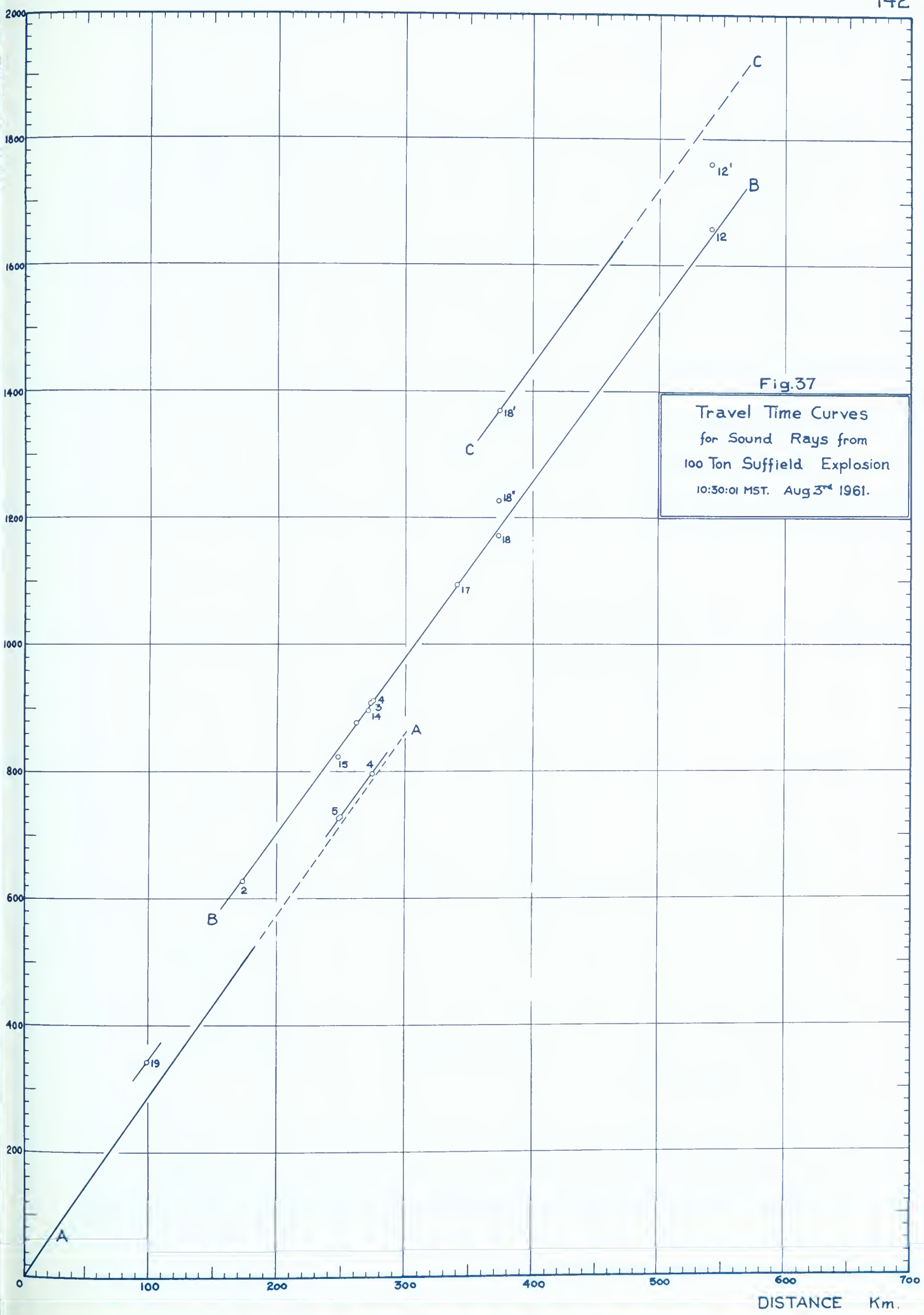


Fig.37

Travel Time Curves
for Sound Rays from
100 Ton Suffield Explosion
10:30:01 MST. Aug 3rd 1961.

DISTANCE Km.

The line A-A through the origin (Fig. 37) is the travel time curve for direct sound rays travelling along the Earth's surface with velocities of 349.2 meters per second, appropriate to the prevailing ambient air temperature of 30.0°C . Since low level propagation toward the west was suppressed by the wind conditions prevailing near the ground, no records of the direct wave could be obtained by observers in the area west of the shot point. This is clearly shown in Fig. 38, reproduced from a paper by Johnson et al. (1962). It is seen also that a direct wave could propagate freely toward the east and, to some extent, into narrow sectors north and south. Aural reports received by Johnson substantially confirmed the predicted pattern of low level propagation.

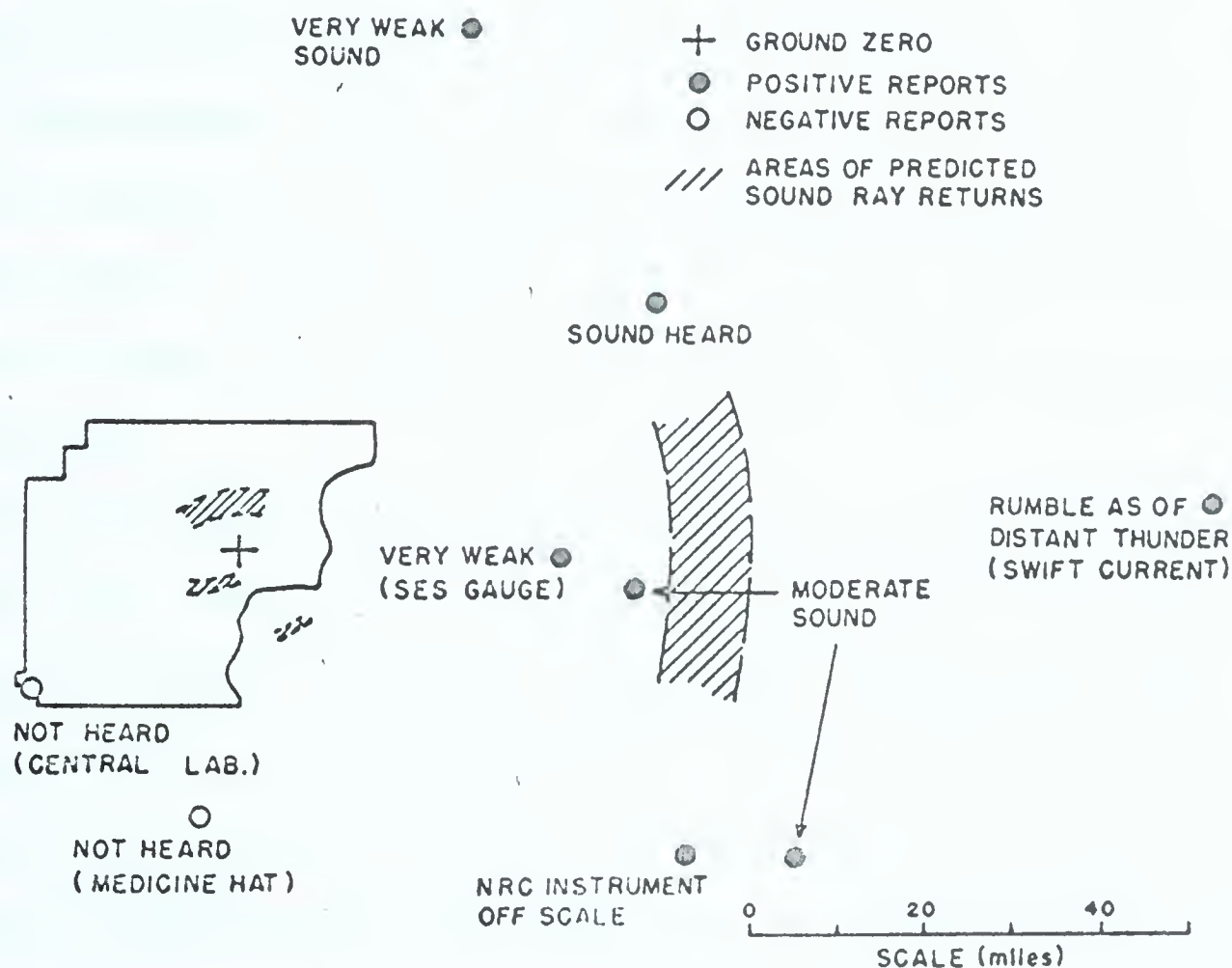


Fig. 38: Low Level Propagation of Blast Waves.

Areas of predicted returns based on winds at 0844 MST and temperatures forecast for 1030 MST. From Johnson (1962).

The direct wave could not be identified on any of the records taken by observers of the University of Alberta network. The observation sites were either too far away, or too noisy (high surface winds) to permit positive identification of possible direct waves.

As mentioned earlier, Fig. 38 shows also that meteorological conditions were favourable for low-level propagation toward the northerly quadrant. Signals were, indeed, recorded at two quiet locations to the north (Record 5) and north-northwest (Record 4). These two points plot just above the curve A-A of the direct wave. This is as it should be: the wave did not propagate along a straight line, but skipped along the ground, within the lowest one or two kilometer layer of the atmosphere; the apparent velocity is, therefore, a little higher, and the travel time a little longer than what one would expect to observe for the rectilinear rays of the direct wave.

The line B-B is the travel time curve for the first (or single) skip arrivals of anomalous sound: rays refracted once and returned to ground by the high temperatures and easterly winds of the mesoincline. Significantly, no positive returns were received by any of the recording sites located in Saskatchewan and Manitoba. The prevailing easterly circulation in the mesosphere was evidently of sufficient strength to suppress all high level propagation toward the east - the usual state of affairs in midsummer.

Since the observing stations were not arrayed along a line or other regular pattern, the plotted data points will not fall perfectly on a single straight line. To wit, the slope of the travel time curve will vary a little for waves propagating in different directions. This is largely due to the variation of the wind component with change in bearing, but sloping layers, non-uniform horizontal wind and temperature distributions, and other inhomogeneities may all, at times, influence the travel time significantly. The curve B-B is thus in essence a composite or mean of many possible curves. The associated apparent velocity (the reciprocal of the slope) is 362.5 meters per second; in the absence of wind, the corresponding air temperature at a ray apex would need to be at least 326.5°K. , or some 23 degrees higher than the temperature at the Earth's surface. While not impossibly high, it is evident from the asymmetry of the observed pattern of returns that the effect of the upper winds was appreciable, and the peak temperature correspondingly lower. A complete temperature and wind profile consistent with the observed travel times will be deduced later, in section 5.4.

The short segment of line C-C is the travel time curve for second or double skip arrivals, i.e. anomalous signals which, on reaching the ground, have been reflected, and then refracted a second time. This curve must remain subject to some speculation, since it is largely based on the existence of a single point, marked 18' (see also Record 18, page 133). Station 12, located near Edson, should have supplied another

point, but the record ended at 10:59:49 MST, some 50 seconds short of the required time. It is, nevertheless, possible to construct this section of curve with some confidence. Halving the time and distance values of point 18' will transpose it quite close to the "main line" B-B, a sure test for double skip arrivals. The slope of the double skip curve is, of course, the same as that of line B-B.

Point 12' falls between the lines and poses rather more of a problem, for the halved time-distance test must evidently fail in this case. The portion of Record 12 (page 130), marked tentatively 'double skip arrival', has all the earmarks of a genuine arrival, and has been accepted as such. Lack of other evidence all but fails to decide the issue one way or other, and one can only ask: if it is not an arrival from the 100 ton explosion, what is it?

Point 18" presents much the same sort of problem. It is unlikely that points 12' and 18" should be joined by a straight line, for the resulting apparent velocity would be very low. It is far more likely that they are elements of a system of travel time curves of slightly different slopes. A similar splitting or "fine structure" of curves has been observed before, notably by Hale and Johnson (1953). One can only assume that a denser network of observing stations would have resolved the complications of the present case in a similar manner.

Record 19, secured by the University of Michigan party some 99 kilometers west-southwest of the shot point, poses a rather different problem. This peculiar signal* registered itself on a set of standard Benioff seismometers (vertical and horizontal components), but there can be little doubt as to its authenticity. This arrival came as a complete surprise, for the manner of its recording seemed highly unorthodox if not mysterious. But early bafflement dispelled when it was discovered that Gutenberg had used seismographs of similar design as long ago as 1938, to record the blast of naval guns fired off the coast of California. This phenomenon is now called an acoustically induced ground wave, and will be considered later.

But a deeper mystery remains. The signal is evidently not a direct wave, and hardly an anomalous wave in the presently accepted sense of the term, for its range is so extraordinarily short, a mere 99 kilometers. Since there is no reason to doubt the reliability of the aerological data and related information, it is difficult to see how this arrival could possibly be explained in terms of simple atmospheric refraction. A high temperature layer near 30 kilometers would solve the problem well enough, but there is no evidence whatever that such a layer was present at the time of the trial. On the contrary, all radio-sondes show this to be a region of very low temperature. A marked dis-

*This record is not reproduced in the present work.

continuity in the wind field could return a ray by total reflection, but the observed wind data do not support this suggestion in any way.

Cox (1947, 1949) has reported similar signals on two occasions; one wave returned to ground at a distance of 137 kilometers, at the time of the Heligoland explosion. But such ray paths are still as much a mystery as ever. It is quite possible that such rays cannot be accounted for at all in terms of geometrical acoustics. The correct explanation may well lie elsewhere: wave acoustics comes most readily to mind, and a detailed investigation of diffraction effects into the shadow zone would seem to hold considerable promise.

5.4 Winds and Temperatures in the Upper Air

The pattern of acoustic arrivals, and the shape of the anomalous zone of audibility (shown in section 5.5) suggested from the very outset that the upper winds would have to be considered in any realistic interpretation of the observed data. Preliminary calculations which neglected the effect of the winds substantiated this view, but, as a first approximation, a theoretical travel time curve for rays in quiet air was computed and compared with the observed curve. These early estimates turned out to be quite useful, however, and indicated clearly the presence of a sound speed maximum near 57 km, a result which agreed rather well with a second estimate, the height of a reflecting layer, derived from the slope and intercept time by a standard seismological method.

The first problem to be considered in some detail concerned the deduction of plausible wind and temperature distributions above the level reached by standard radiosondes. Extra radiosondes were launched at the time of the explosion by the Meteorological Service of Canada aerological stations at Edmonton, Calgary, The Pas, and Suffield, and the U.S. Weather Bureau Stations at Great Falls and Glasgow, Montana. The variations of temperature with altitude registered during these ascents are shown on the next page, on Fig. 39. Considering the large volume sampled by the sondes, it is evident that the horizontal temperature field was remarkably uniform, and undisturbed by frontal systems. Since the wind field over the test area was also free of major zones of shear and curvature (see Fig. 23, p.123) the assumption of horizontal homogeneity seems amply justified.

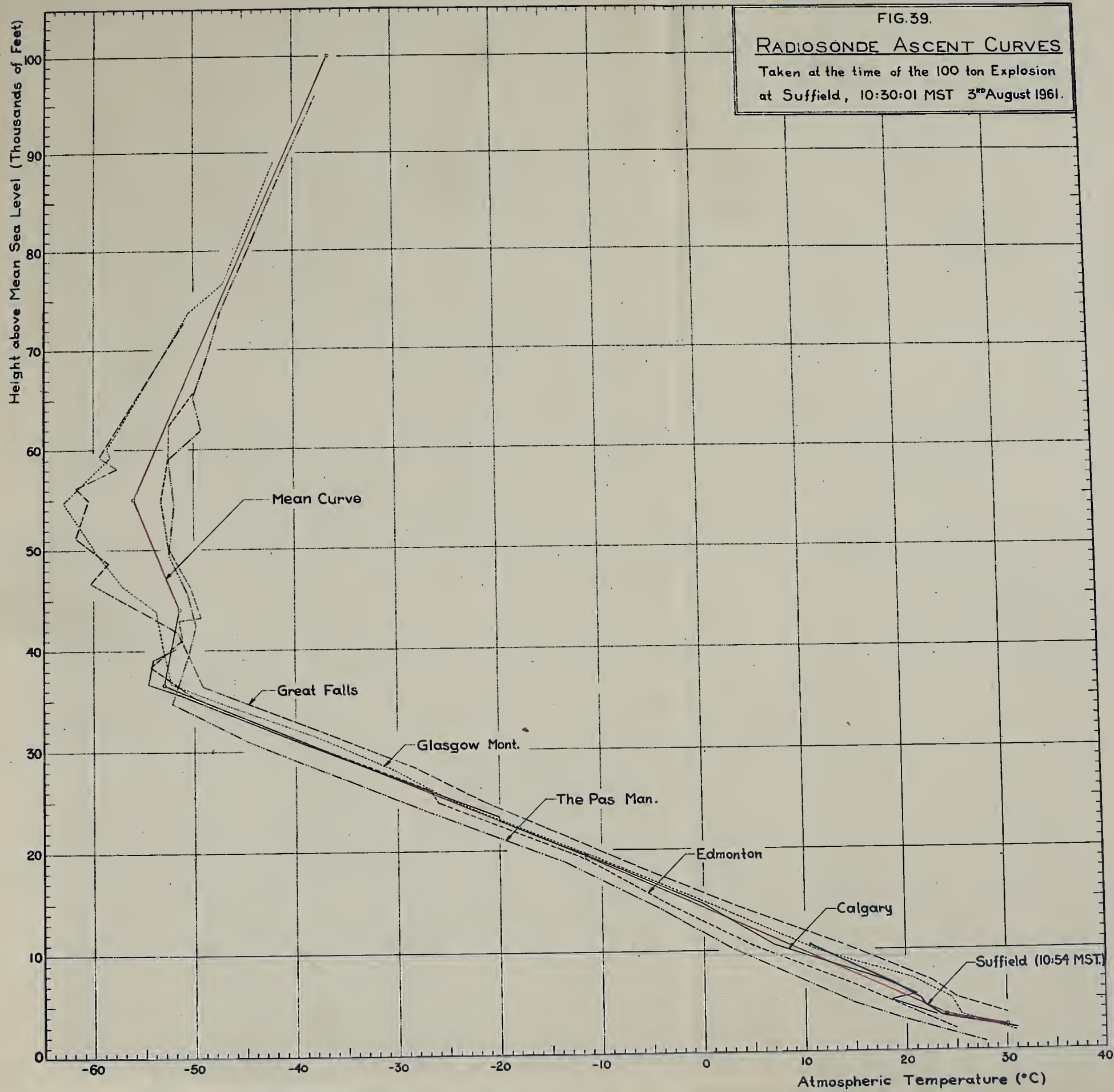
Three of the sondes penetrated to levels above 70 thousand feet, including one which terminated just short of 96 thousand feet, or 29 kilometers. The temperature and wind distributions in the troposphere and most of the stratosphere may hence be considered to be known with good accuracy. For purposes of computation, a mean curve was deduced from the six soundings, giving due regard to the geographical distributions of the ascents with respect to Suffield. This curve is shown in red in Fig. 39.

In Fig. 40, the mean curve has been extended to a height of 60 kilometers. Also shown are the observed mean winds to 30 kilometers, and the deduced winds to 60 kilometers.

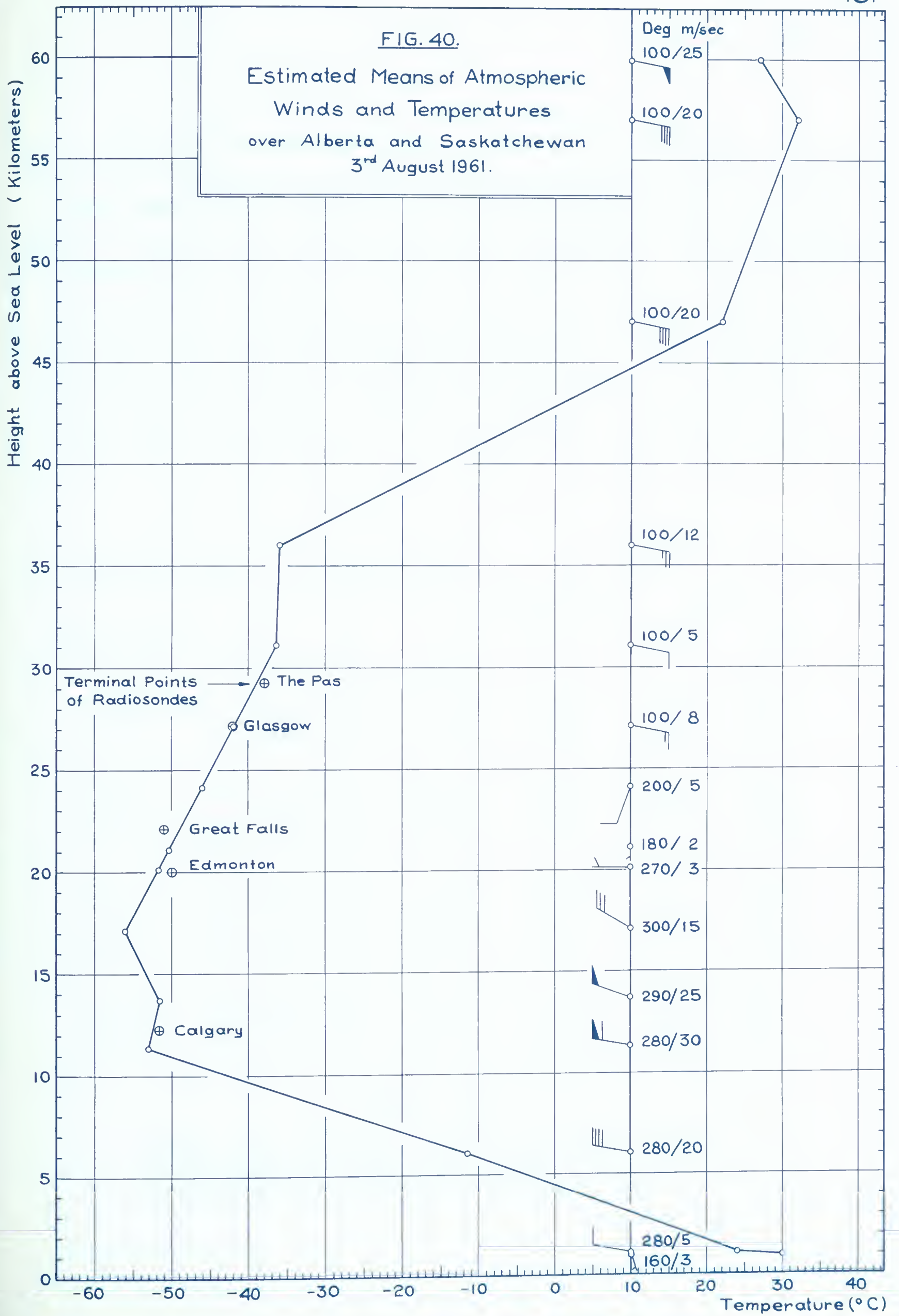
FIG. 39.

RADIOSONDE ASCENT CURVES

Taken at the time of the 100 ton Explosion
at Suffield, 10:30:01 MST 3rd August 1961.



NOTE: This drawing will become a fold-out in a book of size 8 1/2 x 11. Please reduce the drawing so that the height becomes 100,000 feet. Allow a 2-inch margin on the left (after reduction) for binding, and a 1 inch margin on the right.



The winds and temperatures above the 30 kilometer level were derived by trial and error, and successive approximation. Using reasonable estimates of wind and temperature, ray paths were computed for various angles of emergence. The resulting time-distance data was then plotted and compared with the observed travel time curve. The winds were introduced largely on the basis of their effect on the shapes of the shadow zone and the first zone of anomalous audibility, to be discussed in the next section. The mean and deduced temperature and wind data which best fit the observations are listed below:

ALTITUDE (MSL).		TEMPERATURE		WIND		
kilometers	feet	°K.	°C.	Direction	Speed	
				degrees	knots	m/sec.
0.778	2 550	303.2	30.0	160	6	3.0
1.09	3 570	297.2	24.0	280	10	5.0
6.07	19 930	261.7	-11.5	280	40	20.0
11.35	37 220	220.2	-53.0	280	60	30.0
13.68	44 800	221.7	-51.5	290	50	25.0
17.10	56 100	217.2	-56.0	300	30	15.0
20.10	65 950	221.4	-51.8	220	6	3.0
21.10	69 250	222.8	-50.4	180	4	2.0
24.10	79 450	227.0	-46.2	200	10	5.0
27.10	89 000	231.1	-42.1	100	16	8.0
31.10	101 700	236.7	-36.5	100	10	5.0
36.00	118 200	237	-36	100	24	12
47.00	154 200	295	22	100	40	20
57.00	187 000	305	32	100	40	20
60.00	196 800	300	27	100	50	25

Table VIII

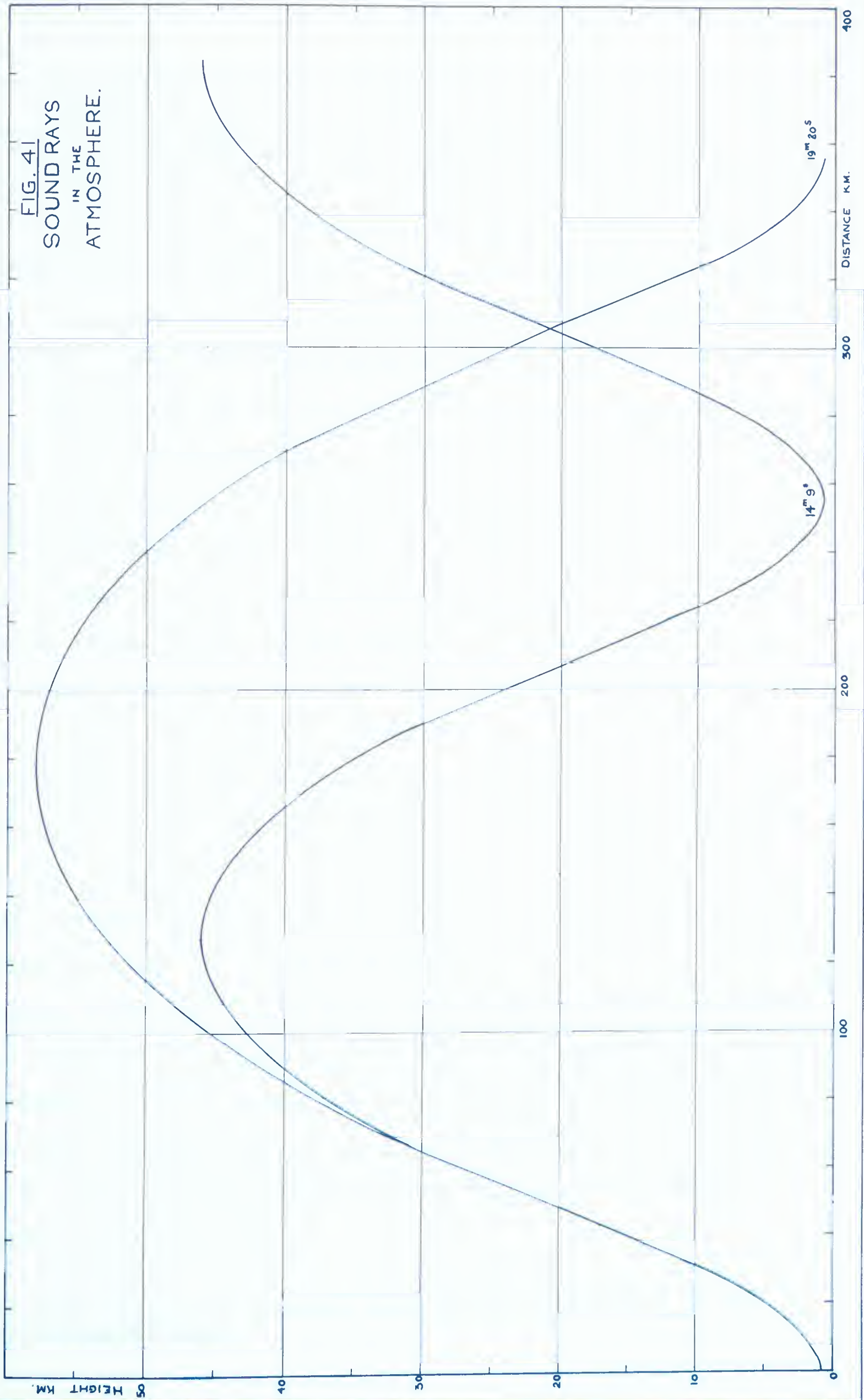
Winds and Atmospheric Temperatures to 60 km.
from anomalous sound measurements made on
the Southwestern Praries, 3 August, 1961.

5.5 Ray Paths and the Pattern of Acoustic Zones

The wind and temperature data given in Table VIII were used to compute families of rays for bearing angles around the compass, at intervals of 15 degrees. The first ray in each family is a ray which starts out horizontally from the site of the explosion, and lands usually close to the inner boundary of the first zone of audibility. The last ray in a family is a terminal ray which returns near the tenuous outer boundary of the first anomalous zone. Such rays emerge from the shot point usually with angles of 20 to 30 degrees and are, in the limit, refracted to infinity. For reasons of computer economy, rays well within the bounding envelope were usually chosen as limiting rays.

The typical, if somewhat schematical ray paths shown in Fig. 41 were plotted from data derived from preliminary computations in quiet air. The detailed computation of the acoustic zones was made in moving air, however, neglecting only the wind component transverse to the ray trajectory. A plot of these results is shown in Fig. 42a and, in more detail, in Fig. 42b. The pattern shown is derived entirely from single skip rays, and represents the intersection of the bounding envelope of rays with a horizontal plane tangent to the Earth's surface at the Suffield shot point. The curves shown in Fig. 42b are in essence isoclinic lines, defining the intersections with the tangent plane of cones of rays which emerge from the shot point at all possible angles.

FIG. 41
SOUND RAYS
IN THE
ATMOSPHERE.



The spacing of the isoclinic lines is, of course, a direct measure of the energy density received at the surface. The sharp inner boundary is clearly marked, but the outer boundary is ill-defined and slowly fades away into silence. However, as mentioned in Section 2.14, there will normally be considerable overlap between the first, second and third zones, particularly downwind* from the site of the explosion.

Not all reports of positive signal fall within the predicted zone of audibility. Point 2 falls just within the shadow zone and is probably due to diffraction into the bounds of the geometric zone. Point 12 is well outside the area of good signal strength, but it is almost certainly within the strong signal region of the second, and just possibly within the third zone as well. The first zone could be made to enclose Point 12 more definitively by one of several means: temperature and wind modifications near 60 km; a sonic duct between 50 and 60 km; tilted layers in the upper atmosphere, or even a marginal ray. Computation to fractional degrees could extend the zone well to the west, but the predicted intensity near the limit of a critical ray would be, of course, very low. This poses a problem, for the strength of the recorded signal was rather high. An examination of the frequency content of the signal leads one to believe that it was not a marginal ray at all, but a guided ray in a sonic duct. The spectrum of this wave train will be examined later, in Section 5.8.

*To the west in the present case, in the direction of the upper easterlies.

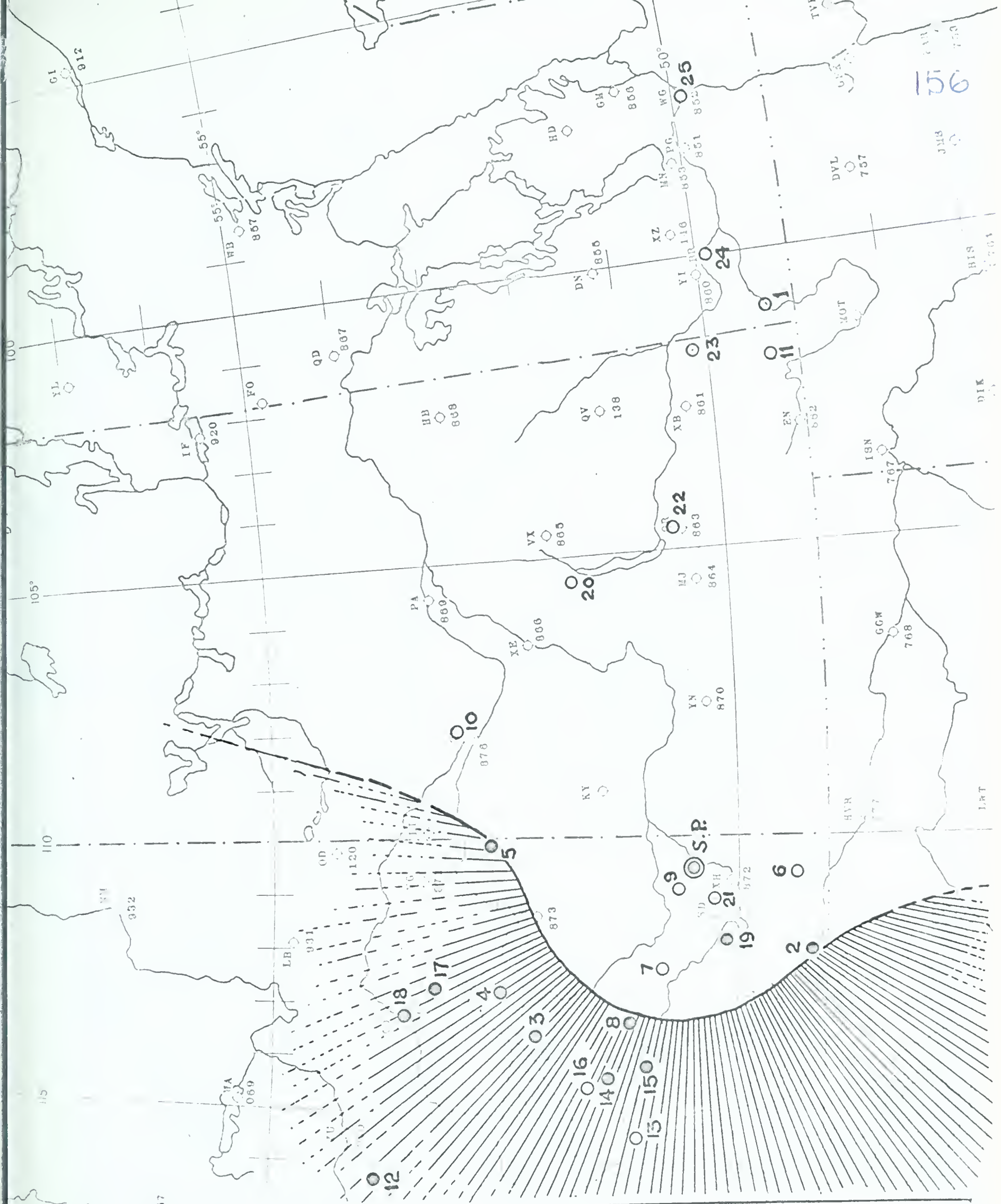


Fig. 42a.

First Zone of Anomalous Audibility.

Suffield 100 ton Explosion

3rd August, 1961

● positive reports

○ negative reports

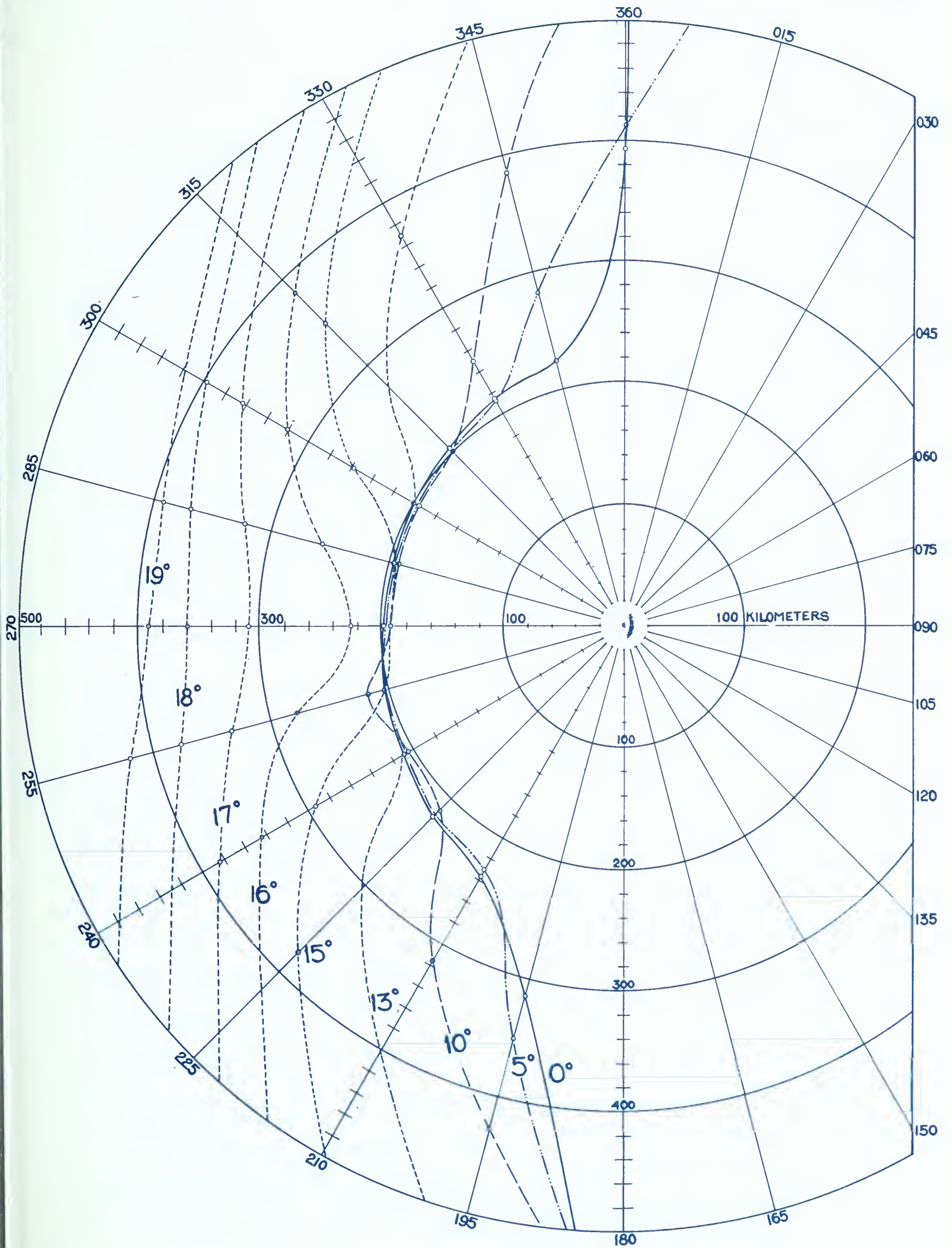


Fig. 42b.

First Zone Isoclines.

5.6 Absorption Curves

The absorption along rays propagating due west from the shot point was computed at 0.5 degree intervals for angles of emergence between zero and the critical value for which rays were no longer refracted back to the ground. The results of this computation are shown in Fig. 43, and a sample of the calculated data is included in Appendix VI. The absorption curves have been plotted for frequencies of 5, 10, 20 and 50 cycles per second. The absorption at 1 and 0.1 cycles per second has been computed also, but for reasons of clarity these results are not shown in the diagram.

The first set of curves (A) shows the great complexity of the energy distribution in the vicinity of the inner boundary of the zone of audibility. This is the region where double and triple events are commonly observed, and the fine structure splitting of travel time curves takes place. The complexity is actually much greater than shown in the diagram. The wave front becomes highly distorted in this region, and the travel time curve will normally contain a number of marked discontinuities, such as a major cusp at the geometric inner boundary, and several minor cusps within the zone itself. At the point of a cusp the travel time curve is split, and the differential $di/d\Delta$ in the absorption equation undergoes a change in sign. Physically, this simply means that the function is no longer single valued, and that multiple arrivals will be received within the

domain of the cusp. It is in this part of the zone that the wave fronts interfere with each other most severely. The observed energy density varies between wide limits even over quite short distances, but the degree of absorption is about the same for all frequencies below 20 cycles per second. Even the 50 cycle curve reflects a faithful family resemblance; the attenuation is simply greater at the higher frequencies, an expected result.

The other set of curves, spread over four sections of Fig. 43, shows the absorption between rays which strike the inner boundary, and the critical ray which is refracted to infinity. This part of the absorption curve is rather less complicated, except near the inner boundary, at the junction to the principal cusp. But the absorption at individual frequencies differs now widely. The curves for the lower frequencies are still, at least superficially, similar enough, but the attenuation increases markedly for the higher frequencies. Little if any audible sound should be transmitted along a single skip ray to distances in excess of 300 km, a supposition in general accord with observation. However, there is no theoretical objection to audible frequencies being transmitted to distances beyond 400 km. by means of double- or even triple-skip rays. Such rays would not normally penetrate to heights much greater than 45 km, and their attenuation would thus be much less severe.

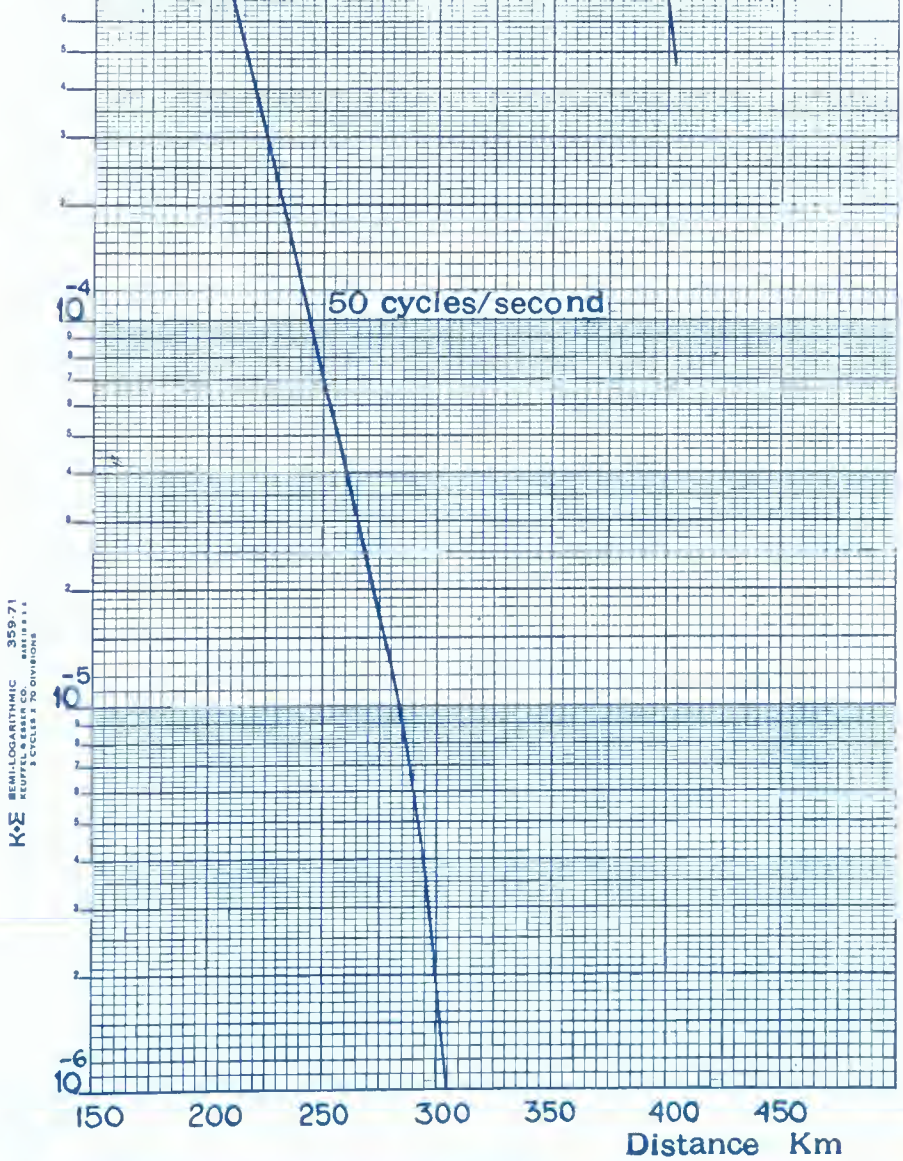
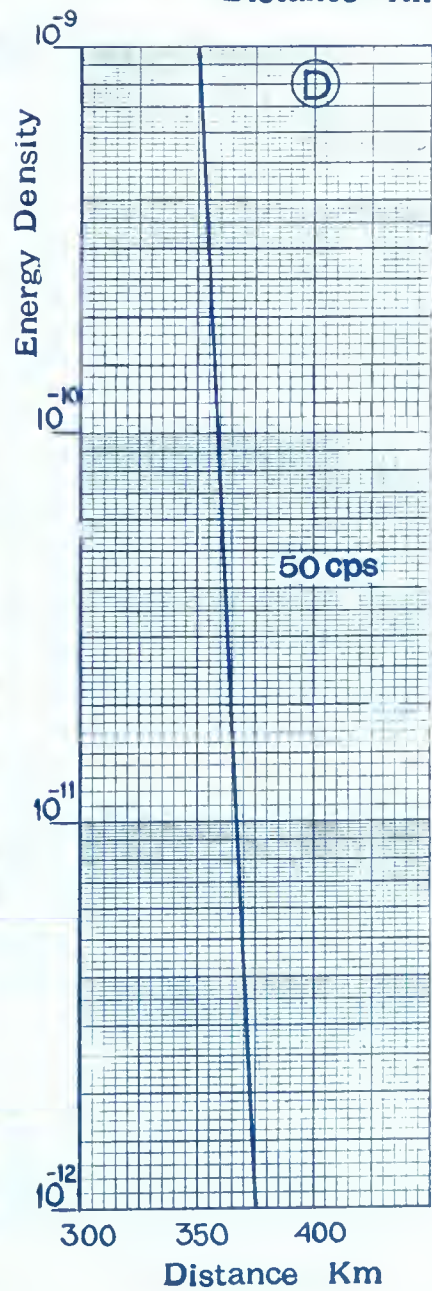
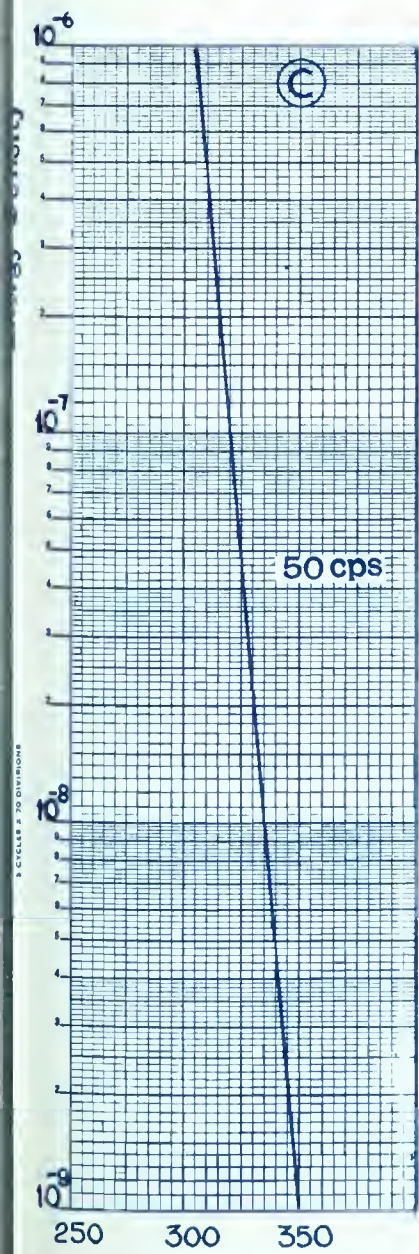
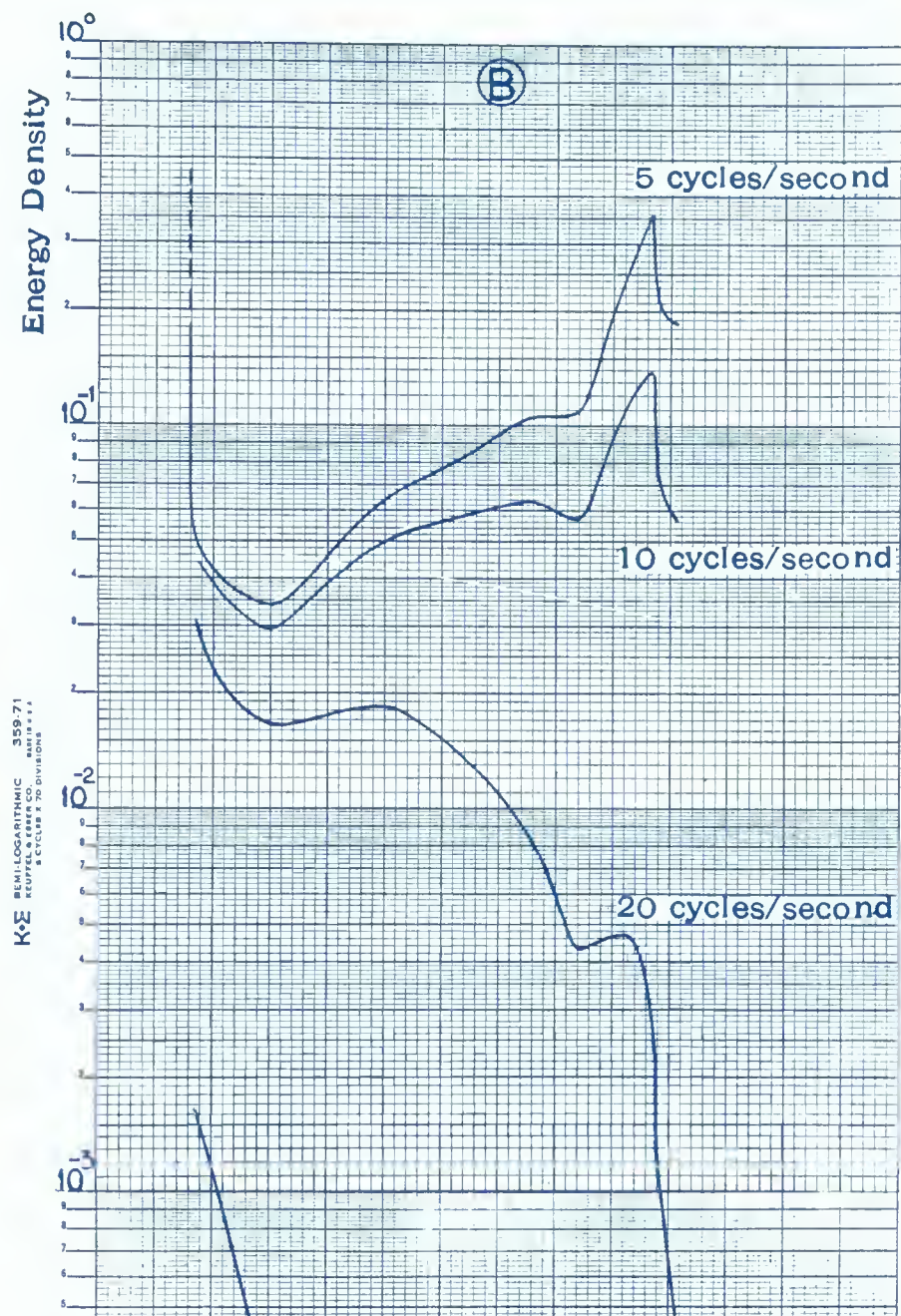
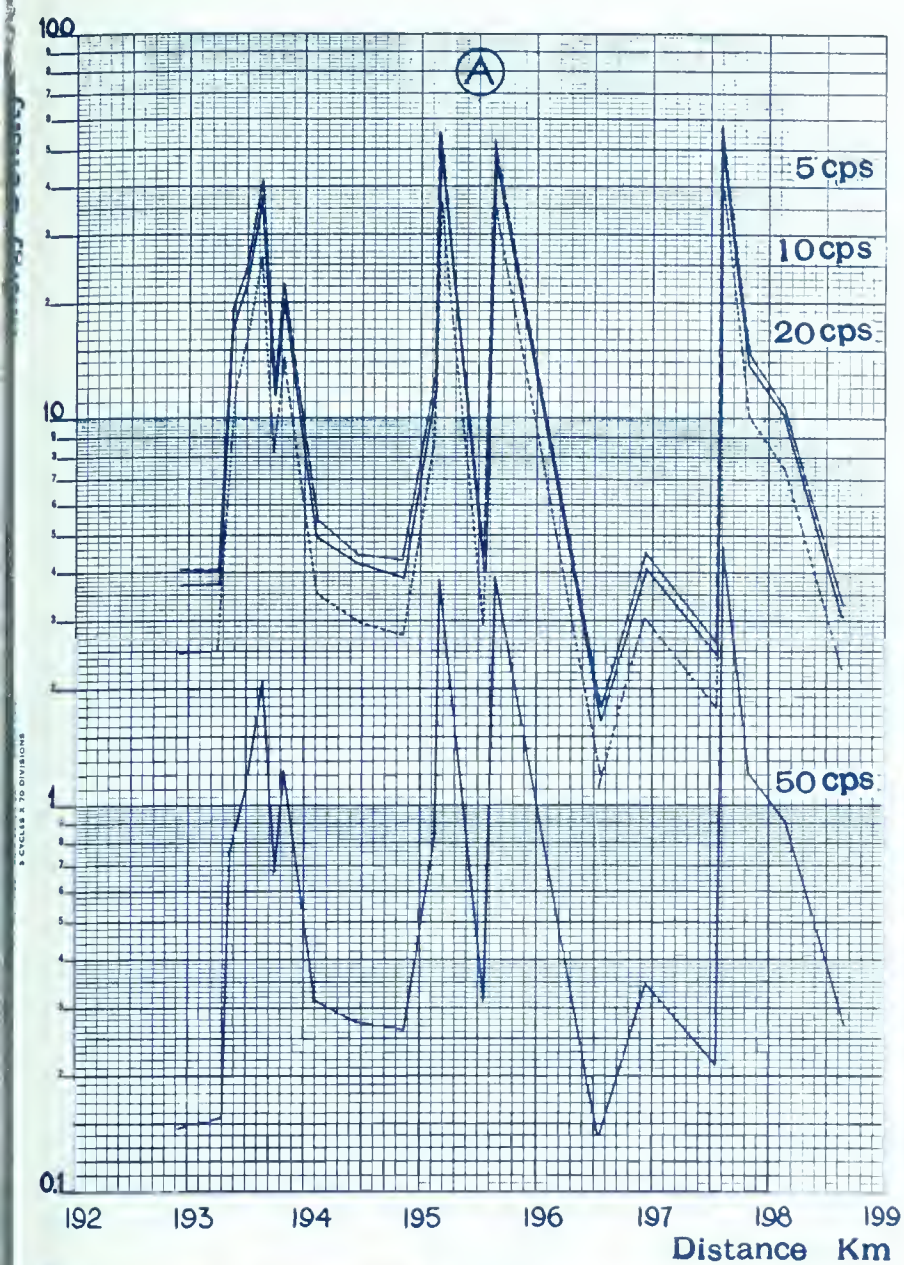


Fig. 43

Absorption Curves.

5.7 Sound Rays in a Standard Atmosphere

With the recent publication of the revised U.S. Standard Atmosphere data to 700 kilometers, it seemed desirable to compute ray paths for a set of typical angles of emergence up to the limit of the critical penetrating ray. The data used in the computation are given below, (Table IX), and are plotted in Fig. 44 on Page 162.

Standard Atmospheric Data*

Height above Mean Sea Level		Atmospheric Temperature	
Feet	Kilometers	°C.	°K.
0.00	00.000	14.99	288.15
36151.14	11.019	-56.51	216.65
65822.69	20.063	-56.51	216.65
105517.09	32.162	-44.51	228.65
155345.88	47.350	- 2.51	270.65
172009.06	52.429	- 2.51	270.65
202067.75	61.591	-20.51	252.65
262444.32	79.994	-92.51	180.65
298549.52	90.999	-92.51	180.65
328080.00	100.000	-62.51	210.65
360888.00	110.000	-12.51	260.65
393696.00	120.000	87.49	360.65
492120.00	150.000	687.49	960.65
524928.00	160.000	837.49	1110.65
557736.00	170.000	937.49	1210.65
623352.00	190.000	1077.49	1330.65
754584.00	230.000	1277.49	1550.65
984240.00	300.000	1557.49	1830.65
1312320.00	400.000	1887.49	2160.65
1640400.00	500.000	2147.49	2420.65
1968480.00	600.000	2317.49	2590.65
2296560.00	700.000	2427.49	2700.65

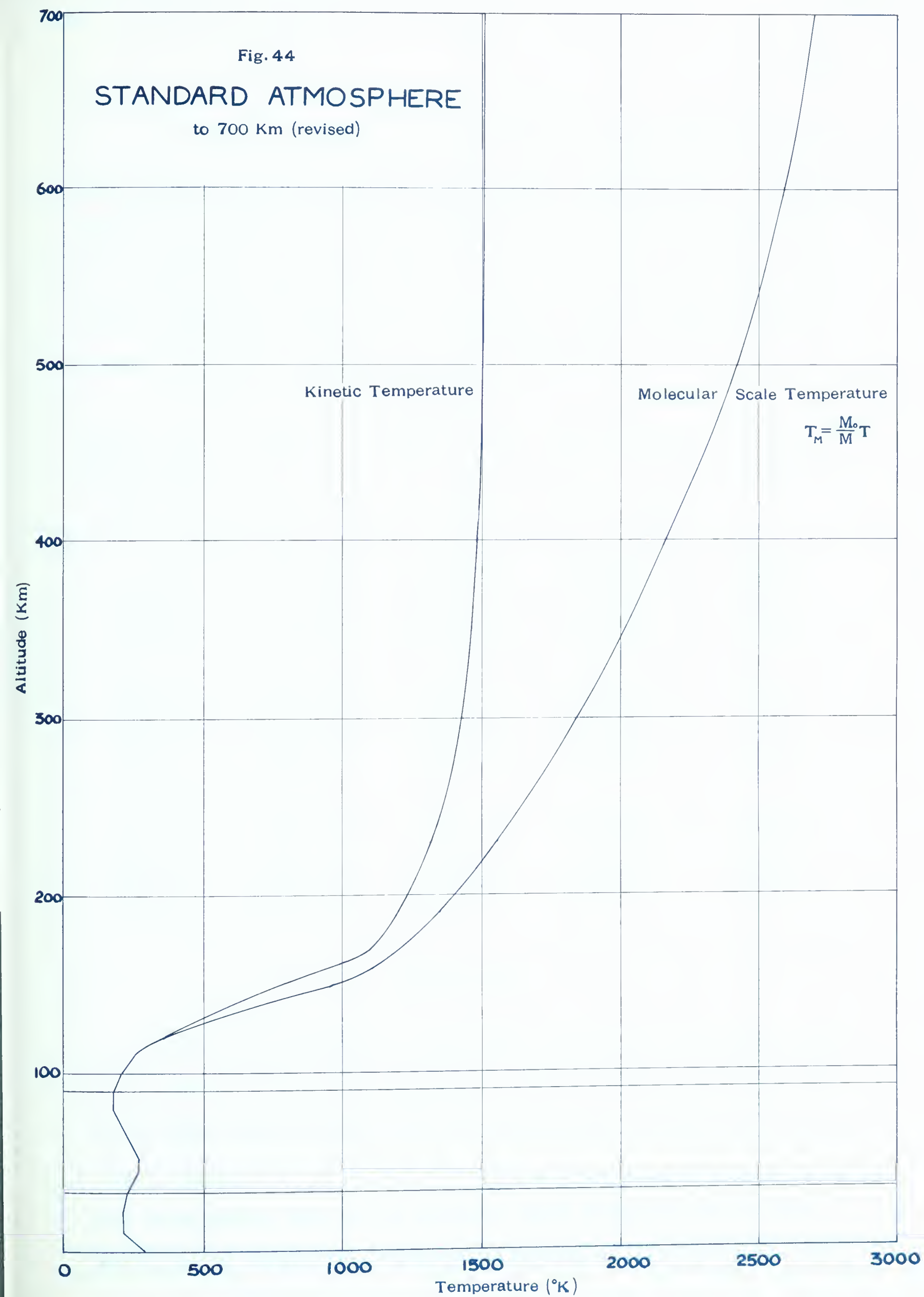
TABLE IX

*See Sissenwine (1962); Champion et al. (1963)

Fig. 44

STANDARD ATMOSPHERE

to 700 Km (revised)



It will be noted that the standard temperatures of the mesosphere are all lower than surface temperature. In the absence of wind, no rays whatever would hence be refracted to ground by the warm layers of the mesopeak, and all rays would penetrate into the ionosphere. The results of the computation are tabulated below (Table X).

Angle of Emergence	Range	Travel Time	Apex Height	Apex Temperature
Degrees	Kilometers	seconds	Kilometers	^o K.
0	540.848	1919.26	112.750	288.2
5.00	516.896	1849.04	112.971	290.4
10.00	481.056	1744.74	113.646	297.1
15.00	441.252	1633.54	114.819	308.9
20.00	406.382	1533.08	116.568	326.4
25.00	376.140	1451.00	119.016	350.9
30.00	333.846	1341.94	121.178	384.3
35.00	303.288	1265.18	123.440	429.5
40.00	280.978	1213.12	126.521	491.1
45.00	266.210	1181.04	130.784	576.4
50.00	259.996	1168.56	136.840	697.5
55.00	265.092	1177.44	145.765	876.0
60.00	347.110	1303.14	164.207	1152.8
65.00	857.380	1982.48	245.715	1613.5
70.00	2481.916	3746.20	525.259	2463.6

TABLE X.

It is seen that all rays with angles of emergence less than about 70 degrees should be refracted to ground by the very high temperatures of the ionosphere. Detailed absorption computations for the Standard Atmosphere curve have not yet been made, but it is evident that frequencies in the aural range would be attenuated severely, particularly for

angles of emergence greater than 30° . Rays leaving the ground at angles greater than 70° would, of course, not be returned at all.

The significance of the results for the present purpose is this: the range of the ray decreases for increasing angles of emergence up to about 55 degrees, and the rays penetrate to progressively greater altitudes as the range shortens. A ray which lands relatively close to the source will hence suffer much more severe attenuation than a ray which lands much further away. The high frequency content of distant arrivals may thus well be greater than that of arrivals landing closer to the source. It should be noted, however, that this is no longer true for steep rays which leave the source at angles greater than 55 degrees.

5.8 Power Spectra

Much of what can be said in the present context about the value of power spectrum analysis must needs be tentative, for at least two reasons:

- (1) the recording equipment was not expressly calibrated for frequency response and sensitivity, and
- (2) some of the spectra had to be obtained in a roundabout and "noisy" way.

The condition of the EVP-5 pressure phones was not always known, particularly in connection with the 100 ton test. Field tests made on the phones in 1962 (see Section 4.4) showed wide variations in response and sensitivity, from

excellent to nil. The frequency response curve of the EVP-5 supplied by the manufacturer is probably quite representative in the case of new phones, but characteristics undoubtedly change with time and abuse. It is only fair to say, therefore, that the tests were largely carried out with uncalibrated instruments. This was not so much a case of simple oversight, but, by the very nature of the project, almost a necessity, considering the make-up of the far-flung net of stations, and the great variety of equipment used. Initially at least, the primary goal of the experiment was simply the determination of arrival times and the geographic distribution of sonic returns; and almost any detector might do. Frequency analysis came, as it were, as an afterthought.

With two exceptions, all traces analysed were recorded with EVP-5 pressure phones; analyses of microbarograph and geophone traces are so marked on the appropriate diagrams.

Since the recording characteristics were apt to be quite different, little direct comparison should probably be made between different sets of curves. But some deductions can justifiably be made regarding the sequence and intensity of events recorded at any particular site. The spectral curves in each set are all drawn to the same scale, but the scale varies from set to set, and is not standardized in any way.

Not all records were suitable for power spectrum analysis. Only the high speed records showed enough resolution to make analysis worthwhile. The slow speed B.A. record of August 15,

1962, was so interesting, however, that an enlargement of it was analysed. But this involved certain difficulties: enlargement had broadened the galvanometer traces to such an extent that the traces had to be sketched in by hand onto transparent paper, and the half-inch-wide timing intervals subdivided into 10 parts. No doubt, this process introduced a good deal of noise into the spectrum and it is largely for this reason that the analysis is of dubious value. Nevertheless, two EVP-5 traces and one S-36 geophone were analysed in this way.

Lacking calibration, the term "power density" as used here, denotes strictly a relative quantity with little more than qualitative meaning. Rigorously, the power density is defined as the quantity $P(\nu)$ in the expression

$$P = \int_0^{\infty} P(\nu) d\nu \quad , \quad 5-1$$

where P is the total acoustical power arriving at the detector, and ν is the frequency.

Allowing for all these deficiencies, several conclusions may still be drawn from the analysis of the frequency content. Since the response of the phones is still high at 30 cycles and essentially flat to 100 cycles per second, the loss of high frequency content must largely be due to absorption, in full agreement with Schrödinger's theory. The sharp drop in response below 3 or 4 cycles, on the other hand, would seem to be largely due to the lack of low frequency response of phone and amplifier. This is shown rather clearly in Fig. 45.

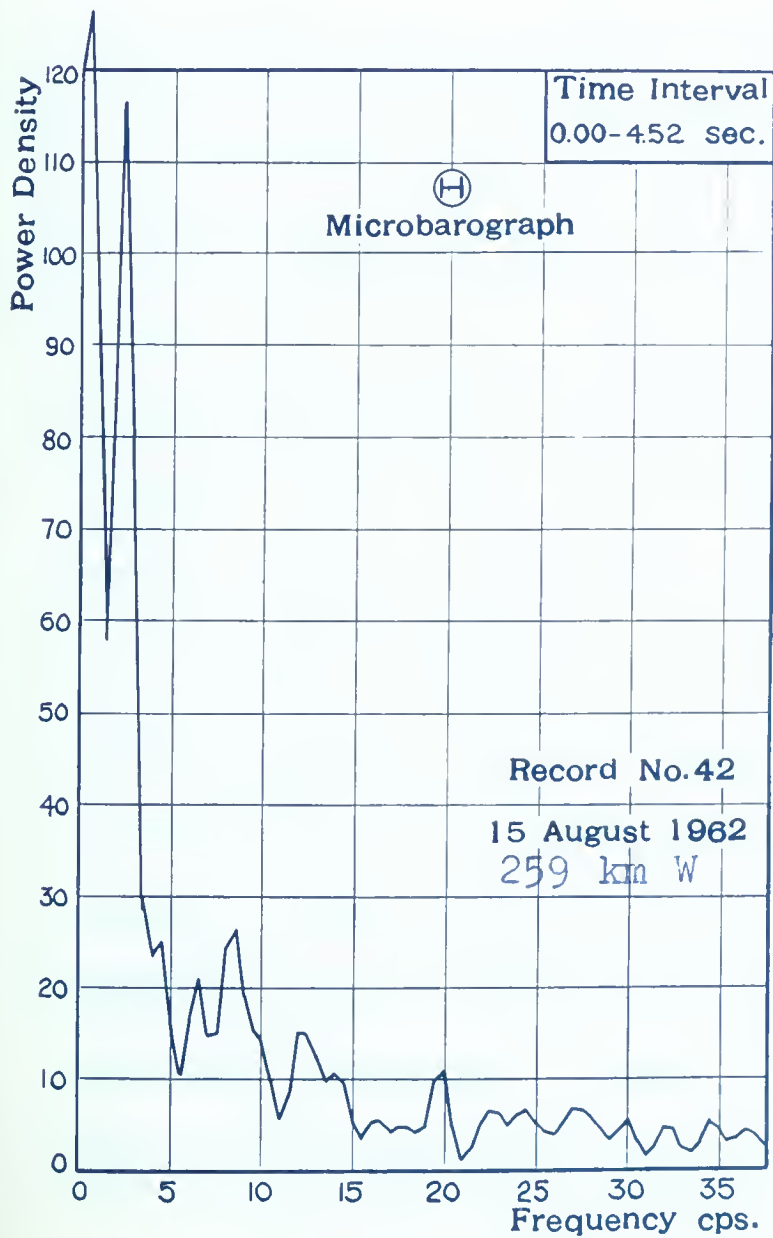
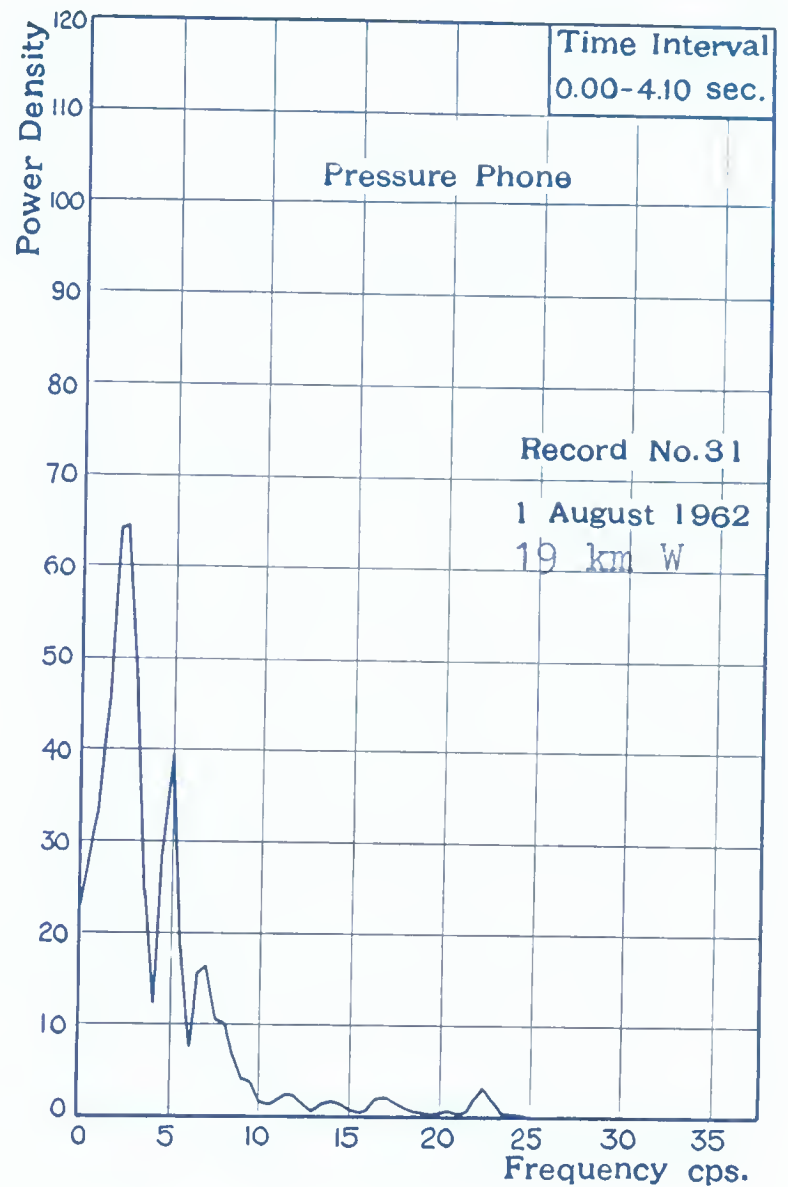
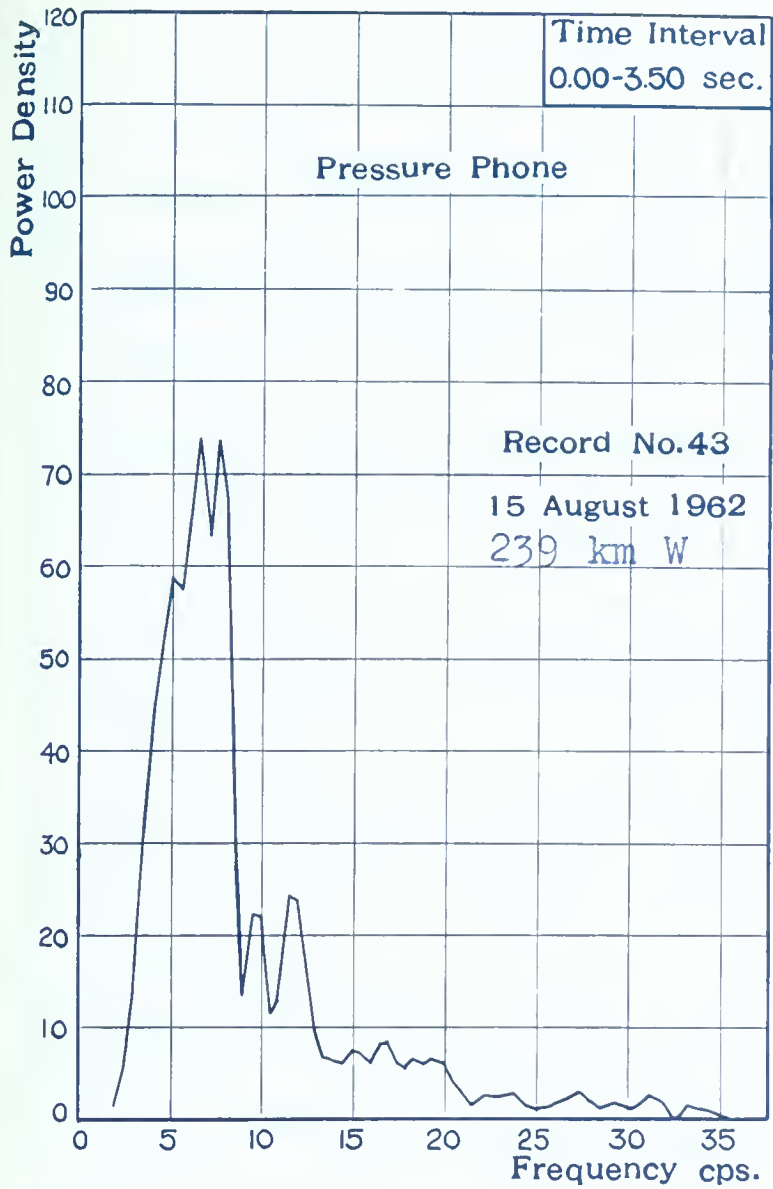
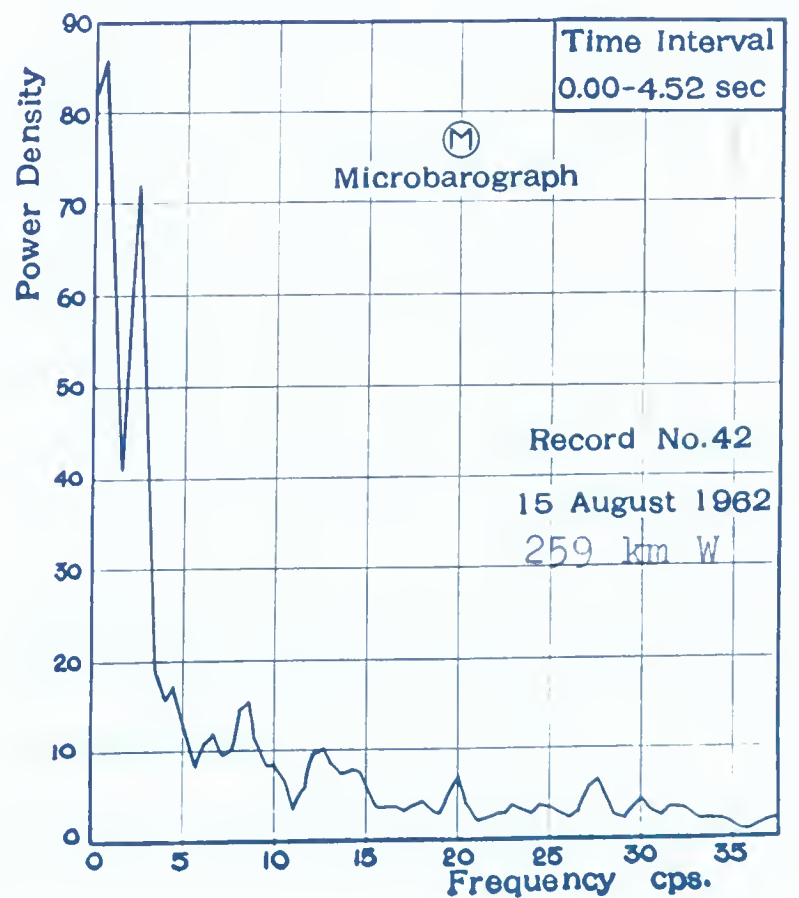


Fig. 45

Power Spectra.

 $Df = 0.5 \text{ cps}$
 $Dt = 0.01 \text{ sec}$


Of the four curves shown, two are analyses of the same microbarograph trace, one digitized by hand, and the other mechanically.* Record 43, taken 239 km from the 20-ton explosion, shows a very sharp drop in response below 5 cps. Record 31, 19 km from a 5 ton blast, also shows the drop, but since the intensity of the wave was so much higher (the blast was heard clearly by the observers) even the lower frequencies registered with good amplitude. The microbarograph response was clearly excellent also at the lowest frequencies and at a considerable distance (259 km) where the intensity of the wave was comparatively low.

The peculiar "line spectrum" structure shown by most of the curve is, however, another matter. If the reasonable assumption is made that the energy of the decaying shock wave is converted to "white" acoustical energy, the appearance of the spectrum defies explanation in terms of existing theory. Does the atmosphere absorb some frequencies selectively, and if so, why? It is evidently not simply a case of dispersion, for some low frequencies are missing altogether, while higher frequencies may be present in abundance.

Fig. 46, page 170, shows an interesting set of curves from records of the 20 ton blast, when three recording stations were set up in line well to the west of the shot point. The spectral curves seem to confirm an observation made by Cox at the time of the Helgoland explosion - that the high frequency content of a wave may increase with

*Marked (H) and (M) on the graphs.

increasing distance. At first glance, this phenomenon would seem to go counter to existing theory, but it may well be that at least part of the effect can be explained in terms of ray acoustics. The diversion of Section 5.7 on the Standard Atmosphere is an attempt to do so.

It is difficult to say more about this effect without the help of further evidence. The shift of frequency in the present records is rather small, (about 1 cps) and may be insignificant, in view of the uncertainties in instrumentation. In addition, the analysis of Record 43 suffers from the deficiencies mentioned earlier. However, since the analysed portion of the record contains three distinct events, the spectrum should be fairly complicated. At any rate, the signals analysed all show three major peaks, and the displacement is such as to give added support to Cox's Helgoland observations.

The last spectrum in Fig. 46 is that of an induced ground wave detected by an S-36 TI geophone. There can be little doubt that a great deal of digital noise was introduced into this record, by the very character of the trace. But the interesting point to note is the almost total loss of frequencies below 5 cps. Considering the good frequency response of the geophones near 2 cps., this is indeed remarkable, and one can only assume that no lower frequencies were generated by the inducing aerial wave.

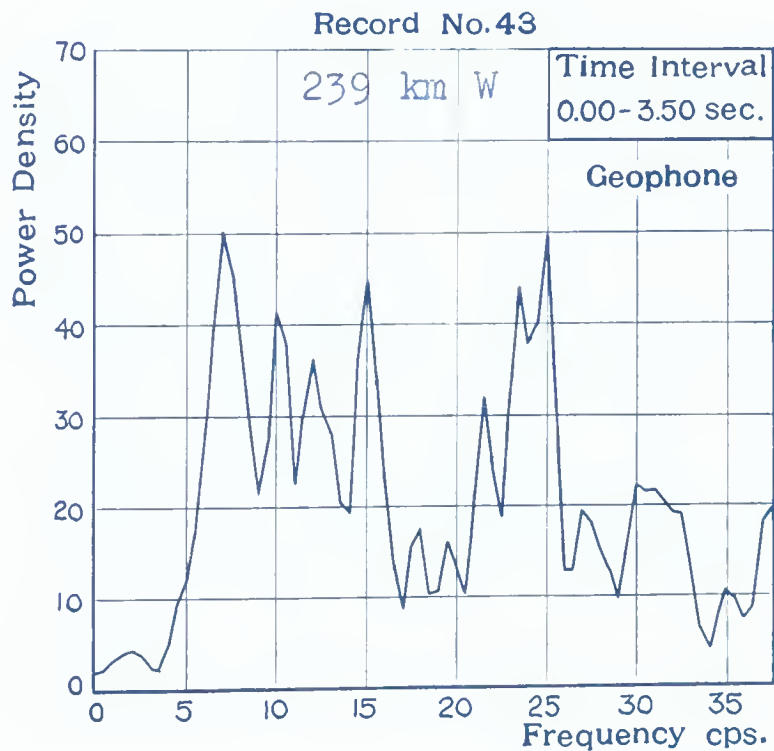
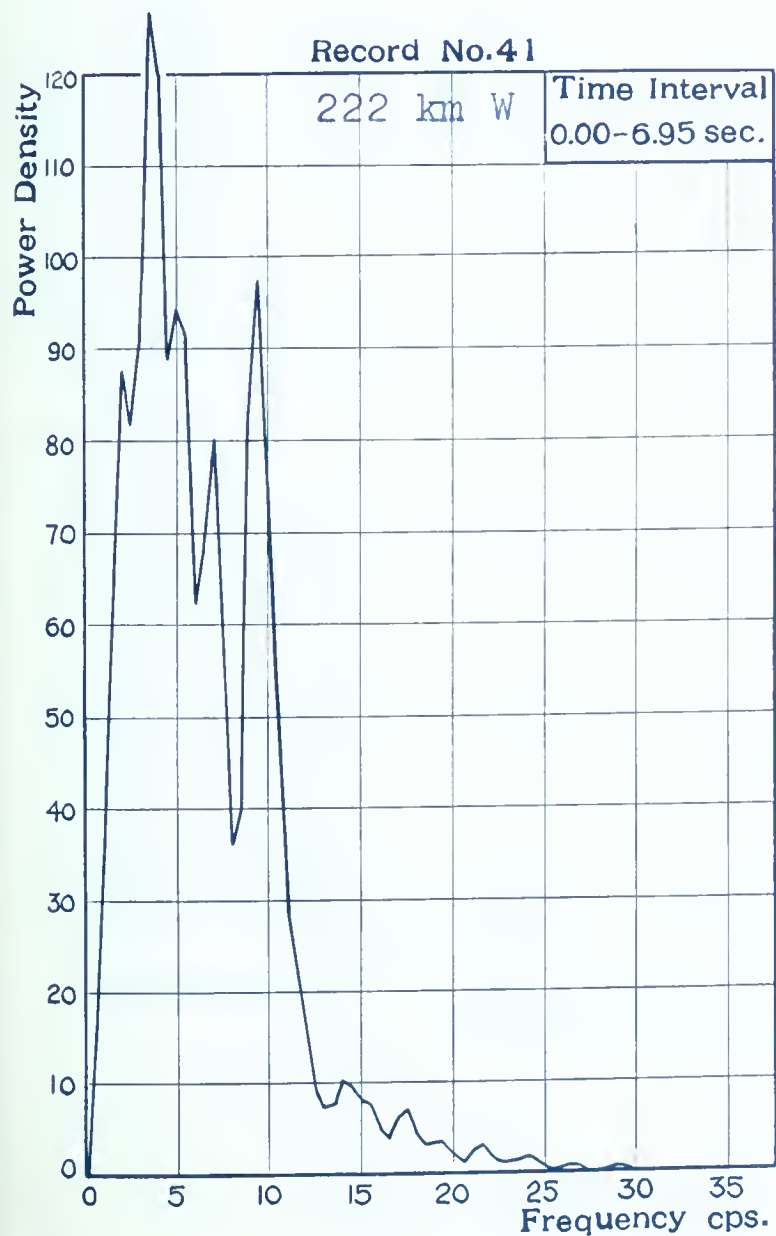
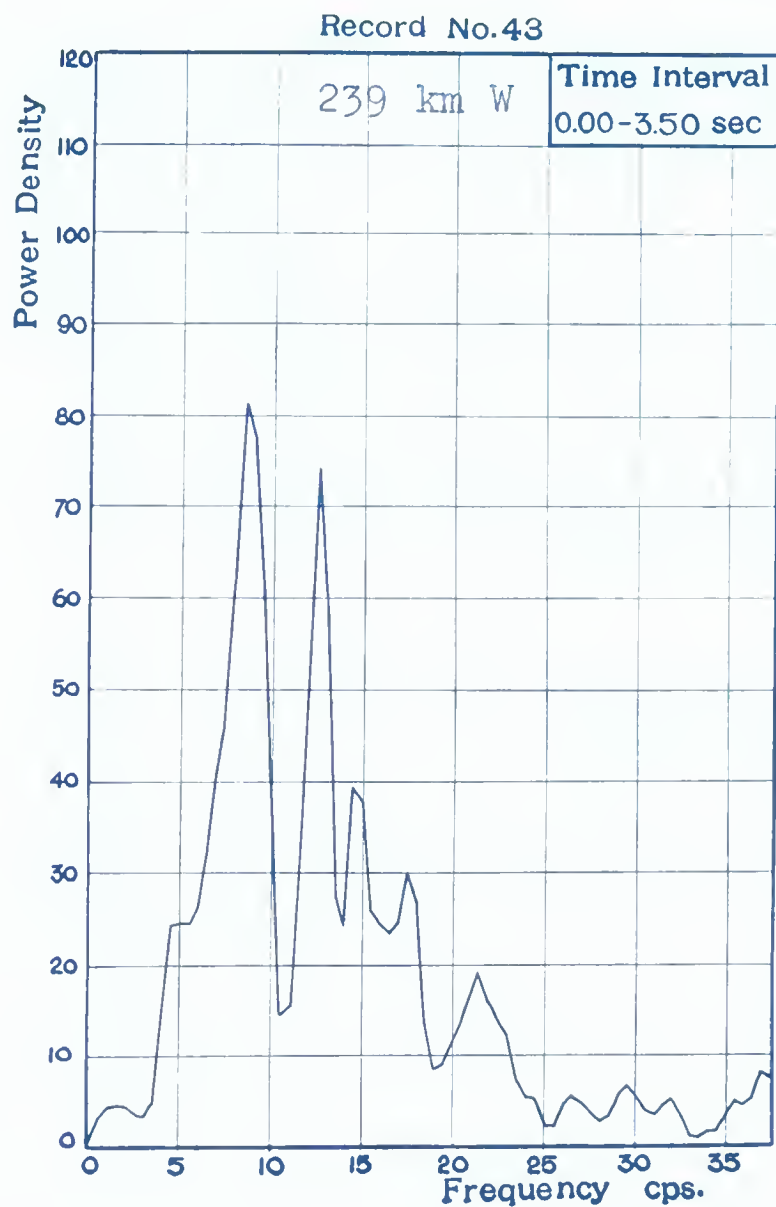
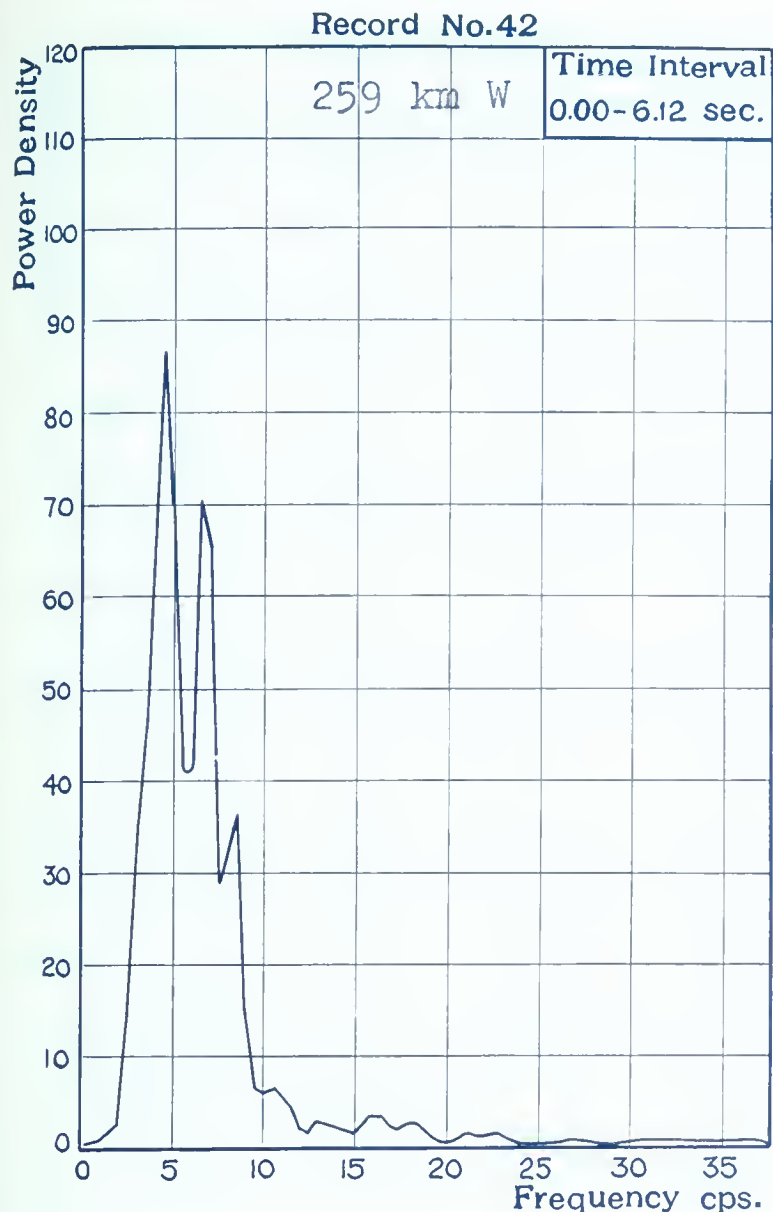


Fig. 46

Power Spectra.

Single Skip Arrivals

15 August 1962

 $Df = 0.5$ cps $Dt = 0.01$ sec

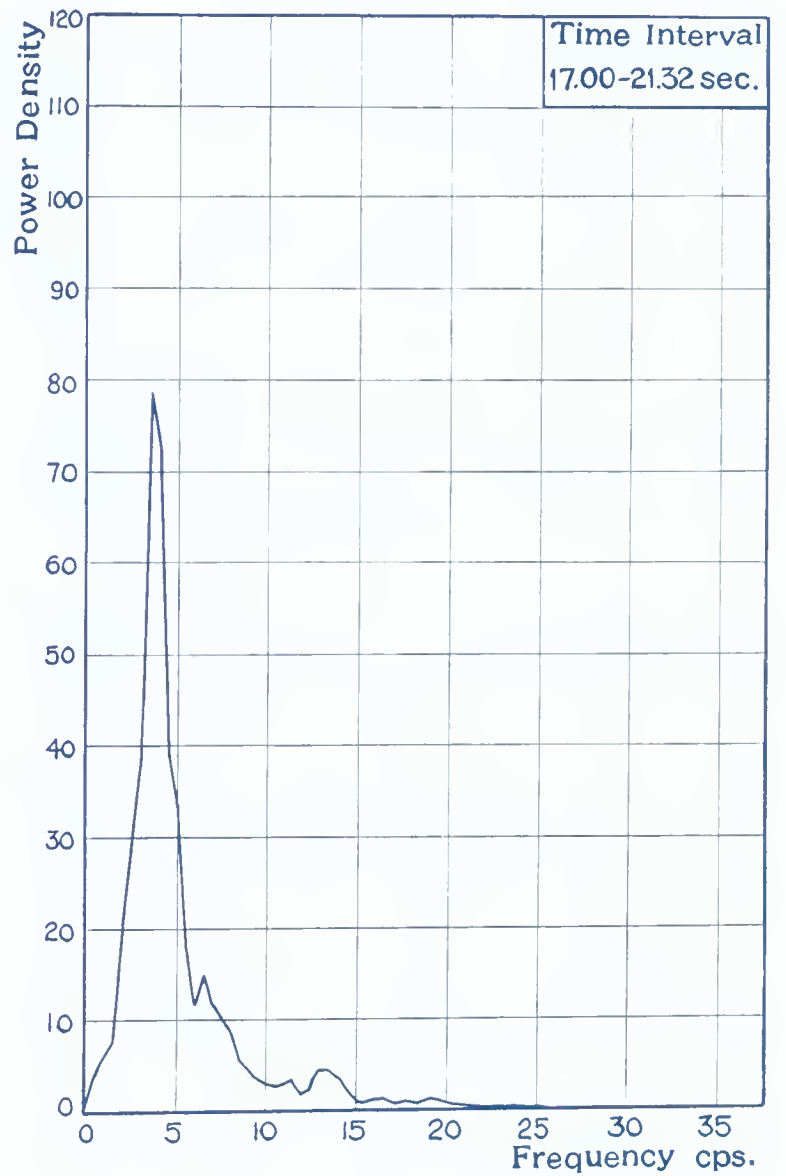
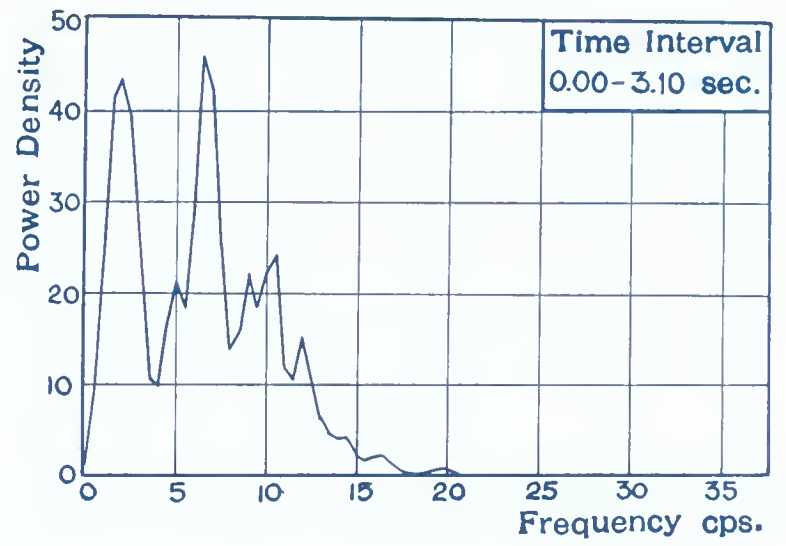
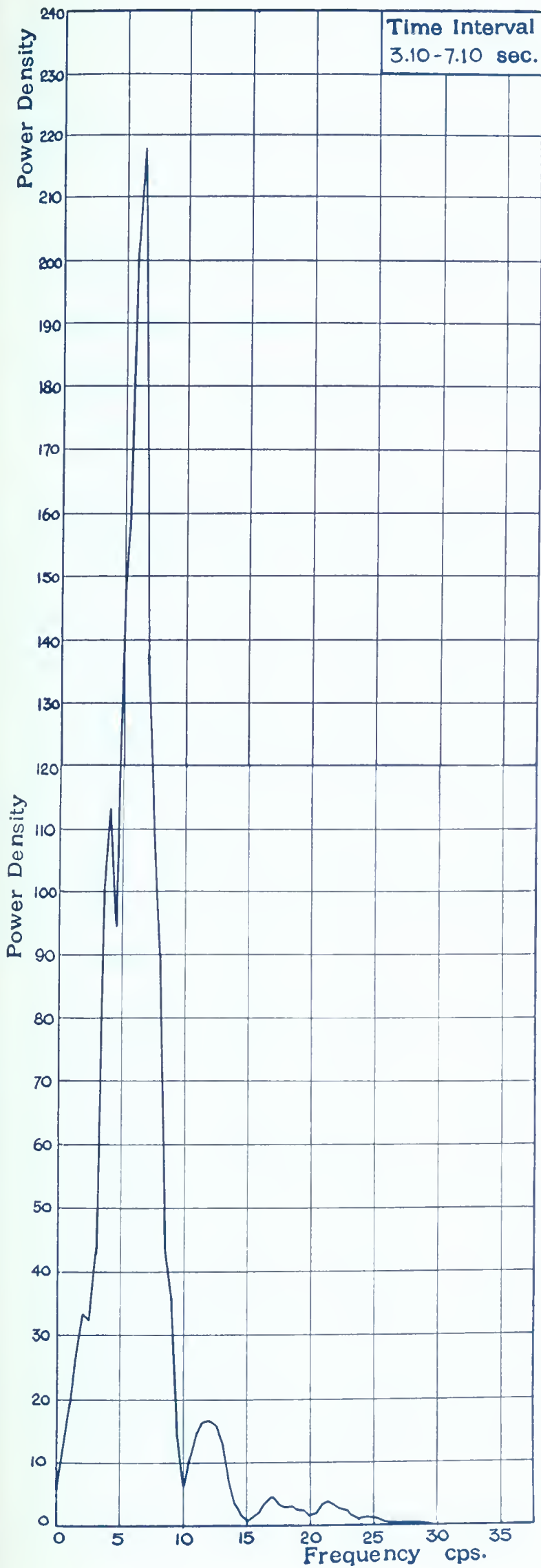


Fig. 47

Power Spectra.

Record No.32 192 km W 1 August 1962

Df = 0.5 cps

Dt = 0.01 sec

The high speed records that follow were all analysed pulse by pulse, from an arbitrary zero reference time (0.00 in the graphs) near the beginning of the first discernable pulse. Of the 1961 records, four long wave trains were analysed in this way. The three separate events of record 32 (1962) were also analysed pulse by pulse; this interesting spectrum is shown in Fig. 47 on the previous page.

The remaining sets of spectra are presented below without individual comment, but one or two tentative generalizations may be in order:

The leading pulse in each train is usually of very low intensity and low frequency. Second and subsequent pulses increase in amplitude and frequency content to a cut-off value near 30 cps. The trailing pulses are usually again of much lower amplitude and their spectra are flatter, with few sharp peaks, particularly in the higher frequencies. It is evident also that the total length of the signal increases with distance, a fact noted before by other observers. This phenomenon is not a case of true acoustic dispersion, but due to the superposition of signals which have arrived along different trajectories.

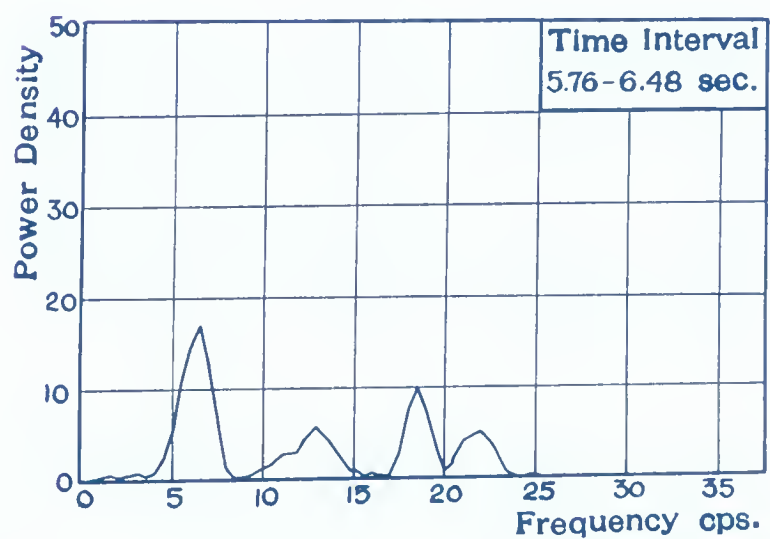
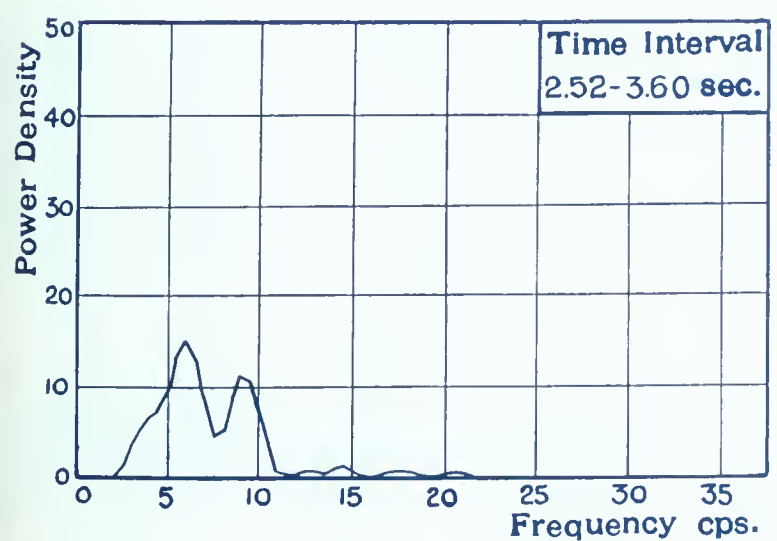
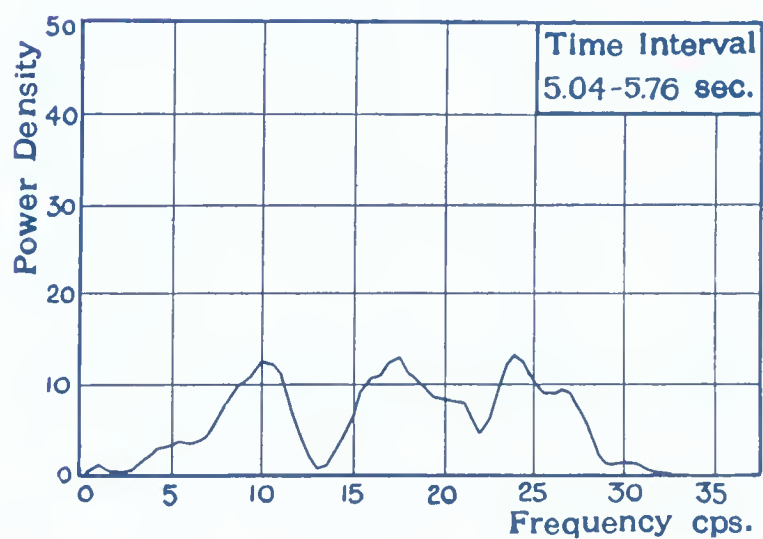
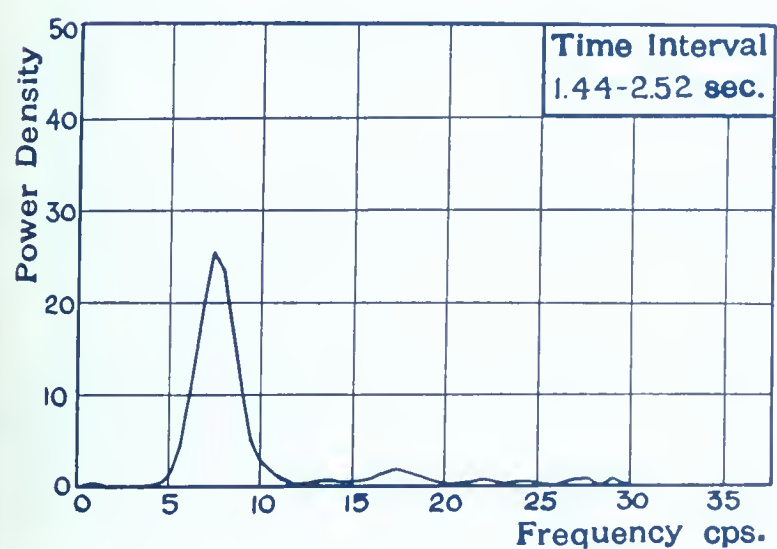
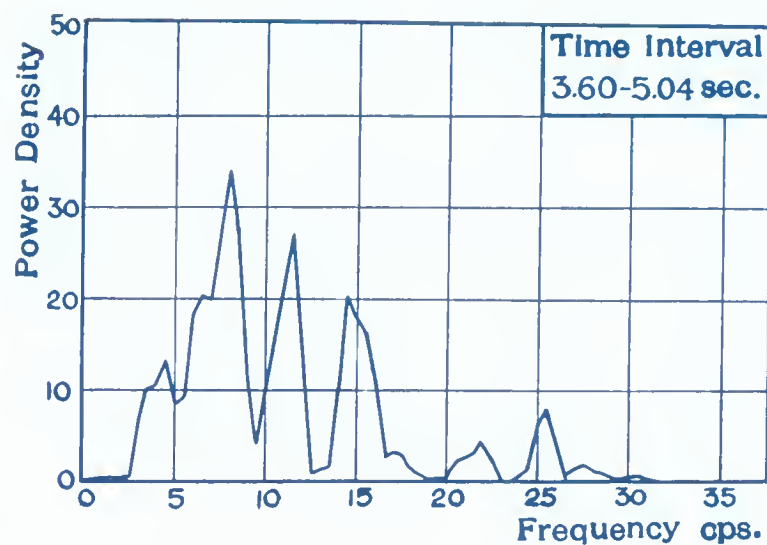
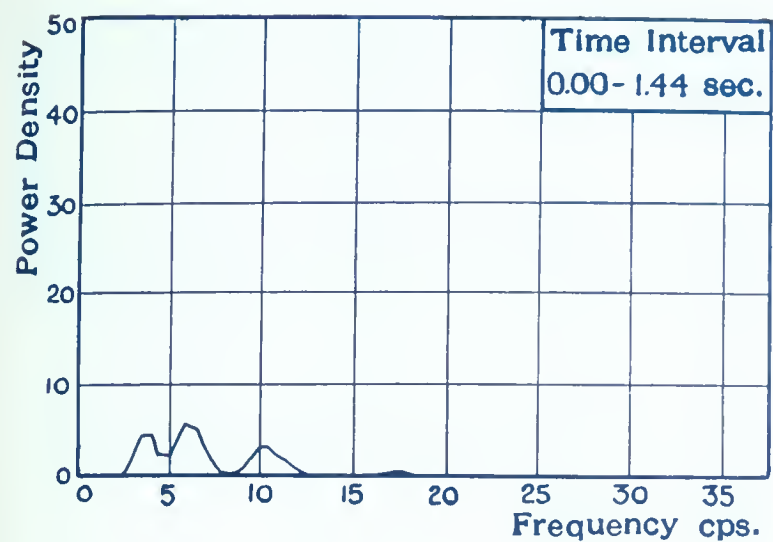


Fig. 48

Power Spectra.

Record No. 14

271 km WNW

3 August 1961

 $Df = 0.5$ cps $Dt = 0.01$ sec

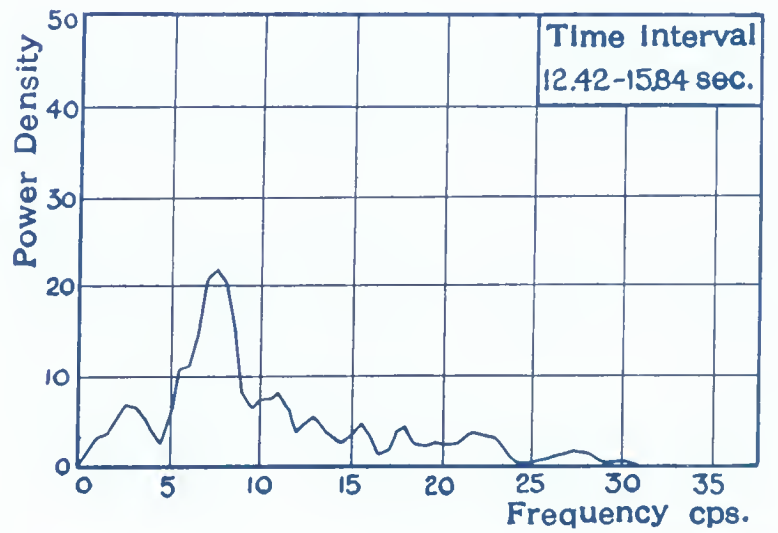
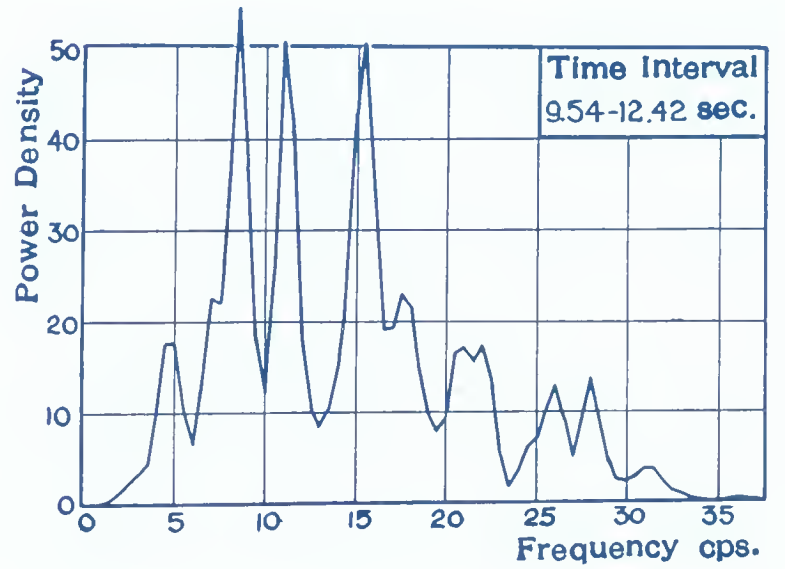
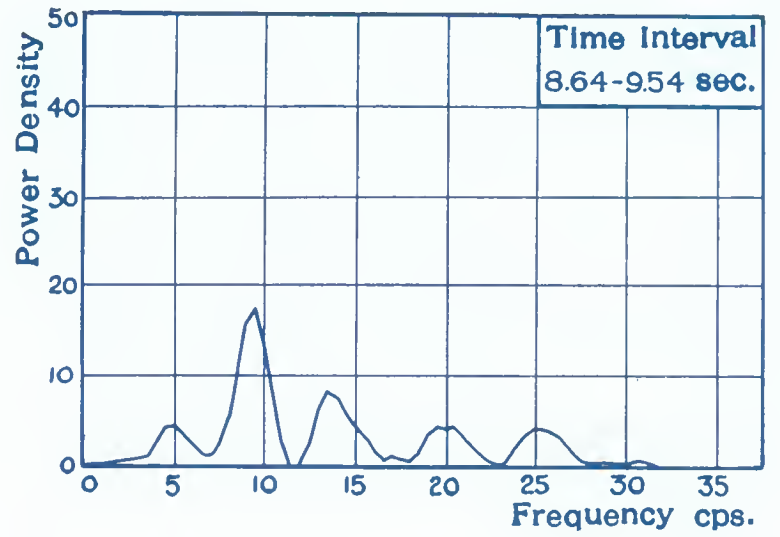
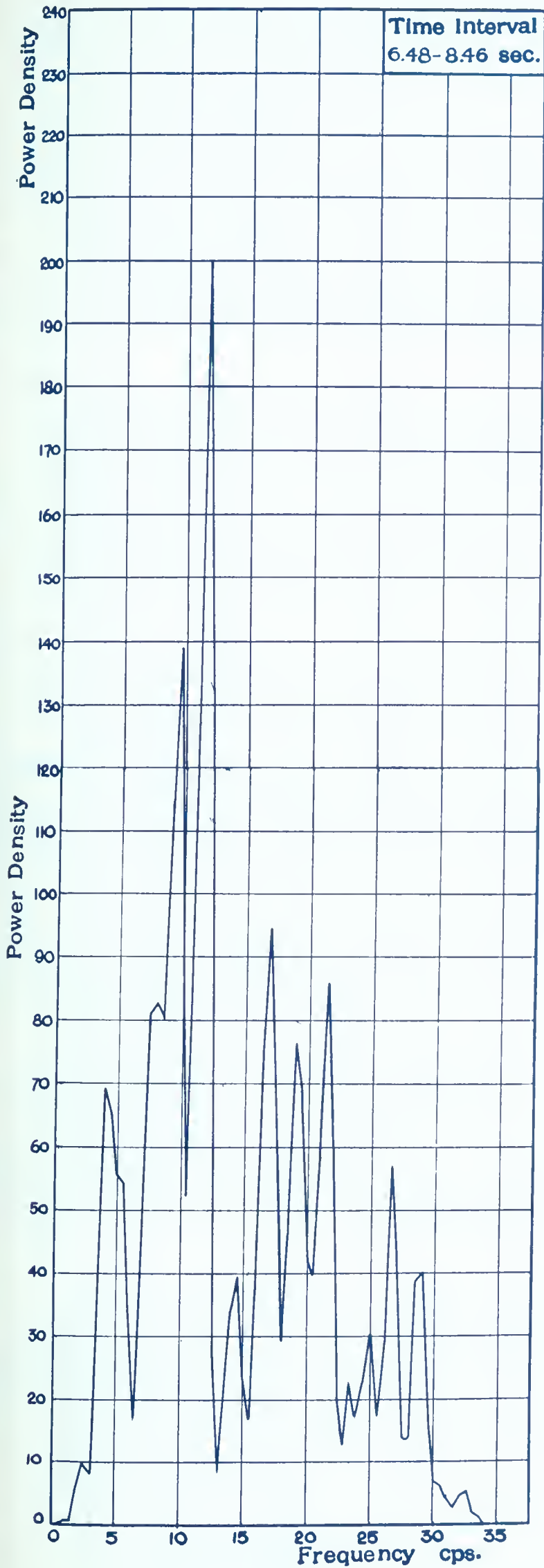


Fig. 49

Power Spectra.

Record No. 14

3 August 1961

271 km WNW

 $Df = 0.5$ cps $Dt = 0.01$ sec

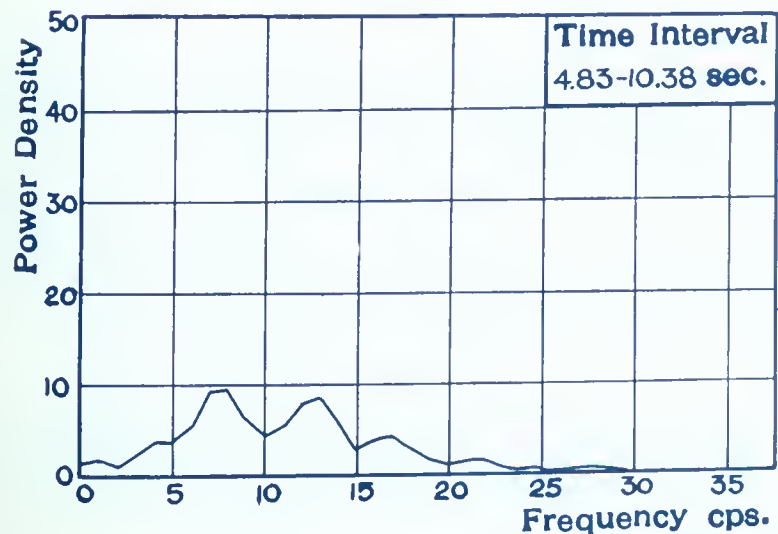
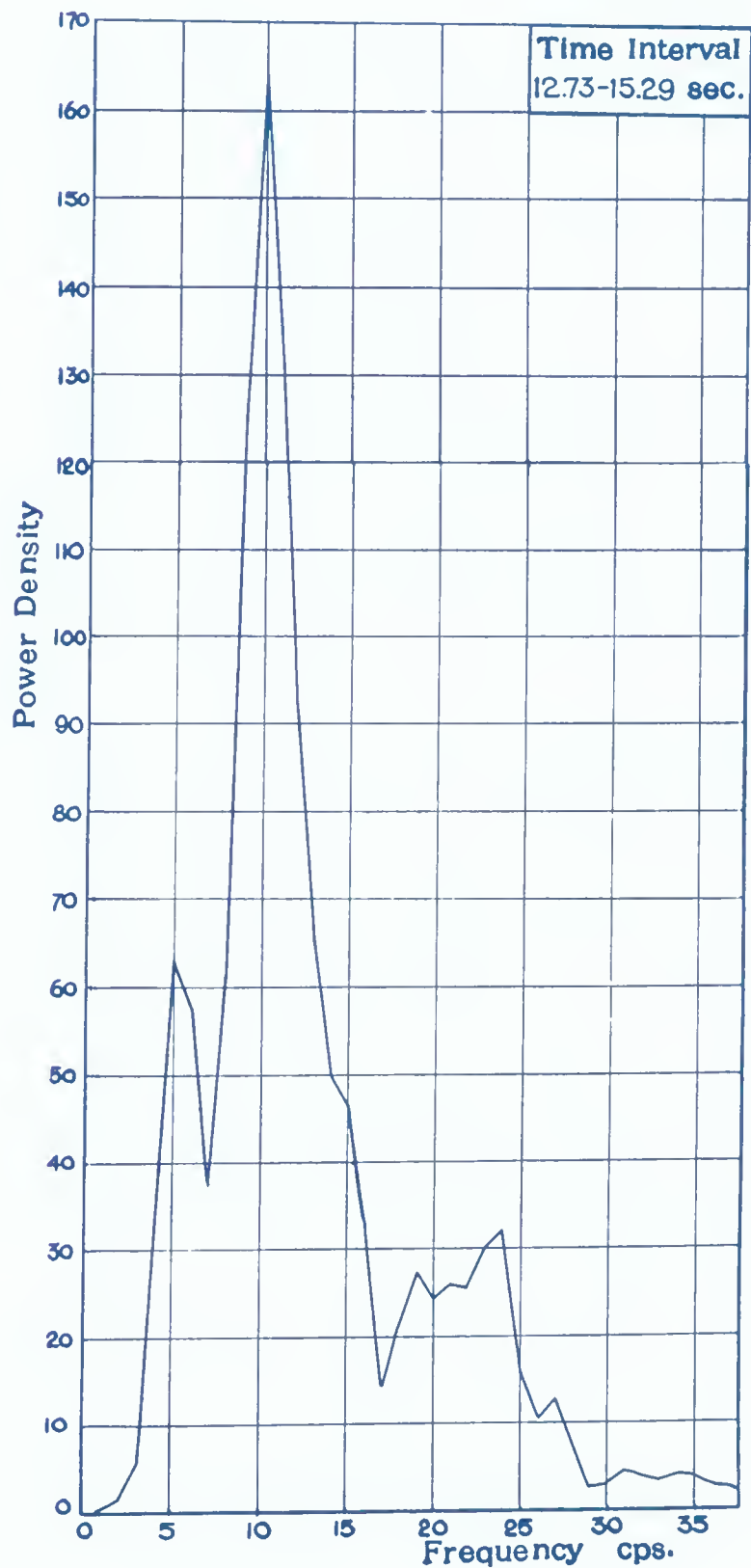
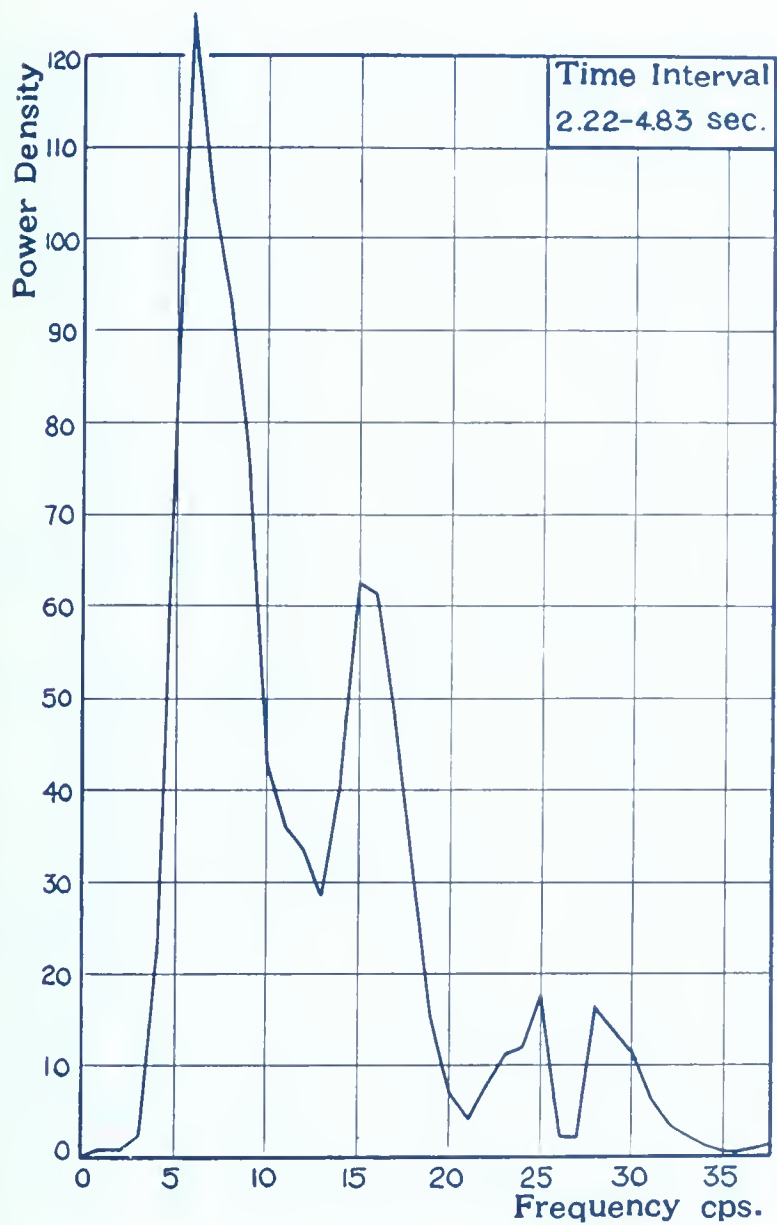
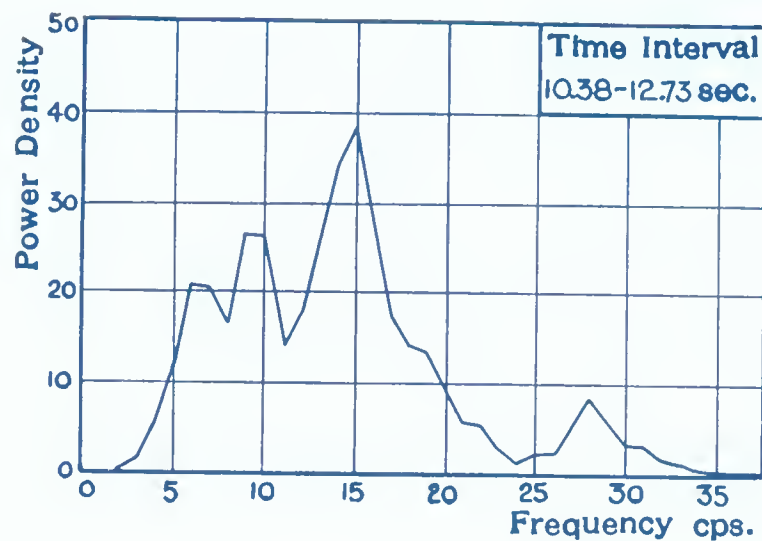
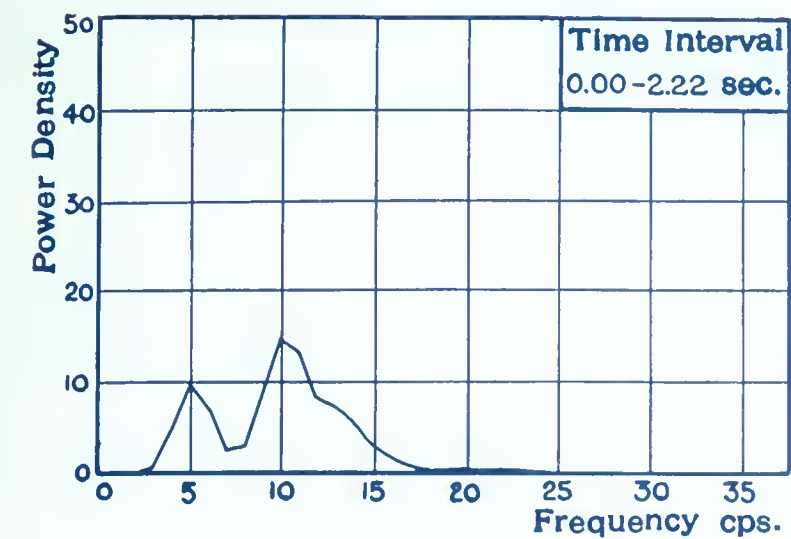


Fig. 50
Power Spectra.

Record No.4 275 km NW 3 August 1961

Df = 1.0 cps

Dt = 0.005 sec

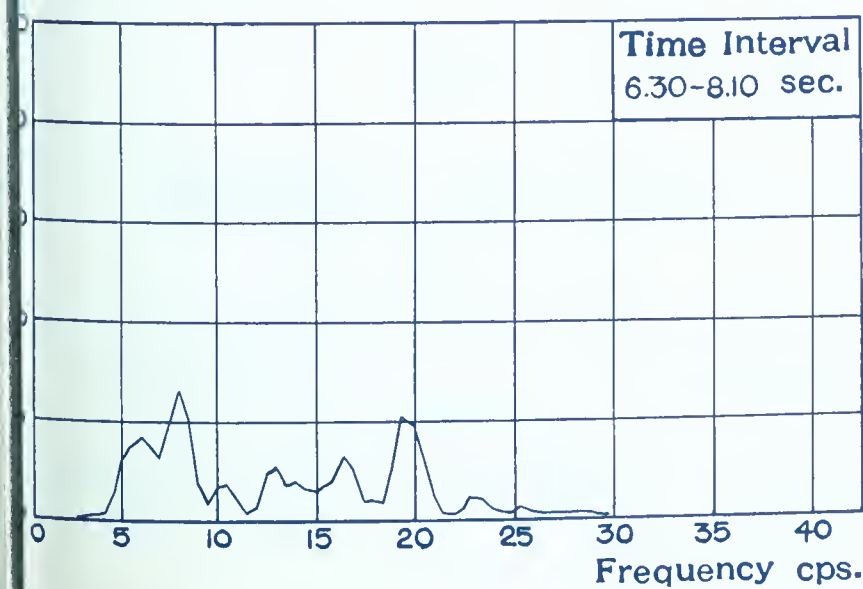
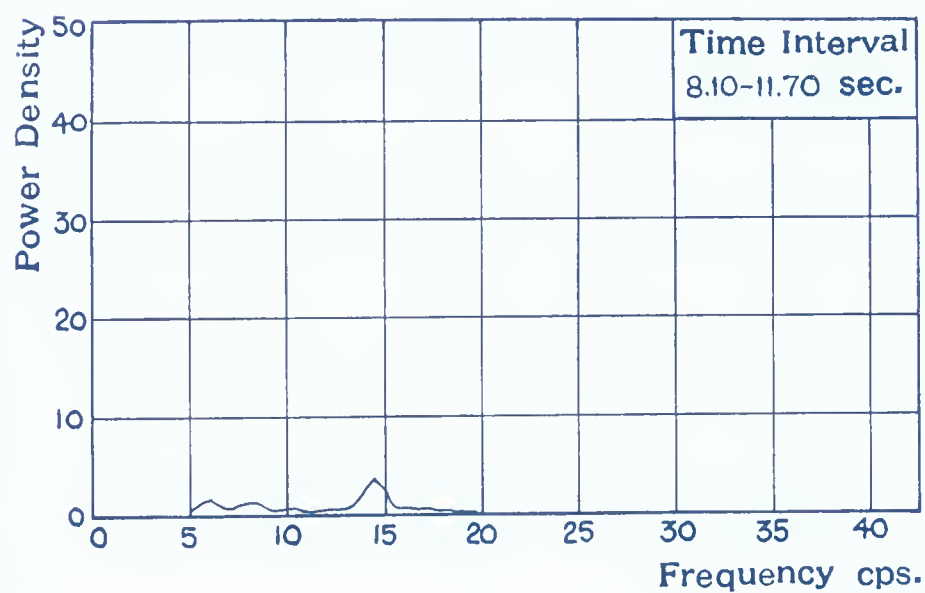
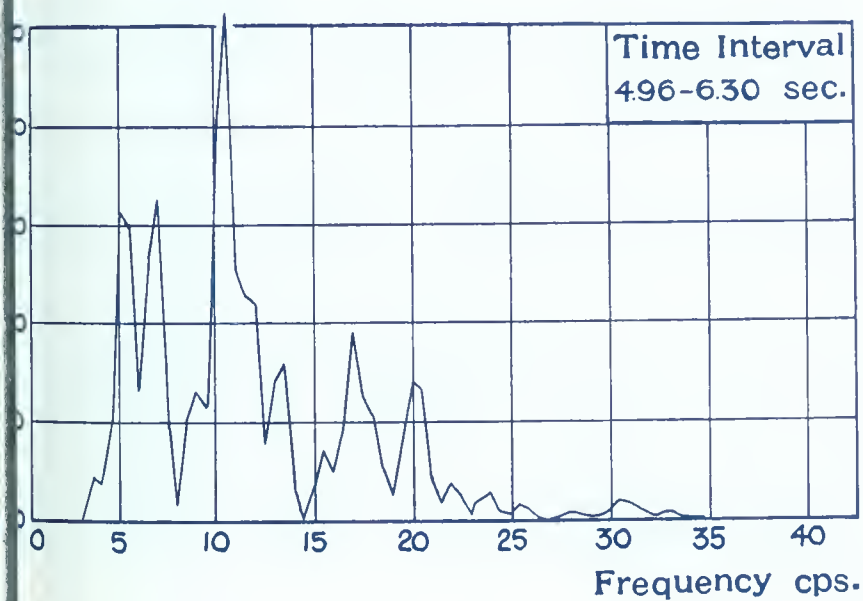
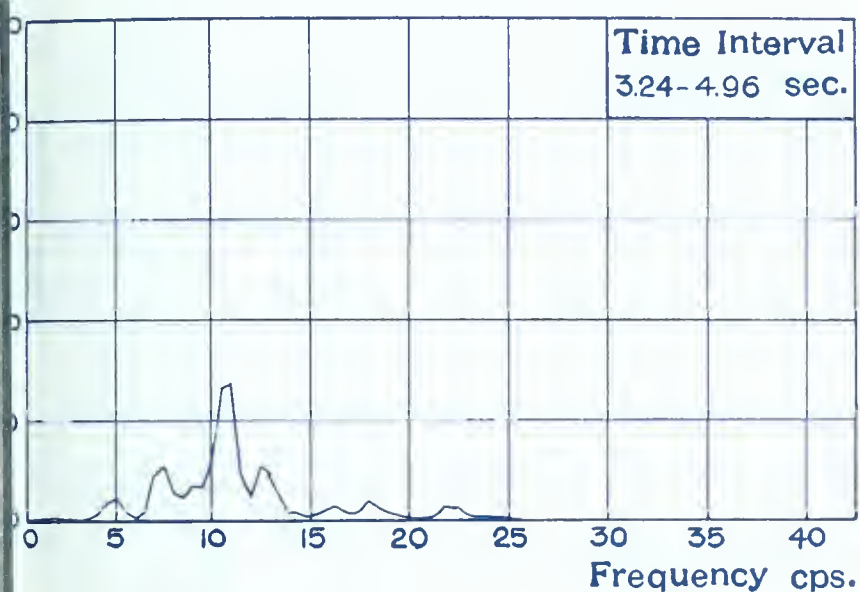
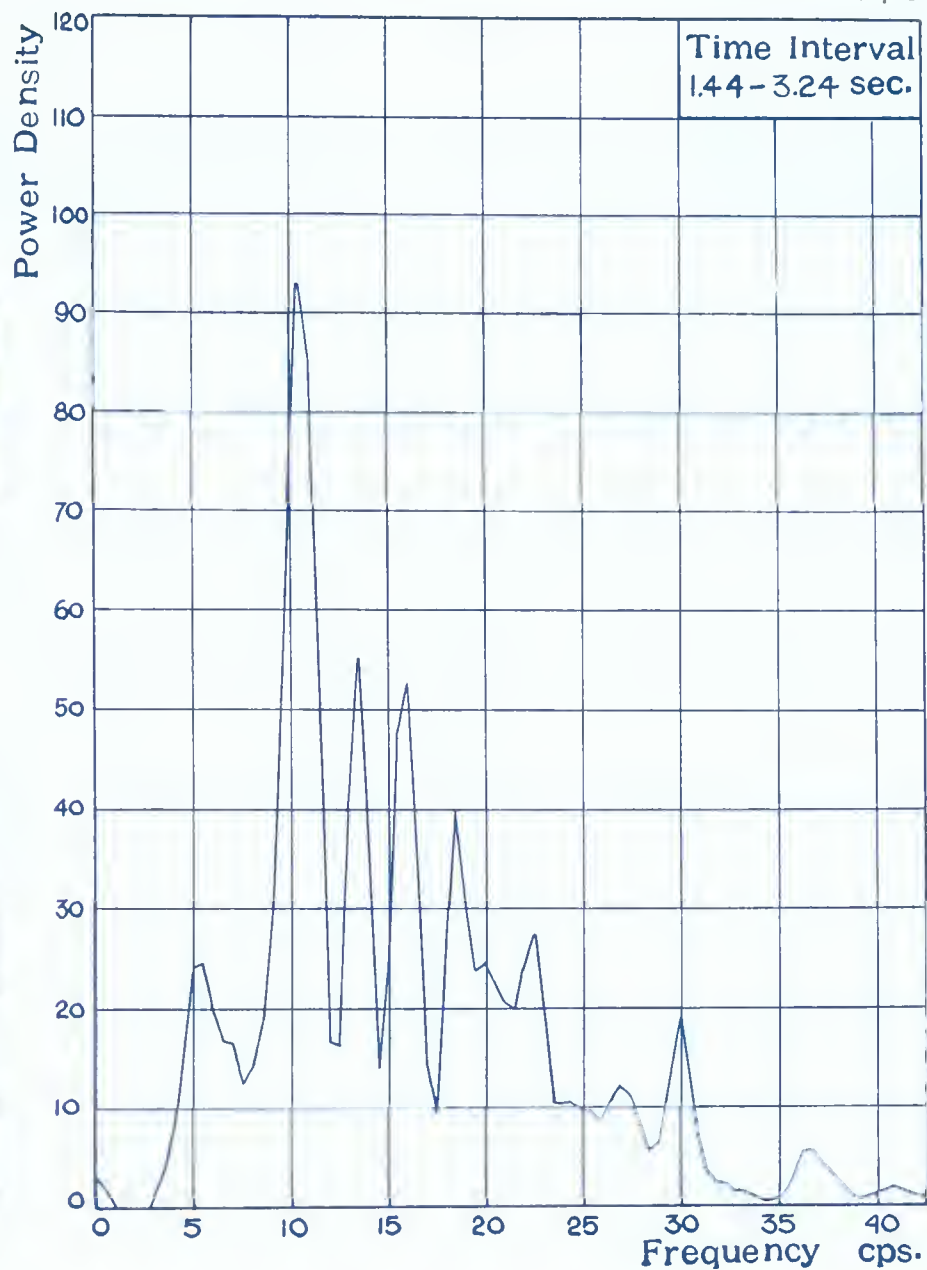
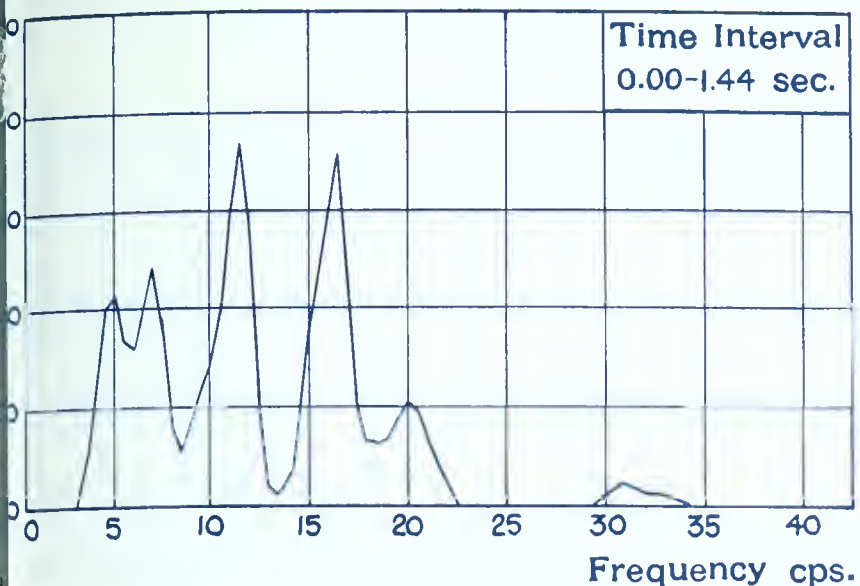


Fig. 51.

Power Spectra.

First Skip Arrivals.

Record 12 540 km NW 3 Aug 1961.

$Df = 0.5$ cps $Dt = 0.01$ sec

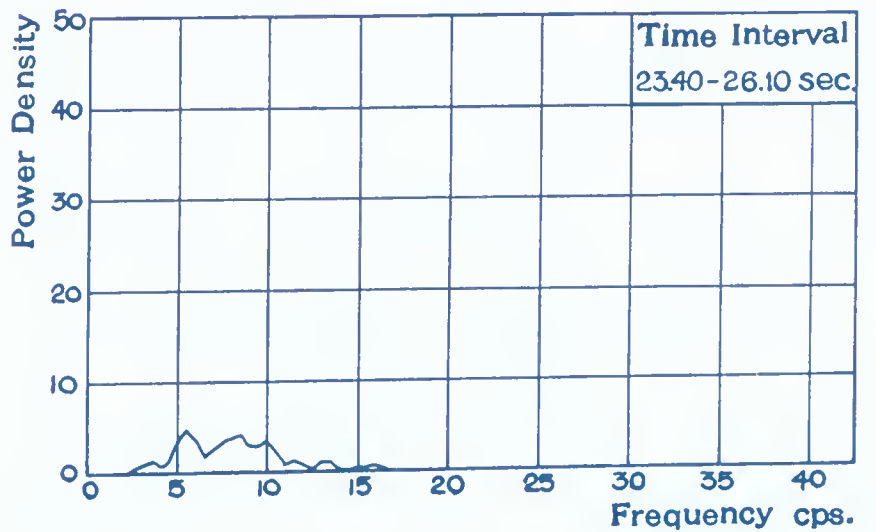
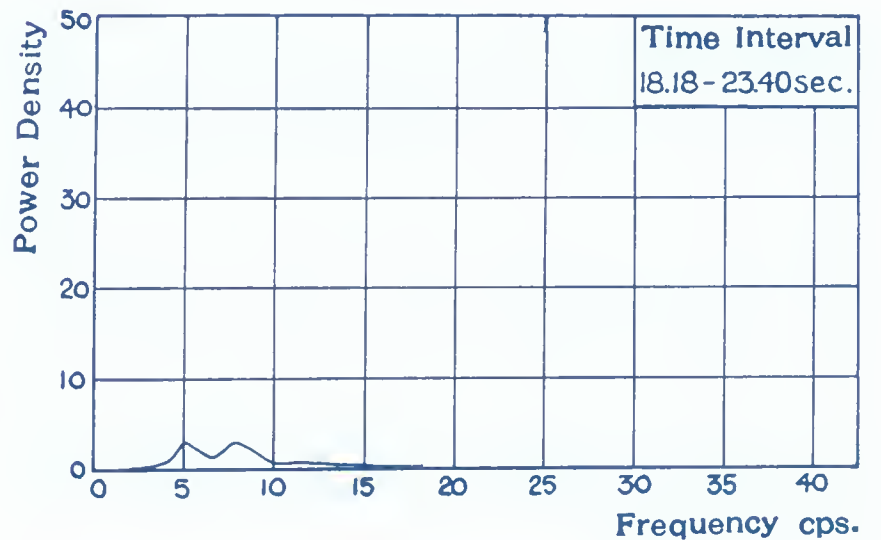
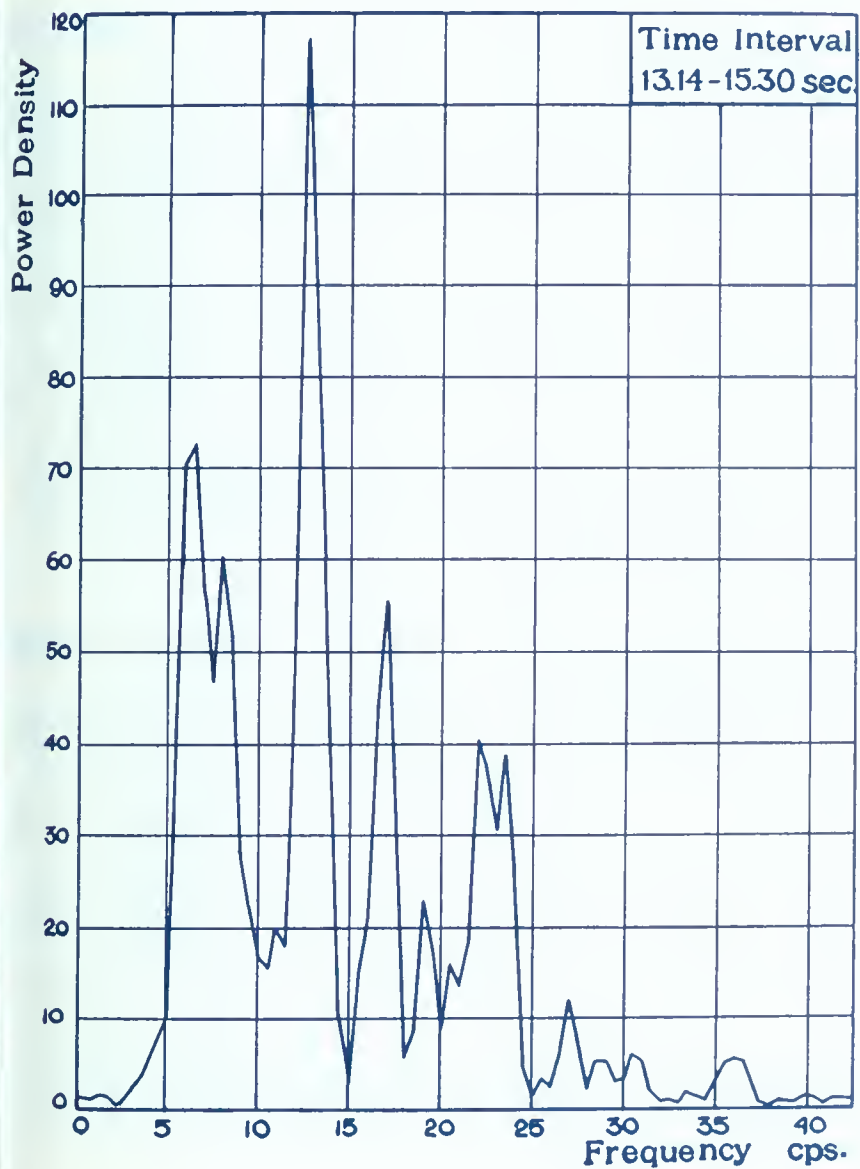
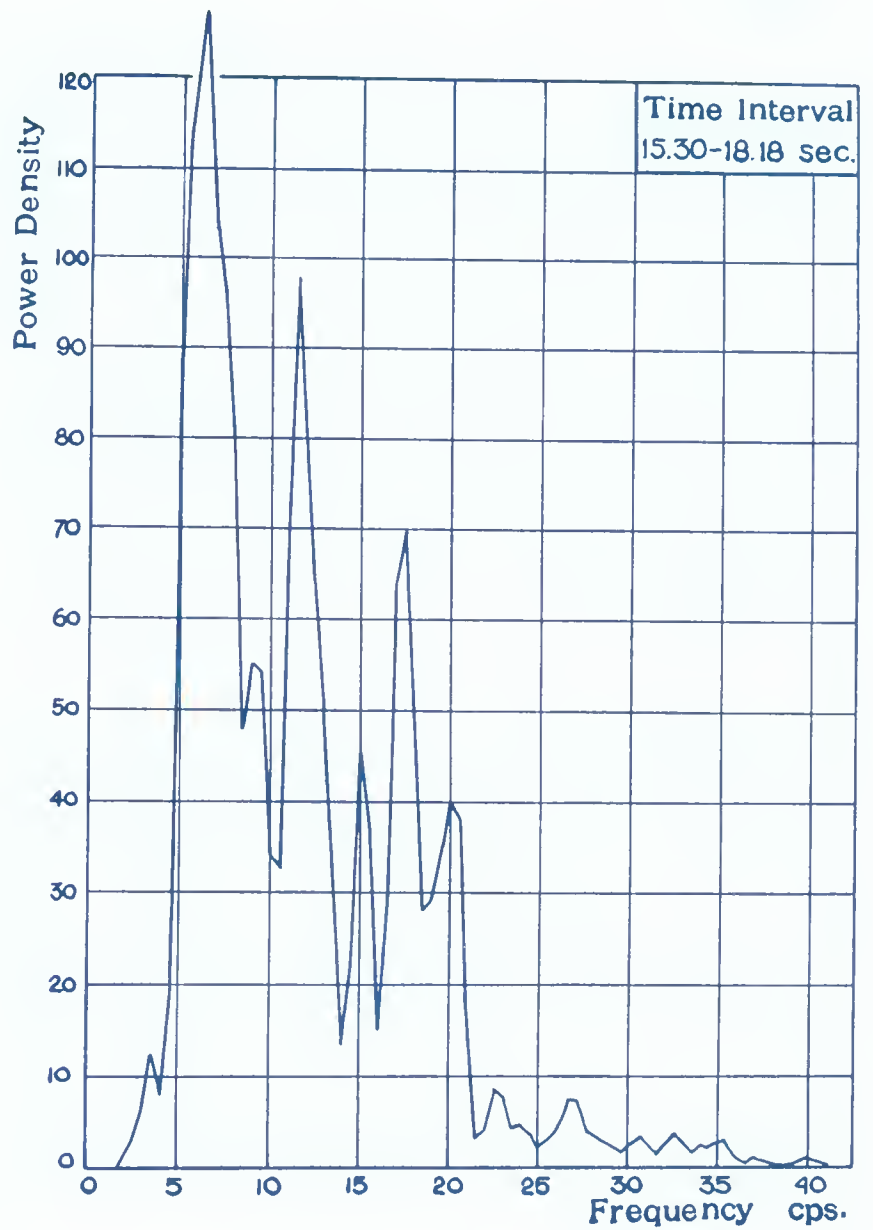
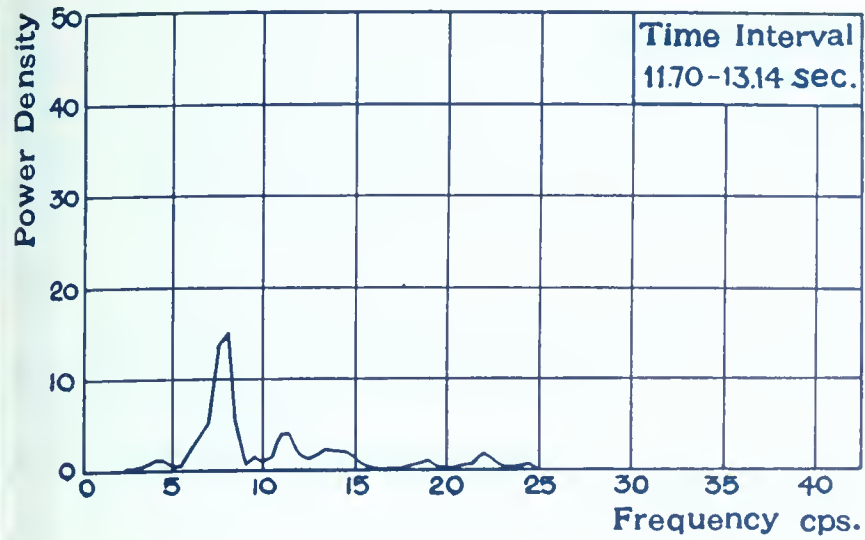


Fig. 52

Power Spectra.

Single Skip Arrivals

Record No. 12

3 August 1961

540 km NW

$Df = 0.5$ cps

$Dt = 0.01$ sec

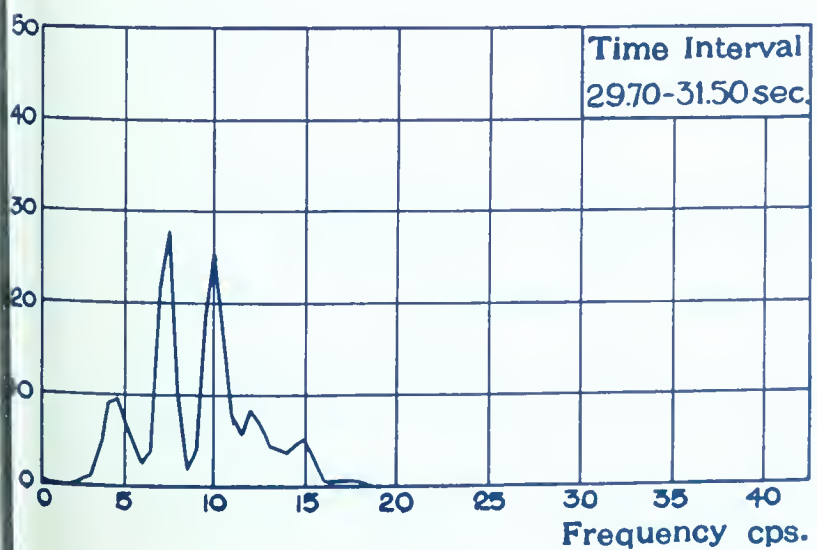
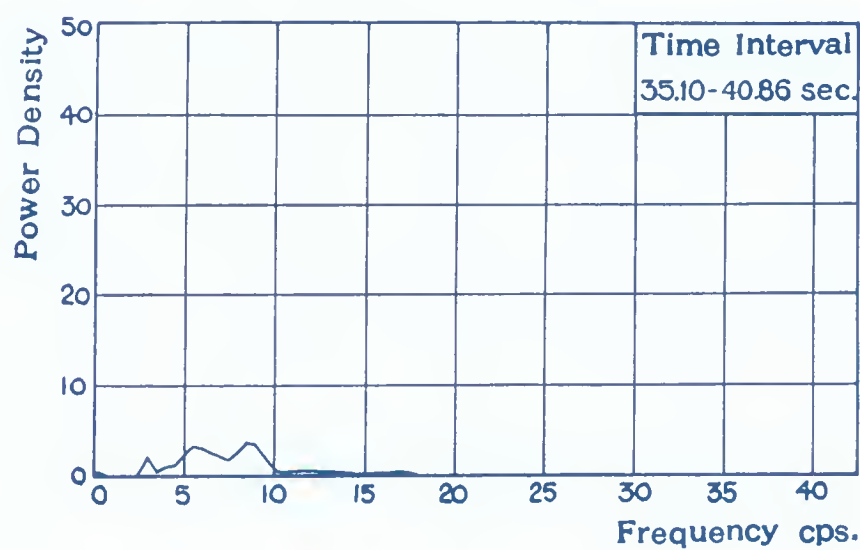
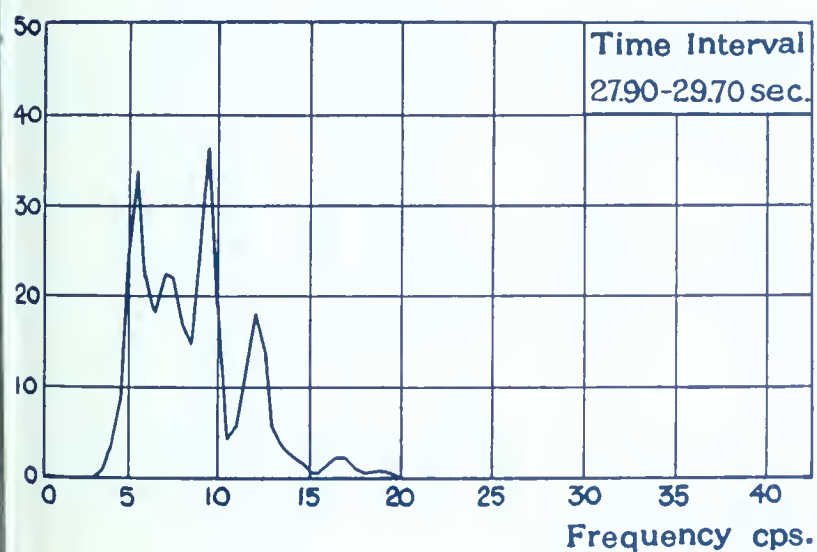
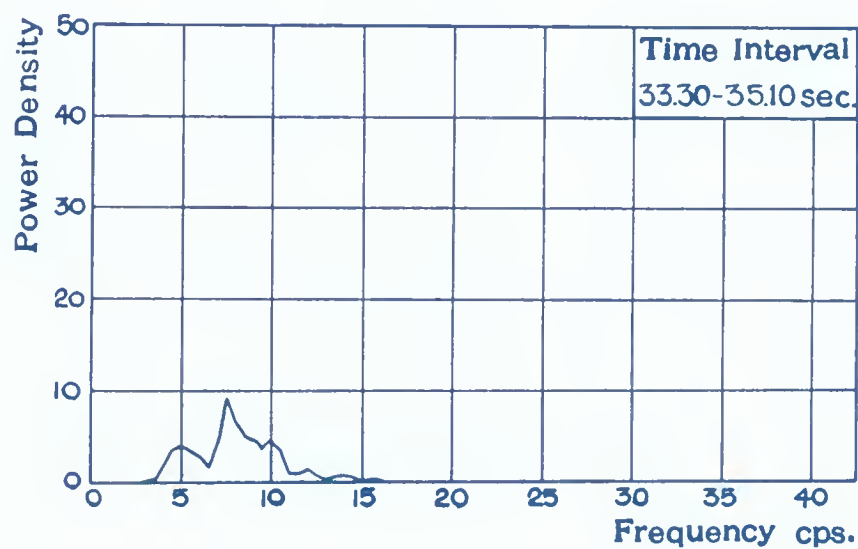
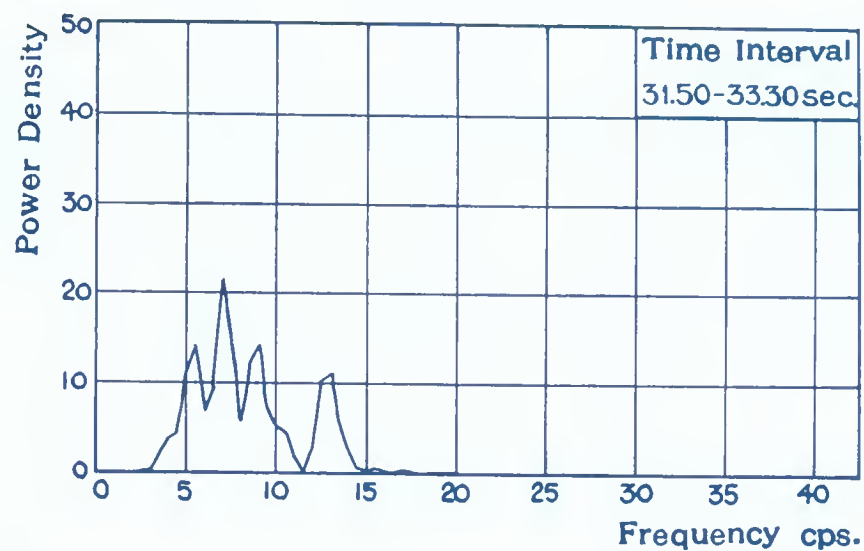
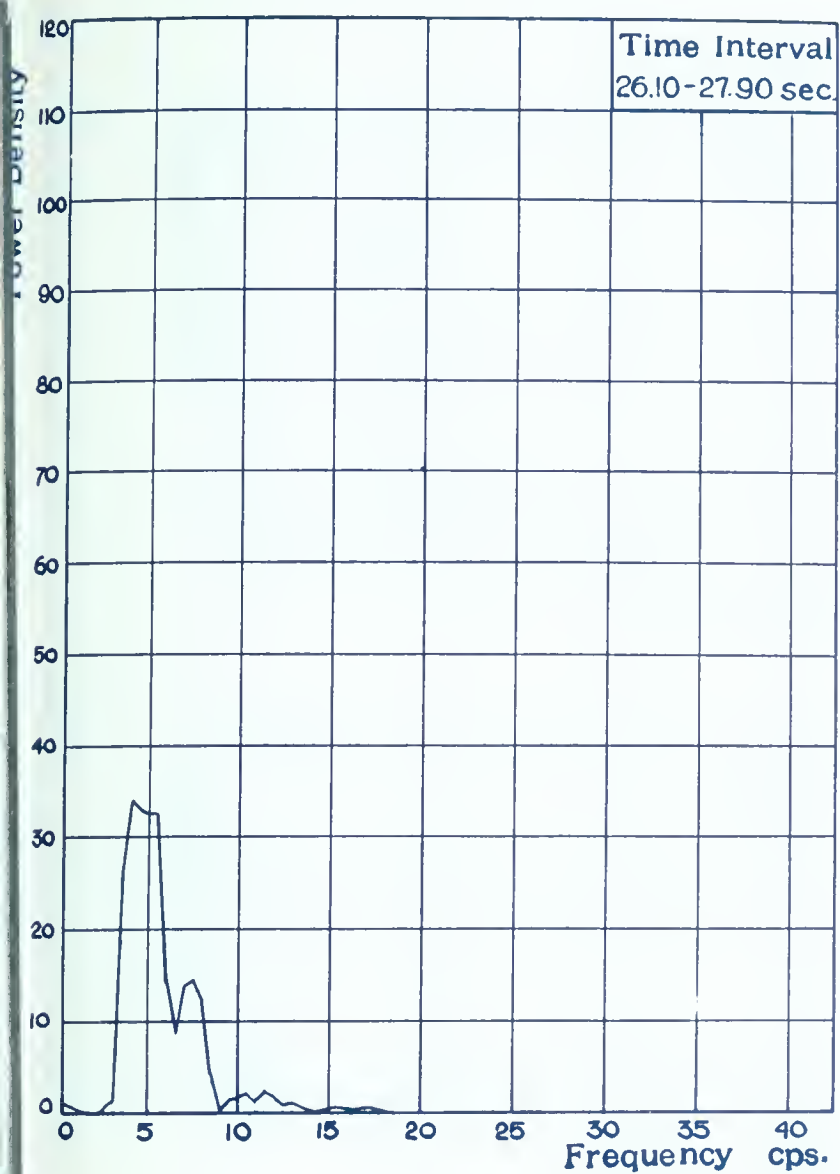


Fig. 53

Power Spectra.

Single Skip Arrivals

Record No. 12 540 km NW 3 August 1961

$\Delta f = 0.5$ cps

$\Delta t = 0.01$ sec

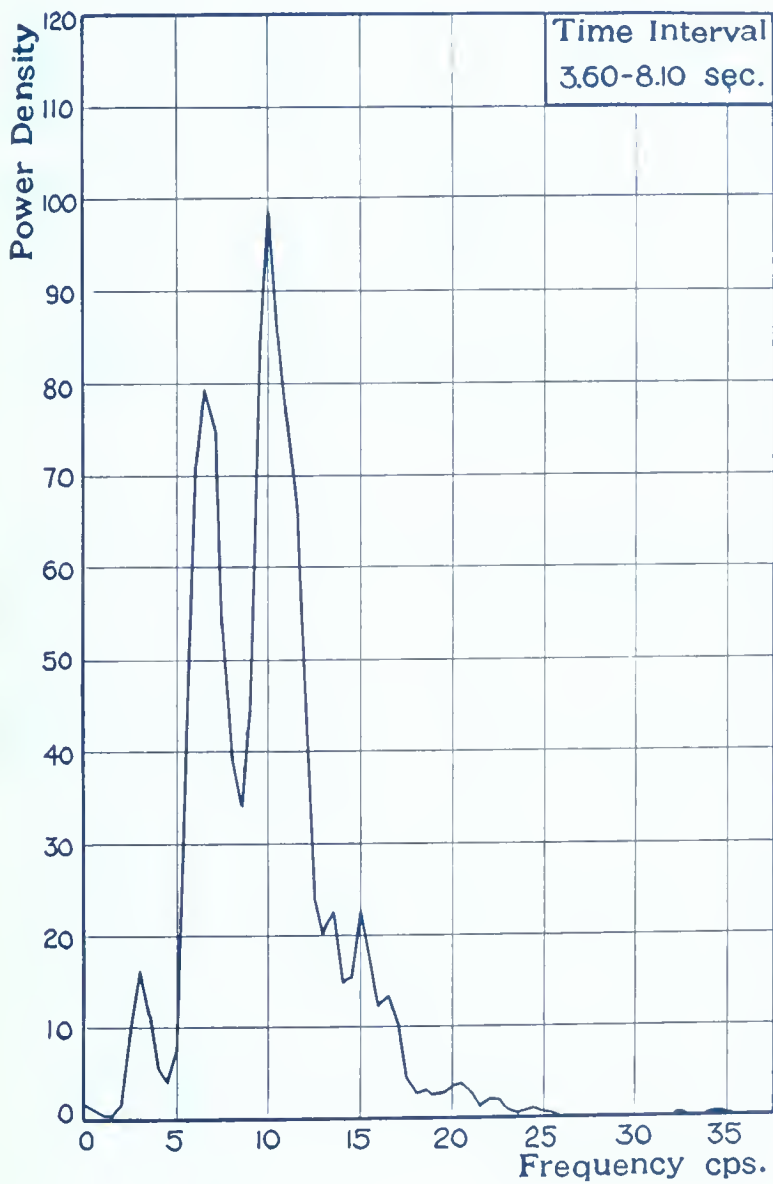
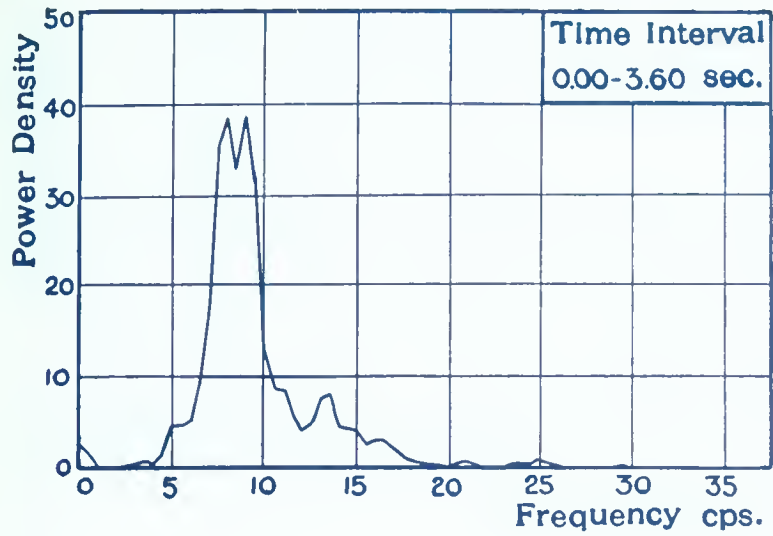


Fig. 54

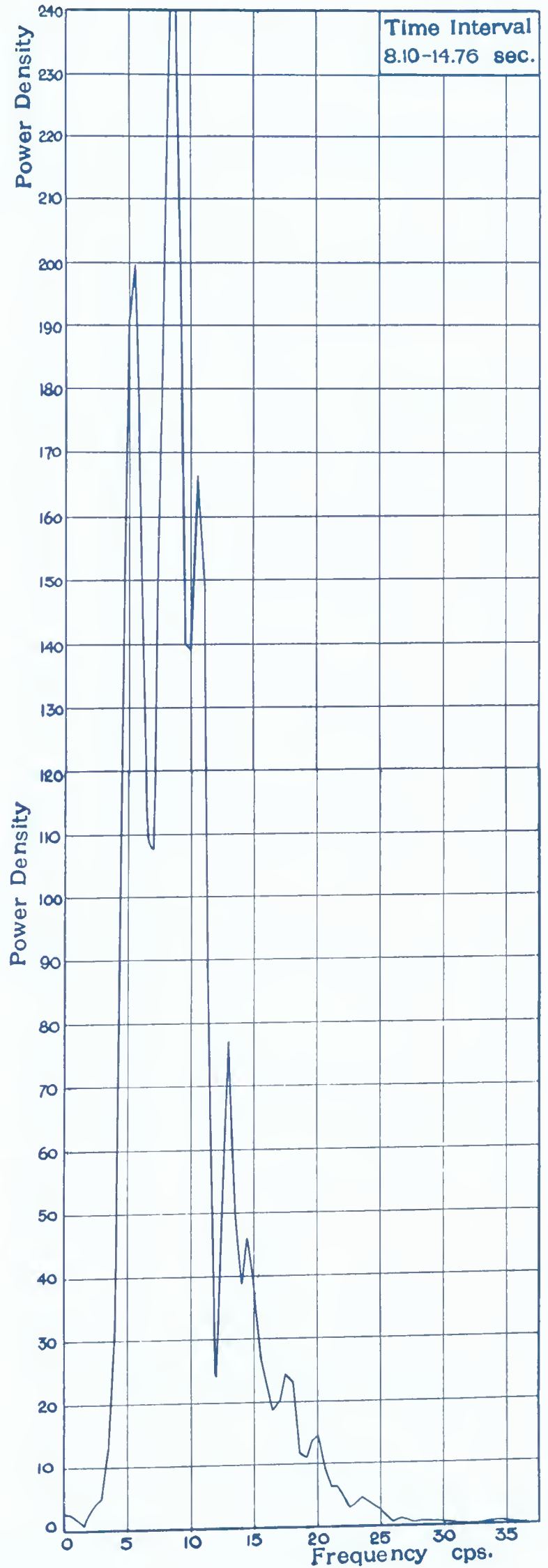
Power Spectra.

Double Skip Arrivals

Record No. 12 540 km NW 3 August 1961

$Df = 0.5$ cps

$Dt = 0.01$ sec



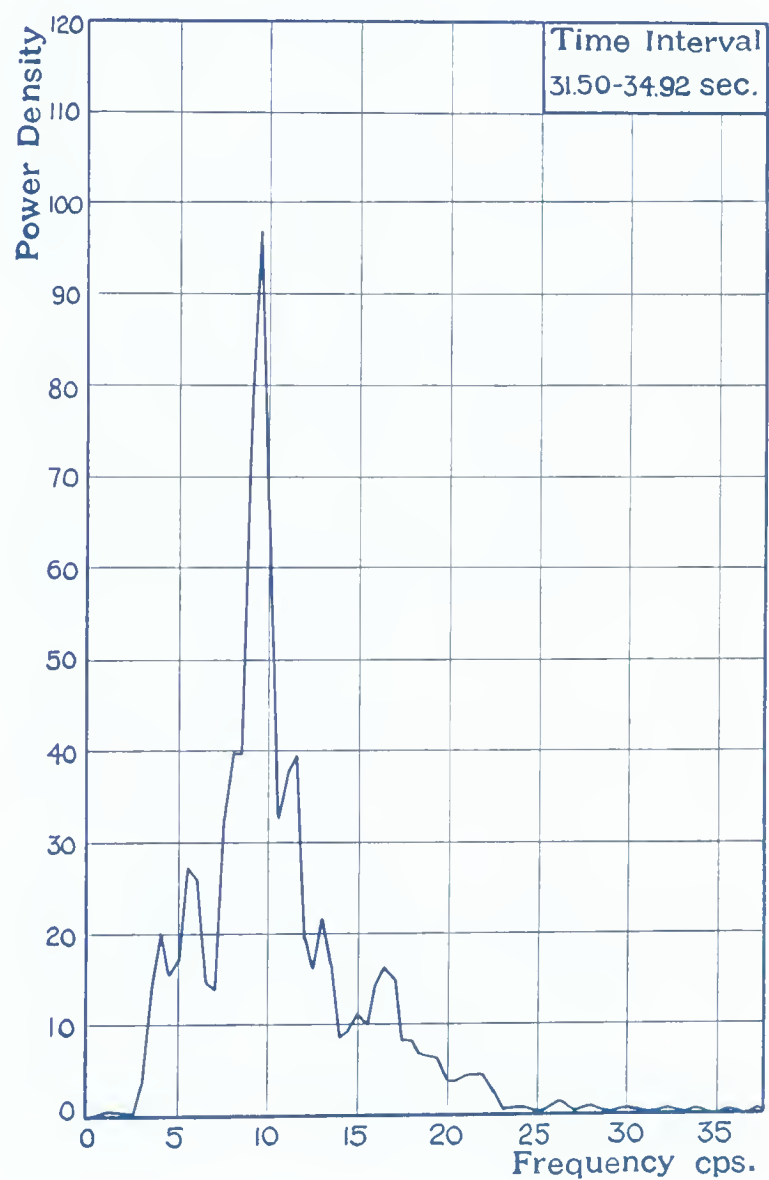
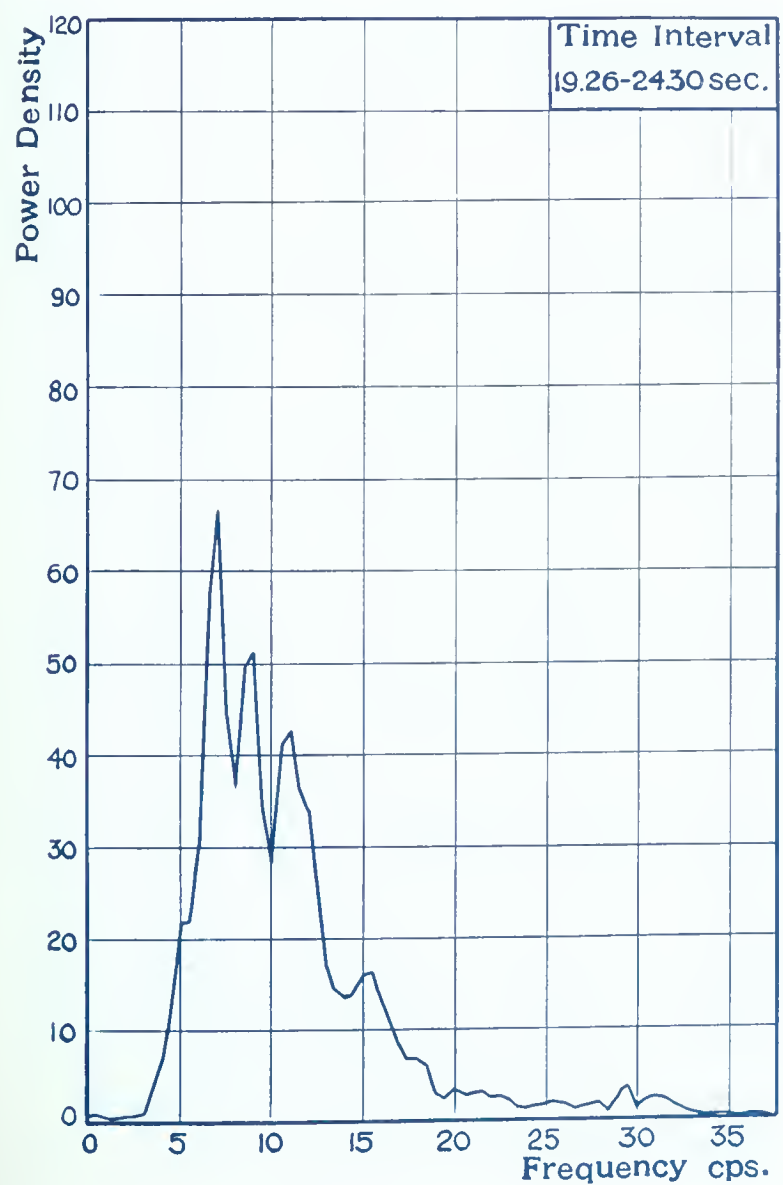
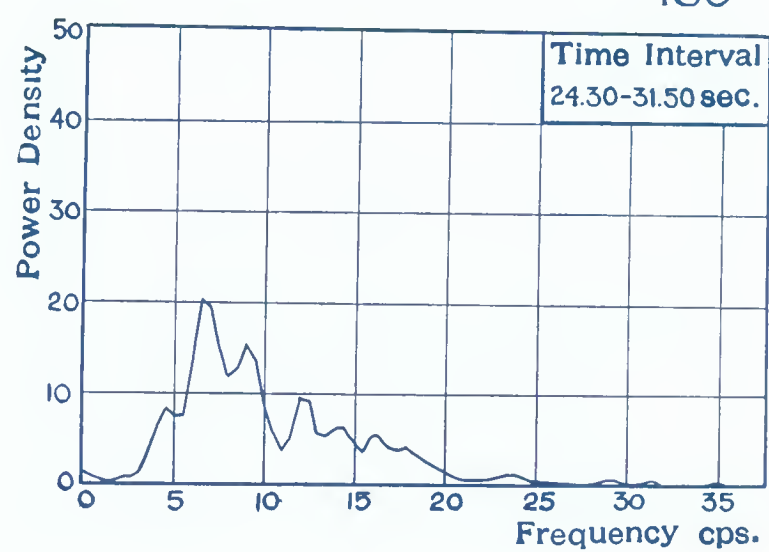
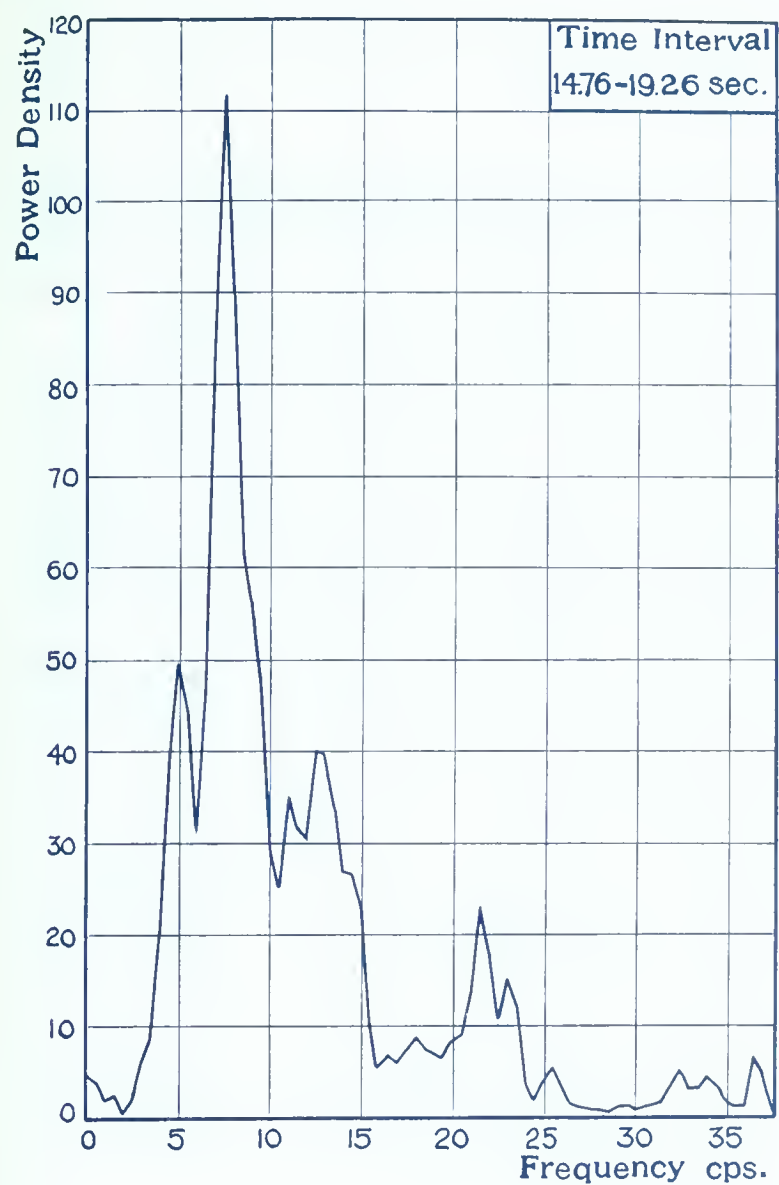


Fig. 55

Power Spectra.

Double Skip Arrivals

Record No. 12 540 km NW 3 August 1961

$Df = 0.5$ cps

$Dt = 0.01$ sec

5.9 Induced Ground Waves

The field observations of the 1962 season are summarized in Table XI below. Since the number of observations was small, it was not possible to construct travel time curves, but the events will plot quite closely near appropriate sections of the 1961 curve, a not unexpected result.

August 1st, 1962

Record No.	Time of Observation MST	Travel Time sec.	Distance from Explosion, km.
31	13:30:51.75	51.75	18.0
32	13:40:56.32	656.32	191.7
	13:40:59.97	659.97	191.7
	13:41:13.78	673.78	191.7

Time of explosion 13:30:00.0 MST

August 15th, 1962

Record No.	Time of Observation MST	Travel Time sec.	Distance from Explosion, km.
41	14:12:40.05	760.05	221.0
42	14:14:57.88	897.88	258.8
43	14:13:32.75	812.75	238.7
	14:14:03.31	843.31	238.7

Time of explosion 14:00:00.0 MST

TABLE XI.

One of the interesting sidelights of the 1962 season was the recording of the induced ground wave. This wave, first noted apparently by Gutenberg, has since been discussed in several papers, e.g. Oliver and Isacks (1961), and Cook et al, (1962).

These waves were first observed on August 1st at two sites, and again on August 15th at two of the three sites, well to the west of Suffield. The latter observations were of special interest, for only the two distant sites recorded the wave. Computing the angle of descent for the aerial waves at the three sites led to the following result:

			Distance (km)	Angle of descent (degrees)
TR.1	U. of A.	Record 41	221.0	4.3
	B. A. Oil Co.	" 43	238.7	20.2
TR.2	U. of A.	" 42	258.8	18.8

Taking these results at face value, it would appear that there is a greater likelihood of an induced ground wave at great distances for steeper angles of descent. There may well be other reasons, but it is rather odd that the nearest station should fail to register a ground wave when it recorded a strong aerial wave. However, since the ground wave is generated by the impact of the aerial wave, the intensity of the wave produced should be a function of the angle of incidence. Cook (1962) notes specifically that the amplitude of the induced wave depends on the frequency and pressure of the aerial

wave. Thus when the frequency of the air wave is near the frequency of observed ground vibration, the amplitude of the induced wave is much greater than if the vibrations were of widely different frequency. Reference is made to Cook's paper for further details.

5.10 A note on weather and the effects of sloping layers

Had it been necessary to carry out full scale computations on the data of the 20 ton blast (15th August, 1962), the assumptions of horizontal homogeneity would have been difficult to justify, except as a very crude first approximation. Maps of the prevailing weather situation are given in Appendix V. It is readily seen that the low level circulation is dominated by a large anticyclone of Arctic origin, and the flow aloft is complicated by the presence of a jet stream between Edmonton and Calgary. By contrast, the weather on August 1st would have been excellent for analysis, with weak circulations well up into the stratosphere.

The frontal system present on August 15th suggested, however, another problem, that of the effect of sloping layers on sound propagation. A sloping layer in the atmosphere usually indicates the presence of horizontal temperature gradients. An estimate of the effect of such layers on long distance sound propagation would be of some practical interest if some way could be found to take account of the horizontal variation of temperature.

The thought occurred that sloping layers could be introduced quite easily above the tropopause by means of a little artifice.

Since the lower stratosphere is usually close to being isothermal, it would be a simple matter to introduce a thin isothermal wedge into this region; the layers above the wedge are then tilted with respect to the ground, but quasi-horizontal, and still laterally homogeneous with respect to the upper plane of the wedge. A horizontal temperature gradient will now exist in the layers with respect to the earth's surface but the extra work of computation is quite trivial. The computation in the sloping layers proceeds the same as in horizontal layers, and appropriate rays are simply joined to the rays below the tropopause by straight section of ray through the isothermal wedge.

The effect of such layers on the horizontal range of a ray is quite appreciable: differences of 100 to 150 km between down- and upslope rays are typical for wedge angles of one half of one degree, and differences of over 200 km are usual for angles of one degree. Larger angles produce even greater effects, but there is little reason to suppose that such stratification could prevail in nature. Nevertheless, slopes of the order of 30 minutes are quite feasible, particularly in connection with frontal systems, and the effect on anomalous propagation would seem to be appreciable.

5.11 Conclusions, Afterthoughts, and Suggestions for Future Investigation.

As is the case with so much of geophysical investigation, the results presented here are tentative and not unique. On the whole, simplicity was given precedence over complexity when such seemed reasonable, and the observations were fitted to theory in the simplest possible way. It is evident that a greater number of observations might lead to significant changes in the structure of the travel time curves and acoustic zones. A rocket probe might well show a wealth of upper air detail which the acoustic method could only hint at, whether by luck or pedantry is hard to say.

The basic problem in all such investigation is largely instrumental, the gathering of good and necessary, if not sufficient data. But granted even sufficient data, the grand task of solving the aerodynamical problem in sufficient detail remains - a monumental obstacle and a magnificent challenge. Until that far-off time, who is to say what is truly sufficient and truly necessary, and what is attainable and what is just possible. But to the meteorologist who loves his craft, the whorled complexity of the moving air will always remain a source of unending excitement and, often enough, of sheer delight or bafflement, depending on the time of day, and the mood of the moment.

In the main, the present investigation attempted merely to probe a little among the lesser innards of the mighty engine, the littler tubes of flux, and shed a little sound where

none was heard before. And it may be that little more was done than to knock off a few bits of dusty scale from the busy boiler. But be that as it may, the cobbler beloves his leather, and a great deal of work remains. And now even the cobbler may make haste, for the advent of the high speed computer has made at least the tedious possible by doing away not only with tedium, but with the limitations of time itself.

Thus one can only hope that the forthcoming 500 ton blast at Suffield will furnish the necessary first rate material to help the work along. With good instrumentation, it should be possible to settle the problem of frequency change with distance and, with a great deal of good fortune, the problem of the slow apparent velocity. These curious waves have been observed sporadically by several observers, particularly Johnson and Hale (1953). Record 17 of the 100 ton test shows such a wave with an apparent velocity of only 320 meters per second, some 30 meters per second below the normal velocity of sound appropriate to the ambient temperature at the time of the test. One can only speculate, but it may just be that such waves move through a sonic duct by multiple reflection and at a speed appropriate to the mean wind and temperature in the duct, somewhat in the manner of a gravity wave (see Hunt et al, 1960).

It should be well worthwhile to program ray path computations by taking full account of both horizontal wind components. Every attempt should be made to obtain records of high resolution, and refraction equipment should be used

as much as possible. Frequency analysis should lead to a better understanding of atmospheric absorption, and this in turn should make it possible to deduce the properties of the upper air in greater detail. A few more barographs should be constructed for use in the field; a permanent installation at the observatory would seem to be in order also, and of continuing interest in recording airborne noises and natural sounds.

The Meteorological Service of Canada should take the lead in setting off a few tons of TNT in the Arctic. So little is yet known of the structure of the mesosphere at high latitudes and few, if any, propagation studies have been carried out since the time of Wölken's 1934 experiments at Novaya Zemlya. Workers in ozone should find such studies of particular interest.

It may well be argued that anomalous propagation studies have become somewhat anachronistic in this day of rocketry and satellites, and in a sense this is true. But satellites still don't orbit well or long at heights of 60 km, and a rocket samples only a small volume of atmosphere, even if it samples it directly and well. At any rate, one or two tons of TNT should always be much cheaper, even for peaceful soundings of the air, and the volume sampled will be so much greater, but with the details blurred. Still, it is not too difficult to envisage some future synthesis, a meteorological network of rocket probes interspersed with piles of TNT. And much may yet be learned from TNT alone, as the work of Facy (1962), a contribution to I.G.Y., has shown.

At all events, the anachronism has yet a charm all its own: in the history of science it will always be remembered as the pioneer means, the seismic method for probing the upper air - the perfect example of the unity of science and the fruitful interchange among its many disciplines which, superficially, at first seem totally unrelated.

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The first part of the paper is devoted to a general discussion of the problem. It is shown that the problem is of great importance in the theory of the differential equations of the second order. The second part of the paper is devoted to the study of the properties of the solutions of the differential equations of the second order. It is shown that the solutions of the differential equations of the second order are of great importance in the theory of the differential equations of the second order. The third part of the paper is devoted to the study of the properties of the solutions of the differential equations of the second order. It is shown that the solutions of the differential equations of the second order are of great importance in the theory of the differential equations of the second order. The fourth part of the paper is devoted to the study of the properties of the solutions of the differential equations of the second order. It is shown that the solutions of the differential equations of the second order are of great importance in the theory of the differential equations of the second order. The fifth part of the paper is devoted to the study of the properties of the solutions of the differential equations of the second order. It is shown that the solutions of the differential equations of the second order are of great importance in the theory of the differential equations of the second order. The sixth part of the paper is devoted to the study of the properties of the solutions of the differential equations of the second order. It is shown that the solutions of the differential equations of the second order are of great importance in the theory of the differential equations of the second order. The seventh part of the paper is devoted to the study of the properties of the solutions of the differential equations of the second order. It is shown that the solutions of the differential equations of the second order are of great importance in the theory of the differential equations of the second order. The eighth part of the paper is devoted to the study of the properties of the solutions of the differential equations of the second order. It is shown that the solutions of the differential equations of the second order are of great importance in the theory of the differential equations of the second order. The ninth part of the paper is devoted to the study of the properties of the solutions of the differential equations of the second order. It is shown that the solutions of the differential equations of the second order are of great importance in the theory of the differential equations of the second order. The tenth part of the paper is devoted to the study of the properties of the solutions of the differential equations of the second order. It is shown that the solutions of the differential equations of the second order are of great importance in the theory of the differential equations of the second order.

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APPENDIX

ANALYSIS OF VARIANCE

The following table shows the results of the analysis of variance for the data given in the preceding table. The table is arranged in the usual manner, with the sources of variation in the first column, the degrees of freedom in the second column, the sum of squares in the third column, the mean squares in the fourth column, and the F-ratios in the fifth column.

The first source of variation is between the groups, and the degrees of freedom are 2. The sum of squares is 10.0, and the mean square is 5.0. The F-ratio is 5.0, which is significant at the 1% level. The second source of variation is within the groups, and the degrees of freedom are 18. The sum of squares is 18.0, and the mean square is 1.0. The F-ratio is 1.0, which is not significant. The third source of variation is between the treatments, and the degrees of freedom are 1. The sum of squares is 1.0, and the mean square is 1.0. The F-ratio is 1.0, which is not significant. The fourth source of variation is within the treatments, and the degrees of freedom are 17. The sum of squares is 17.0, and the mean square is 1.0. The F-ratio is 1.0, which is not significant.

TABLE 1. Analysis of Variance

Source of Variation		df	SS	MS	F	Significance
Between Groups	1	2	10.0	5.0	5.0	0.01
Within Groups	1	18	18.0	1.0	1.0	0.10
Between Treatments	1	1	1.0	1.0	1.0	0.10
Within Treatments	1	17	17.0	1.0	1.0	0.10

APPENDIX II

Special Cases of the Ray Path Integral

The symbols used below are defined on pp.63-66 of Chapter II.

$$\begin{aligned}
 (1) \quad & \alpha \neq 0 \\
 & c = 0 \\
 & R = \sqrt{a + bz} \\
 & K = a\alpha^2 + b\alpha \\
 & x = D \int_0^z \frac{zdz}{R} + E \int_0^z \frac{dz}{R} + F \int_0^z \frac{dz}{(1 - \alpha z)R} \\
 & = DI_1 + EI_0 + FI_2
 \end{aligned}$$

where $I_0 = \frac{2}{b} (R - \sin e_0)$

$$I_1 = [(2bz - 4a)R + 4a \sin e_0] / 3b^2$$

$$I_2 = \frac{1}{\sqrt{K}} \log \left(\frac{-\alpha R - \sqrt{K}}{-\alpha R + \sqrt{K}} \right) \Bigg|_0^z \quad \text{for } K > 0$$

and

$$I_2 = \frac{1}{\sqrt{-K}} \arctan \left(\frac{-\alpha R}{\sqrt{-K}} \right) \Bigg|_0^z \quad \text{for } K < 0$$

1. Introduction

The purpose of this paper is to study the properties of the function

defined on the interval $[0, 1]$ by the formula

$$f(x) = \frac{1}{x} \int_0^x t \sin t \, dt.$$

$$f(x) = \frac{1}{x} \int_0^x t \sin t \, dt \quad (1)$$

$$0 < x < 1$$

$$f(0) = \lim_{x \rightarrow 0} f(x) = 0$$

$$f(1) = \int_0^1 t \sin t \, dt = -\cos t \Big|_0^1 = 1 - \cos 1$$

$$\lim_{x \rightarrow 0} \left(x - \frac{1}{x} \right) \left(x + \frac{1}{x} \right) = 0$$

$$\lim_{x \rightarrow 0} \left(x - \frac{1}{x} \right) = -\infty$$

$$\lim_{x \rightarrow 0} \left(x - \frac{1}{x} \right) = -\infty \quad \text{Hence}$$

$$\lim_{x \rightarrow 0} \left(x - \frac{1}{x} \right) = -\infty \quad \text{Hence}$$

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$$\lim_{x \rightarrow 0} \left(x - \frac{1}{x} \right) = -\infty$$

$$\lim_{x \rightarrow 0} \left(x - \frac{1}{x} \right) = -\infty \quad \text{Hence}$$

$$(2) \quad \alpha = 0 \quad R = \sqrt{\sin^2 e_o - 2\lambda z + \lambda^2 z^2}$$

$$c \neq 0$$

$$x = \int_0^z \frac{(C_o \cos e_o + w_o)}{C_o R} dz + \int_0^z \frac{(\beta - \lambda)z}{C_o R} dz - \int_0^z \frac{\lambda\beta}{C_o} \frac{z^2}{R} dz$$

$$= [\cos e_o + w_o/C_o] I_o + [(\beta - \lambda)/C_o] I_1 - (\lambda\beta/C_o) I_2$$

where I_o is given by (2-66a) or (2-66b), and

I_1 by (2-67), but the last integral is now

$$I_2 = \frac{z}{2c} R + \frac{3b}{4c^2} (\sin e_o - R) + \left[\frac{3b^2 - 4ac}{8c^2} \right] \cdot I_o$$

$$(3) \quad \alpha = 0 \quad R = \sqrt{\sin^2 e_o - 2\lambda z}$$

$$c = 0$$

$$x = \int_0^z \frac{C_o \cos e_o + (w_o + \beta z)(1 - \lambda z)}{C_o R} dz$$

$$= [\cos e_o + w_o/C_o] I_o + [(\beta - \lambda)/C_o] I_1 - (\lambda\beta/C_o) I_2$$

where $I_o = \frac{2}{b} (R - \sin e_o)$

$$I_1 = [(2bz - 4a)R + 4a \sin e_o] / 3b^2$$

$$I_2 = \frac{2}{15b^3} [(8a^2 - 4abz + 3b^2 z^2)R - 8a^2 \sin e_o]$$

APPENDIX IIIInstrumentation of University of Alberta
Field PartiesSuffield 100 ton Explosion, 3 Aug. 1961.Regina:

Magneto-telluric truck
1 Microbarograph
Coils
Earth Current Spread
4 Galvanometer amplifiers
Tape recorder
Generator
1 Times Chronometer
1 Radio Receiver

Moosomin:

1 EVP-5 Pressure Phone
1 Photron amplifier
1 Brush recorder (2 channel) (To operate
off 60 cycle line)
1 Times Chronometer
1 Radio receiver

Brandon:

International truck
EVP-5 Pressure Phone
1 Times Chronometer
1 Photron amplifier
1 TI recorder (60 cycle line operation)

Winnipeg: 1 Microbarograph
 1 Times Chronometer
 1 Photron amplifier
 1 Varian recorder (60 cycle line operation)

Suffield: Magneto - telluric equipment
 1 EVP-5 Pressure Phone
 1 Brush recorder (2 channel)
 1 WWV Receiver

Edmonton: 1 Brush recorder - 6 channel
 1 Times Chronometer
 2 Radio Receivers

Quantities Recorded:

Regina: Pressure
 Magnetic Field (2 components)
 Earth currents (1 component)
 CBK
 Time

Moosomin: Pressure
 Time

Brandon and
Winnipeg: Pressure
 Time

Suffield: Pressure
 Magnetic Field
 WWV
 Time

Edmonton: Time
 CBX
 CKUA

CALCULATION OF SHORT GEODETIC DISTANCES.

COEFFICIENTS A FOR GIVEN MEAN LATITUDE.
(LENGTHS IN KILOMETERS OF 1 MINUTE OF THE PARALLEL)

	46	47	48	49	50
00	1.291108	1.267653	1.243808	1.219582	1.194982
01	1.290720	1.267258	1.243407	1.219175	1.194569
02	1.290332	1.266864	1.243006	1.218767	1.194155
03	1.289944	1.266469	1.242605	1.218360	1.193742
04	1.289556	1.266074	1.242204	1.217953	1.193328
05	1.289168	1.265680	1.241803	1.217546	1.192915
06	1.288779	1.265285	1.241401	1.217138	1.192501
07	1.288391	1.264890	1.241000	1.216731	1.192088
08	1.288003	1.264495	1.240599	1.216324	1.191674
09	1.287615	1.264101	1.240198	1.215916	1.191261
10	1.287226	1.263706	1.239797	1.215508	1.190846
11	1.286837	1.263310	1.239395	1.215100	1.190432
12	1.286448	1.262915	1.238993	1.214691	1.190017
13	1.286059	1.262519	1.238591	1.214283	1.189602
14	1.285670	1.262123	1.238189	1.213875	1.189188
15	1.285280	1.261728	1.237786	1.213466	1.188773
16	1.284891	1.261332	1.237384	1.213058	1.188359
17	1.284502	1.260936	1.236982	1.212649	1.187944
18	1.284113	1.260540	1.236580	1.212241	1.187529
19	1.283724	1.260145	1.236178	1.211833	1.187115
20	1.283334	1.259748	1.235775	1.211424	1.186700
21	1.282943	1.259351	1.235372	1.211014	1.186284
22	1.282553	1.258955	1.234969	1.210605	1.185869
23	1.282163	1.258558	1.234566	1.210196	1.185453
24	1.281773	1.258161	1.234162	1.209786	1.185038
25	1.281382	1.257764	1.233759	1.209377	1.184622
26	1.280992	1.257367	1.233356	1.208967	1.184206
27	1.280602	1.256971	1.232953	1.208558	1.183791
28	1.280211	1.256574	1.232550	1.208149	1.183375
29	1.279821	1.256177	1.232146	1.207739	1.182960
30	1.279430	1.255779	1.231743	1.207329	1.182544

CALCULATION OF SHORT GEODETIC DISTANCES.

COEFFICIENTS A FOR GIVEN MEAN LATITUDE.
(LENGTHS IN KILOMETERS OF 1 MINUTE OF THE PARALLEL)

	46	47	48	49	50
30	1.279430	1.255779	1.231743	1.207329	1.182544
31	1.279039	1.255381	1.231339	1.206918	1.182127
32	1.278647	1.254984	1.230934	1.206508	1.181711
33	1.278256	1.254586	1.230530	1.206097	1.181294
34	1.277864	1.254188	1.230126	1.205687	1.180877
35	1.277473	1.253790	1.229722	1.205276	1.180461
36	1.277082	1.253392	1.229318	1.204866	1.180044
37	1.276690	1.252994	1.228913	1.204455	1.179628
38	1.276299	1.252596	1.228509	1.204045	1.179211
39	1.275907	1.252198	1.228105	1.203634	1.178794
40	1.275515	1.251800	1.227700	1.203224	1.178377
41	1.275123	1.251401	1.227295	1.202812	1.177959
42	1.274730	1.251002	1.226889	1.202401	1.177542
43	1.274338	1.250603	1.226484	1.201989	1.177124
44	1.273945	1.250204	1.226079	1.201578	1.176707
45	1.273553	1.249805	1.225673	1.201166	1.176289
46	1.273160	1.249406	1.225268	1.200755	1.175871
47	1.272768	1.249007	1.224863	1.200343	1.175454
48	1.272375	1.248608	1.224457	1.199932	1.175036
49	1.271983	1.248209	1.224052	1.199520	1.174619
50	1.271590	1.247809	1.223646	1.199108	1.174200
51	1.271196	1.247409	1.223240	1.198696	1.173782
52	1.270803	1.247009	1.222834	1.198283	1.173363
53	1.270409	1.246609	1.222427	1.197870	1.172944
54	1.270015	1.246209	1.222021	1.197458	1.172525
55	1.269622	1.245809	1.221614	1.197045	1.172107
56	1.269228	1.245409	1.221208	1.196633	1.171688
57	1.268835	1.245009	1.220802	1.196220	1.171269
58	1.268441	1.244609	1.220395	1.195807	1.170851
59	1.268047	1.244208	1.219989	1.195395	1.170432
60	1.267653	1.243808	1.219582	1.194982	1.170013

Appendix 1: The 100 most common words in English

Appendix 1: The 100 most common words in English

Appendix 1: The 100 most common words in English

Rank	Word	Frequency	Rank	Word	Frequency
1	the	1000000	51	and	100000
2	of	800000	52	but	90000
3	to	700000	53	or	80000
4	a	600000	54	in	70000
5	an	500000	55	on	60000
6	is	400000	56	at	50000
7	was	300000	57	from	40000
8	he	200000	58	with	30000
9	she	150000	59	it	20000
10	you	100000	60	that	15000
11	and	90000	61	as	10000
12	or	80000	62	by	5000
13	in	70000	63	to	4000
14	on	60000	64	of	3000
15	at	50000	65	the	2000
16	from	40000	66	is	1000
17	with	30000	67	was	500
18	it	20000	68	he	400
19	that	15000	69	she	300
20	as	10000	70	you	200
21	by	5000	71	and	100
22	to	4000	72	or	50
23	of	3000	73	in	40
24	the	2000	74	on	30
25	is	1000	75	at	20
26	was	500	76	from	10
27	he	400	77	with	5
28	she	300	78	it	4
29	you	200	79	that	3
30	and	100	80	as	2
31	or	50	81	by	1
32	in	40	82	to	1
33	on	30	83	of	1
34	at	20	84	the	1
35	from	10	85	is	1
36	with	5	86	was	1
37	it	4	87	he	1
38	that	3	88	she	1
39	as	2	89	you	1
40	by	1	90	and	1
41	to	1	91	or	1
42	of	1	92	in	1
43	the	1	93	on	1
44	is	1	94	at	1
45	was	1	95	from	1
46	he	1	96	with	1
47	she	1	97	it	1
48	you	1	98	that	1
49	and	1	99	as	1
50	or	1	100	by	1

CALCULATION OF SHORT GEODETIC DISTANCES.

COEFFICIENTS A FOR GIVEN MEAN LATITUDE.
(LENGTHS IN KILOMETERS OF 1 MINUTE OF THE PARALLEL)

MIN	51	52	53	54	55
50	1.170013	1.144685	1.119005	1.092980	1.066618
51	1.169593	1.144260	1.118574	1.092543	1.066176
52	1.169174	1.143834	1.118142	1.092106	1.065733
53	1.168754	1.143409	1.117711	1.091669	1.065291
54	1.168335	1.142985	1.117280	1.091232	1.064848
55	1.167915	1.142558	1.116849	1.090795	1.064406
56	1.167495	1.142132	1.116417	1.090358	1.063963
57	1.167076	1.141707	1.115986	1.089921	1.063521
58	1.166656	1.141281	1.115555	1.089484	1.063078
59	1.166237	1.140856	1.115123	1.089047	1.062636
10	1.165817	1.140430	1.114692	1.088610	1.062193
11	1.165396	1.140003	1.114259	1.088172	1.061749
12	1.164976	1.139577	1.113827	1.087734	1.061306
13	1.164555	1.139150	1.113395	1.087296	1.060862
14	1.164134	1.138724	1.112962	1.086858	1.060419
15	1.163714	1.138297	1.112530	1.086420	1.059976
16	1.163293	1.137871	1.112098	1.085983	1.059532
17	1.162873	1.137444	1.111666	1.085545	1.059089
18	1.162452	1.137018	1.111233	1.085107	1.058645
19	1.162031	1.136591	1.110801	1.084669	1.058202
20	1.161610	1.136164	1.110368	1.084230	1.057758
21	1.161189	1.135737	1.109935	1.083791	1.057314
22	1.160767	1.135309	1.109502	1.083352	1.056869
23	1.160346	1.134882	1.109069	1.082914	1.056425
24	1.159924	1.134454	1.108636	1.082475	1.055981
25	1.159502	1.134027	1.108202	1.082036	1.055536
26	1.159081	1.133599	1.107769	1.081597	1.055092
27	1.158659	1.133172	1.107336	1.081158	1.054648
28	1.158238	1.132744	1.106903	1.080719	1.054203
29	1.157816	1.132317	1.106470	1.080280	1.053759
30	1.157394	1.131889	1.106036	1.079841	1.053314

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Dr. N. P. Gibson	2626 Madison St.	Butte	Mont.	U.S.A.
Dr. O. Q. Howell	2727 Jefferson St.	Helena	Mont.	U.S.A.
Dr. P. R. King	2828 Adams St.	Great Falls	Mont.	U.S.A.
Dr. Q. S. Lee	2929 Washington St.	Missoula	Mont.	U.S.A.
Dr. R. T. Scott	3030 Madison St.	Butte	Mont.	U.S.A.
Dr. S. U. Walker	3131 Jefferson St.	Helena	Mont.	U.S.A.
Dr. T. V. Adams	3232 Adams St.	Great Falls	Mont.	U.S.A.
Dr. U. W. Baker	3333 Washington St.	Missoula	Mont.	U.S.A.
Dr. V. X. Carter	3434 Madison St.	Butte	Mont.	U.S.A.
Dr. W. Y. Evans	3535 Jefferson St.	Helena	Mont.	U.S.A.
Dr. X. Z. Foster	3636 Adams St.	Great Falls	Mont.	U.S.A.
Dr. Y. A. Gibson	3737 Washington St.	Missoula	Mont.	U.S.A.
Dr. Z. B. Howell	3838 Madison St.	Butte	Mont.	U.S.A.
Dr. A. C. King	3939 Jefferson St.	Helena	Mont.	U.S.A.
Dr. B. D. Lee	4040 Adams St.	Great Falls	Mont.	U.S.A.
Dr. C. E. Scott	4141 Washington St.	Missoula	Mont.	U.S.A.
Dr. D. F. Walker	4242 Madison St.	Butte	Mont.	U.S.A.
Dr. E. G. Adams	4343 Jefferson St.	Helena	Mont.	U.S.A.
Dr. F. H. Baker	4444 Adams St.	Great Falls	Mont.	U.S.A.
Dr. G. I. Carter	4545 Washington St.	Missoula	Mont.	U.S.A.
Dr. H. J. Evans	4646 Madison St.	Butte	Mont.	U.S.A.
Dr. I. K. Foster	4747 Jefferson St.	Helena	Mont.	U.S.A.
Dr. J. L. Gibson	4848 Adams St.	Great Falls	Mont.	U.S.A.
Dr. K. M. Howell	4949 Washington St.	Missoula	Mont.	U.S.A.
Dr. L. N. King	5050 Madison St.	Butte	Mont.	U.S.A.
Dr. M. O. Lee	5151 Jefferson St.	Helena	Mont.	U.S.A.
Dr. N. P. Scott	5252 Adams St.	Great Falls	Mont.	U.S.A.
Dr. O. Q. Walker	5353 Washington St.	Missoula	Mont.	U.S.A.
Dr. P. R. Adams	5454 Madison St.	Butte	Mont.	U.S.A.
Dr. Q. S. Baker	5555 Jefferson St.	Helena	Mont.	U.S.A.
Dr. R. T. Carter	5656 Adams St.	Great Falls	Mont.	U.S.A.
Dr. S. U. Evans	5757 Washington St.	Missoula	Mont.	U.S.A.
Dr. T. V. Foster	5858 Madison St.	Butte	Mont.	U.S.A.
Dr. U. W. Gibson	5959 Jefferson St.	Helena	Mont.	U.S.A.
Dr. V. X. Howell	6060 Adams St.	Great Falls	Mont.	U.S.A.
Dr. W. Y. King	6161 Washington St.	Missoula	Mont.	U.S.A.
Dr. X. Z. Lee	6262 Madison St.	Butte	Mont.	U.S.A.
Dr. Y. A. Scott	6363 Jefferson St.	Helena	Mont.	U.S.A.
Dr. Z. B. Walker	6464 Adams St.	Great Falls	Mont.	U.S.A.
Dr. A. C. Adams	6565 Washington St.	Missoula	Mont.	U.S.A.
Dr. B. D. Baker	6666 Madison St.	Butte	Mont.	U.S.A.
Dr. C. E. Carter	6767 Jefferson St.	Helena	Mont.	U.S.A.
Dr. D. F. Evans	6868 Adams St.	Great Falls	Mont.	U.S.A.
Dr. E. G. Foster	6969 Washington St.	Missoula	Mont.	U.S.A.
Dr. F. H. Gibson	7070 Madison St.	Butte	Mont.	U.S.A.
Dr. G. I. Howell	7171 Jefferson St.	Helena	Mont.	U.S.A.
Dr. H. J. King	7272 Adams St.	Great Falls	Mont.	U.S.A.
Dr. I. K. Lee	7373 Washington St.	Missoula	Mont.	U.S.A.
Dr. J. L. Scott	7474 Madison St.	Butte	Mont.	U.S.A.
Dr. K. M. Walker	7575 Jefferson St.	Helena	Mont.	U.S.A.
Dr. L. N. Adams	7676 Adams St.	Great Falls	Mont.	U.S.A.
Dr. M. O. Baker	7777 Washington St.	Missoula	Mont.	U.S.A.
Dr. N. P. Carter	7878 Madison St.	Butte	Mont.	U.S.A.
Dr. O. Q. Evans	7979 Jefferson St.	Helena	Mont.	U.S.A.
Dr. P. R. Foster	8080 Adams St.	Great Falls	Mont.	U.S.A.
Dr. Q. S. Gibson	8181 Washington St.	Missoula	Mont.	U.S.A.
Dr. R. T. Howell	8282 Madison St.	Butte	Mont.	U.S.A.
Dr. S. U. King	8383 Jefferson St.	Helena	Mont.	U.S.A.
Dr. T. V. Lee	8484 Adams St.	Great Falls	Mont.	U.S.A.
Dr. U. W. Scott	8585 Washington St.	Missoula	Mont.	U.S.A.
Dr. V. X. Walker	8686 Madison St.	Butte	Mont.	U.S.A.
Dr. W. Y. Adams	8787 Jefferson St.	Helena	Mont.	U.S.A.
Dr. X. Z. Baker	8888 Adams St.	Great Falls	Mont.	U.S.A.
Dr. Y. A. Carter	8989 Washington St.	Missoula	Mont.	U.S.A.
Dr. Z. B. Evans	9090 Madison St.	Butte	Mont.	U.S.A.
Dr. A. C. Foster	9191 Jefferson St.	Helena	Mont.	U.S.A.
Dr. B. D. Gibson	9292 Adams St.	Great Falls	Mont.	U.S.A.
Dr. C. E. Howell	9393 Washington St.	Missoula	Mont.	U.S.A.
Dr. D. F. King	9494 Madison St.	Butte	Mont.	U.S.A.
Dr. E. G. Lee	9595 Jefferson St.	Helena	Mont.	U.S.A.
Dr. F. H. Scott	9696 Adams St.	Great Falls	Mont.	U.S.A.
Dr. G. I. Walker	9797 Washington St.	Missoula	Mont.	U.S.A.
Dr. H. J. Adams	9898 Madison St.	Butte	Mont.	U.S.A.
Dr. I. K. Baker	9999 Jefferson St.	Helena	Mont.	U.S.A.

CALCULATION OF SHORT GEODETIC DISTANCES.

COEFFICIENTS A FOR GIVEN MEAN LATITUDE.
(LENGTHS IN KILOMETERS OF 1 MINUTE OF THE PARALLEL)

MIN	51	52	53	54	55
30	1.157394	1.131889	1.106036	1.079841	1.053314
31	1.156971	1.131461	1.105602	1.079401	1.052869
32	1.156549	1.131032	1.105167	1.078962	1.052424
33	1.156126	1.130604	1.104733	1.078522	1.051978
34	1.155704	1.130175	1.104299	1.078082	1.051533
35	1.155281	1.129747	1.103865	1.077642	1.051088
36	1.154858	1.129319	1.103431	1.077202	1.050643
37	1.154436	1.128890	1.102996	1.076763	1.050198
38	1.154013	1.128462	1.102562	1.076323	1.049752
39	1.153591	1.128033	1.102128	1.075883	1.049307
40	1.153168	1.127604	1.101693	1.075443	1.048861
41	1.152744	1.127175	1.101258	1.075002	1.048415
42	1.152321	1.126745	1.100823	1.074562	1.047969
43	1.151897	1.126316	1.100388	1.074121	1.047523
44	1.151473	1.125887	1.099953	1.073680	1.047076
45	1.151050	1.125457	1.099518	1.073239	1.046630
46	1.150626	1.125028	1.099083	1.072799	1.046184
47	1.150203	1.124598	1.098648	1.072358	1.045738
48	1.149779	1.124169	1.098213	1.071917	1.045292
49	1.149355	1.123740	1.097778	1.071477	1.044845
50	1.148931	1.123310	1.097342	1.071035	1.044399
51	1.148507	1.122879	1.096906	1.070593	1.043952
52	1.148082	1.122449	1.096469	1.070152	1.043505
53	1.147658	1.122018	1.096033	1.069710	1.043058
54	1.147233	1.121588	1.095597	1.069268	1.042611
55	1.146808	1.121158	1.095161	1.068827	1.042164
56	1.146384	1.120727	1.094725	1.068385	1.041717
57	1.145959	1.120297	1.094289	1.067943	1.041270
58	1.145535	1.119866	1.093853	1.067502	1.040822
59	1.145110	1.119436	1.093417	1.067060	1.040375
60	1.144685	1.119005	1.092980	1.066618	1.039928

CALCULATION OF SHORT GEODETIC DISTANCES.

COEFFICIENTS A FOR GIVEN MEAN LATITUDE.
(LENGTHS IN KILOMETERS OF 1 MINUTE OF THE PARALLEL)

MIN	56	57	58	59	60
00	1.039928	1.012918	0.985595	0.957968	0.930047
01	1.039480	1.012465	0.985137	0.957505	0.929579
02	1.039032	1.012012	0.984678	0.957041	0.929111
03	1.038584	1.011558	0.984220	0.956578	0.928643
04	1.038136	1.011105	0.983762	0.956115	0.928174
05	1.037689	1.010652	0.983303	0.955652	0.927706
06	1.037241	1.010199	0.982845	0.955188	0.927238
07	1.036793	1.009746	0.982387	0.954725	0.926770
08	1.036345	1.009292	0.981928	0.954262	0.926302
09	1.035897	1.008839	0.981470	0.953798	0.925834
10	1.035449	1.008386	0.981012	0.953335	0.925365
11	1.035000	1.007932	0.980552	0.952871	0.924896
12	1.034551	1.007478	0.980093	0.952407	0.924427
13	1.034102	1.007024	0.979634	0.951943	0.923959
14	1.033653	1.006570	0.979175	0.951478	0.923490
15	1.033205	1.006116	0.978716	0.951014	0.923021
16	1.032756	1.005662	0.978257	0.950550	0.922552
17	1.032307	1.005208	0.977797	0.950086	0.922083
18	1.031858	1.004754	0.977338	0.949622	0.921614
19	1.031409	1.004300	0.976879	0.949158	0.921145
20	1.030960	1.003845	0.976420	0.948694	0.920676
21	1.030511	1.003390	0.975960	0.948229	0.920206
22	1.030061	1.002935	0.975500	0.947764	0.919736
23	1.029611	1.002481	0.975040	0.947299	0.919267
24	1.029162	1.002026	0.974580	0.946834	0.918797
25	1.028712	1.001571	0.974120	0.946369	0.918327
26	1.028262	1.001116	0.973660	0.945904	0.917857
27	1.027812	1.000661	0.973200	0.945439	0.917387
28	1.027363	1.000206	0.972740	0.944974	0.916918
29	1.026913	0.999751	0.972280	0.944509	0.916448
30	1.026463	0.999296	0.971820	0.944044	0.915978

CALCULATION OF SHORT GEODETIC DISTANCES.

COEFFICIENTS A FOR GIVEN MEAN LATITUDE.
(LENGTHS IN KILOMETERS OF 1 MINUTE OF THE PARALLEL)

MIN	56	57	58	59	60
30	1.026463	0.999296	0.971820	0.944044	0.915978
31	1.026012	0.998840	0.971359	0.943579	0.915508
32	1.025562	0.998384	0.970898	0.943113	0.915037
33	1.025111	0.997928	0.970437	0.942647	0.914566
34	1.024661	0.997472	0.969976	0.942181	0.914096
35	1.024210	0.997017	0.969515	0.941716	0.913625
36	1.023759	0.996561	0.969054	0.941250	0.913155
37	1.023309	0.996105	0.968593	0.940784	0.912684
38	1.022858	0.995649	0.968133	0.940318	0.912214
39	1.022408	0.995193	0.967672	0.939853	0.911743
40	1.021957	0.994737	0.967211	0.939387	0.911272
41	1.021505	0.994281	0.966749	0.938920	0.910801
42	1.021054	0.993824	0.966287	0.938454	0.910330
43	1.020602	0.993367	0.965826	0.937987	0.909858
44	1.020151	0.992911	0.965364	0.937520	0.909387
45	1.019699	0.992454	0.964902	0.937054	0.908916
46	1.019248	0.991997	0.964441	0.936587	0.908444
47	1.018796	0.991541	0.963979	0.936121	0.907973
48	1.018345	0.991084	0.963517	0.935654	0.907501
49	1.017893	0.990627	0.963055	0.935188	0.907030
50	1.017442	0.990171	0.962594	0.934721	0.906559
51	1.016990	0.989713	0.962131	0.934254	0.906087
52	1.016537	0.989256	0.961669	0.933786	0.905614
53	1.016085	0.988798	0.961206	0.933319	0.905142
54	1.015633	0.988340	0.960743	0.932851	0.904670
55	1.015180	0.987883	0.960281	0.932384	0.904198
56	1.014728	0.987425	0.959818	0.931917	0.903726
57	1.014276	0.986968	0.959356	0.931449	0.903254
58	1.013824	0.986510	0.958893	0.930982	0.902781
59	1.013371	0.986053	0.958431	0.930514	0.902309
60	1.012918	0.985595	0.957968	0.930047	0.901837

CALCULATION OF SHORT GEODETIC DISTANCES.

COEFFICIENTS A FOR GIVEN MEAN LATITUDE.
(LENGTHS IN KILOMETERS OF 1 MINUTE OF THE PARALLEL)

	61	62	63	64	65
00	0.901837	0.873348	0.844590	0.815572	0.786300
01	0.901364	0.872871	0.844108	0.815086	0.785810
02	0.900891	0.872393	0.843626	0.814600	0.785320
03	0.900418	0.871916	0.843145	0.814114	0.784829
04	0.899946	0.871438	0.842663	0.813628	0.784339
05	0.899473	0.870961	0.842181	0.813142	0.783849
06	0.899000	0.870483	0.841699	0.812655	0.783359
07	0.898527	0.870006	0.841217	0.812169	0.782868
08	0.898054	0.869529	0.840735	0.811683	0.782378
09	0.897581	0.869051	0.840254	0.811197	0.781888
10	0.897108	0.868574	0.839772	0.810711	0.781398
11	0.896635	0.868096	0.839289	0.810224	0.780907
12	0.896161	0.867617	0.838807	0.809737	0.780416
13	0.895687	0.867139	0.838324	0.809251	0.779925
14	0.895214	0.866661	0.837842	0.808764	0.779434
15	0.894740	0.866183	0.837359	0.808277	0.778943
16	0.894266	0.865705	0.836876	0.807790	0.778452
17	0.893793	0.865227	0.836394	0.807303	0.777961
18	0.893319	0.864748	0.835911	0.806817	0.777471
19	0.892845	0.864270	0.835429	0.806330	0.776980
20	0.892372	0.863792	0.834946	0.805843	0.776489
21	0.891897	0.863313	0.834463	0.805355	0.775997
22	0.891423	0.862834	0.833980	0.804868	0.775506
23	0.890948	0.862355	0.833496	0.804380	0.775014
24	0.890474	0.861876	0.833013	0.803893	0.774522
25	0.890000	0.861397	0.832530	0.803405	0.774031
26	0.889525	0.860918	0.832047	0.802918	0.773539
27	0.889051	0.860439	0.831563	0.802430	0.773048
28	0.888576	0.859961	0.831080	0.801943	0.772556
29	0.888102	0.859482	0.830597	0.801455	0.772065
30	0.887627	0.859003	0.830114	0.800968	0.771573

CALCULATION OF SHORT GEODETIC DISTANCES.

COEFFICIENTS A FOR GIVEN MEAN LATITUDE.
(LENGTHS IN KILOMETERS OF 1 MINUTE OF THE PARALLEL)

	61	62	63	64	65
30	0.887627	0.859003	0.830114	0.800968	0.771573
31	0.887152	0.858523	0.829630	0.800480	0.771081
32	0.886677	0.858043	0.829146	0.799991	0.770588
33	0.886202	0.857564	0.828662	0.799503	0.770096
34	0.885727	0.857084	0.828178	0.799015	0.769604
35	0.885251	0.856604	0.827694	0.798527	0.769112
36	0.884776	0.856125	0.827210	0.798038	0.768619
37	0.884301	0.855645	0.826726	0.797550	0.768127
38	0.883826	0.855165	0.826242	0.797062	0.767635
39	0.883351	0.854686	0.825758	0.796574	0.767143
40	0.882875	0.854206	0.825274	0.796086	0.766650
41	0.882399	0.853725	0.824789	0.795597	0.766157
42	0.881923	0.853245	0.824304	0.795108	0.765665
43	0.881447	0.852765	0.823819	0.794619	0.765172
44	0.880971	0.852284	0.823335	0.794130	0.764679
45	0.880495	0.851804	0.822850	0.793641	0.764186
46	0.880020	0.851323	0.822365	0.793152	0.763693
47	0.879544	0.850843	0.821881	0.792663	0.763200
48	0.879068	0.850363	0.821396	0.792174	0.762707
49	0.878592	0.849882	0.820911	0.791685	0.762214
50	0.878116	0.849402	0.820426	0.791196	0.761721
51	0.877639	0.848921	0.819941	0.790707	0.761227
52	0.877162	0.848439	0.819456	0.790217	0.760734
53	0.876685	0.847958	0.818970	0.789727	0.760240
54	0.876209	0.847477	0.818485	0.789238	0.759747
55	0.875732	0.846996	0.817999	0.788748	0.759253
56	0.875255	0.846515	0.817514	0.788259	0.758759
57	0.874778	0.846034	0.817028	0.787769	0.758266
58	0.874302	0.845552	0.816543	0.787279	0.757772
59	0.873825	0.845071	0.816057	0.786790	0.757279
60	0.873348	0.844590	0.815572	0.786300	0.756785

Appendix 1: Summary of the 100 most common words

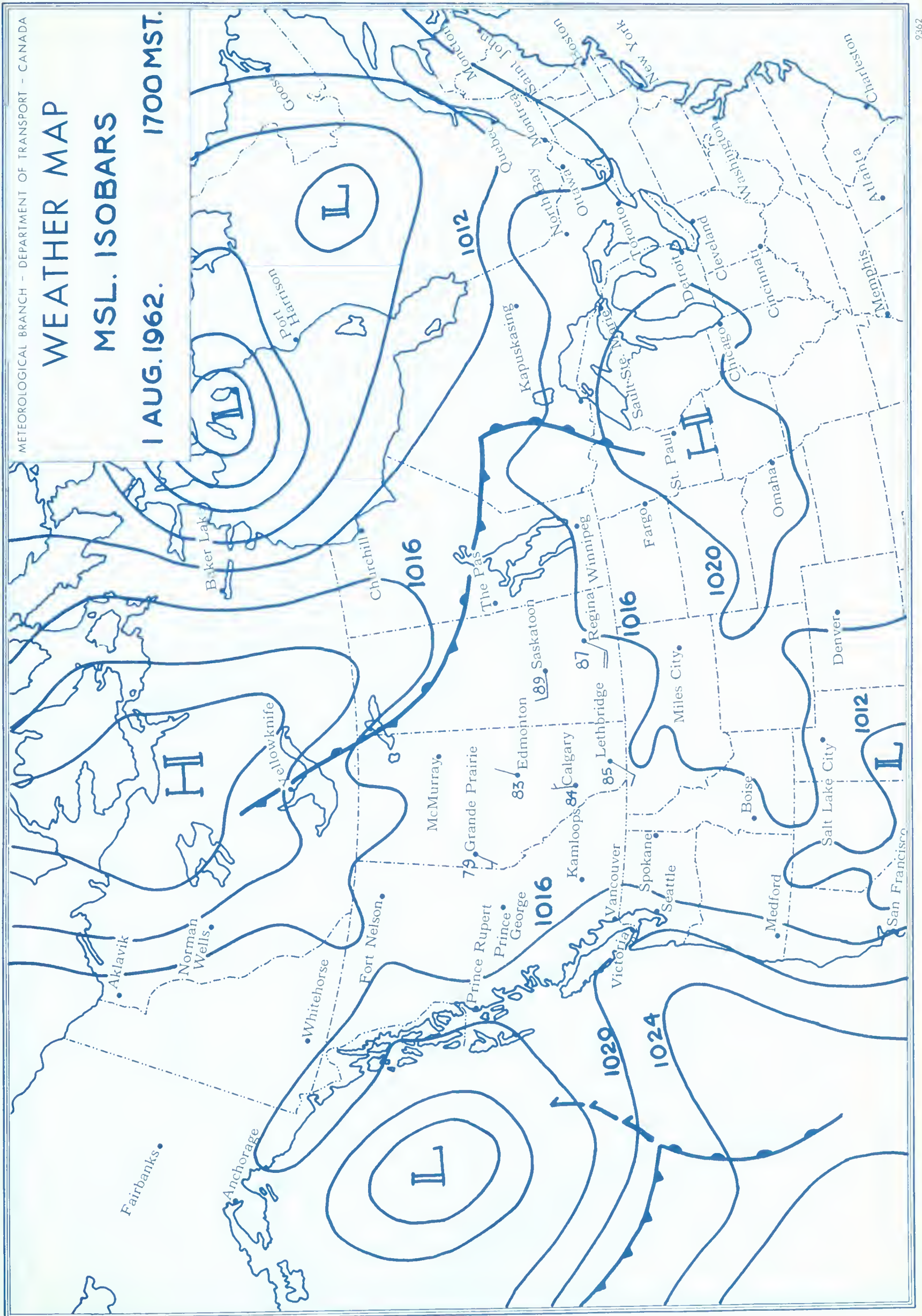
The following table shows the 100 most common words in the corpus, ranked by frequency. The words are listed in the first column, and the frequency of each word is shown in the second column. The words are listed in descending order of frequency.

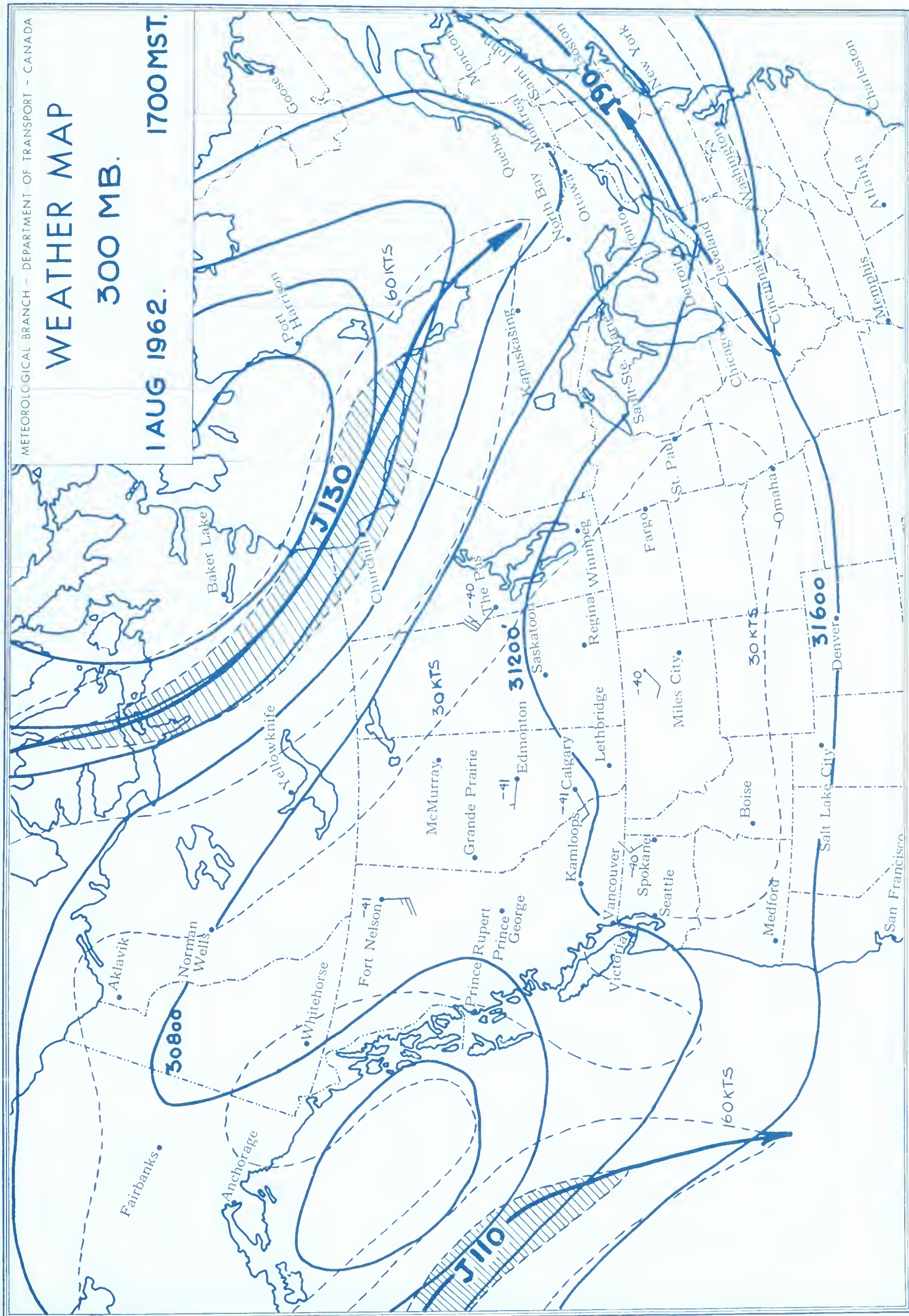
Rank	Word	Frequency	Rank	Word	Frequency
1	the	1000	51	and	100
2	is	800	52	of	90
3	to	700	53	a	80
4	in	600	54	on	70
5	at	500	55	with	60
6	of	400	56	by	50
7	and	300	57	for	40
8	is	200	58	from	30
9	to	150	59	the	20
10	in	100	60	and	10
11	at	80	61	is	5
12	of	70	62	to	5
13	and	60	63	in	5
14	is	50	64	at	5
15	to	40	65	of	5
16	in	30	66	and	5
17	at	20	67	is	5
18	of	15	68	to	5
19	and	10	69	in	5
20	is	5	70	at	5
21	to	5	71	of	5
22	in	5	72	and	5
23	at	5	73	is	5
24	of	5	74	to	5
25	and	5	75	in	5
26	is	5	76	at	5
27	to	5	77	of	5
28	in	5	78	and	5
29	at	5	79	is	5
30	of	5	80	to	5
31	and	5	81	in	5
32	is	5	82	at	5
33	to	5	83	of	5
34	in	5	84	and	5
35	at	5	85	is	5
36	of	5	86	to	5
37	and	5	87	in	5
38	is	5	88	at	5
39	to	5	89	of	5
40	in	5	90	and	5
41	at	5	91	is	5
42	of	5	92	to	5
43	and	5	93	in	5
44	is	5	94	at	5
45	to	5	95	of	5
46	in	5	96	and	5
47	at	5	97	is	5
48	of	5	98	to	5
49	and	5	99	in	5
50	is	5	100	at	5

CALCULATION OF SHORT GEODETIC DISTANCES.

COEFFICIENTS B FOR GIVEN MEAN LATITUDE.
(LENGTHS IN KILOMETERS OF 1 MINUTE OF THE MERIDIAN)

	46	47	48	49	50
00	1.852531	1.852860	1.853188	1.853515	1.853842
10	1.852586	1.852915	1.853243	1.853570	1.853896
20	1.852641	1.852970	1.853297	1.853624	1.853950
30	1.852696	1.853024	1.853352	1.853679	1.854004
40	1.852750	1.853079	1.853406	1.853733	1.854058
50	1.852805	1.853133	1.853461	1.853788	1.854112
60	1.852860	1.853188	1.853515	1.853842	1.854165
	51	52	53	54	55
00	1.854165	1.854487	1.854805	1.855122	1.855433
10	1.854219	1.854540	1.854858	1.855174	1.855485
20	1.854273	1.854593	1.854911	1.855226	1.855536
30	1.854326	1.854647	1.854964	1.855278	1.855588
40	1.854380	1.854700	1.855016	1.855330	1.855639
50	1.854433	1.854752	1.855069	1.855382	1.855691
60	1.854487	1.854805	1.855122	1.855433	1.855742
	56	57	58	59	60
00	1.855742	1.856045	1.856345	1.856640	1.856928
10	1.855793	1.856095	1.856395	1.856689	1.856976
20	1.855844	1.856145	1.856444	1.856737	1.857023
30	1.855894	1.856195	1.856493	1.856785	1.857071
40	1.855945	1.856245	1.856542	1.856833	1.857118
50	1.855995	1.856295	1.856591	1.856881	1.857165
60	1.856045	1.856345	1.856640	1.856928	1.857212
	61	62	63	64	65
00	1.857212	1.857490	1.857762	1.858025	1.858283
10	1.857259	1.857536	1.857806	1.858068	1.858325
20	1.857305	1.857581	1.857851	1.858112	1.858367
30	1.857352	1.857627	1.857895	1.858155	1.858409
40	1.857398	1.857672	1.857938	1.858198	1.858451
50	1.857444	1.857717	1.857982	1.858240	1.858492
60	1.857490	1.857762	1.858025	1.858283	1.858533



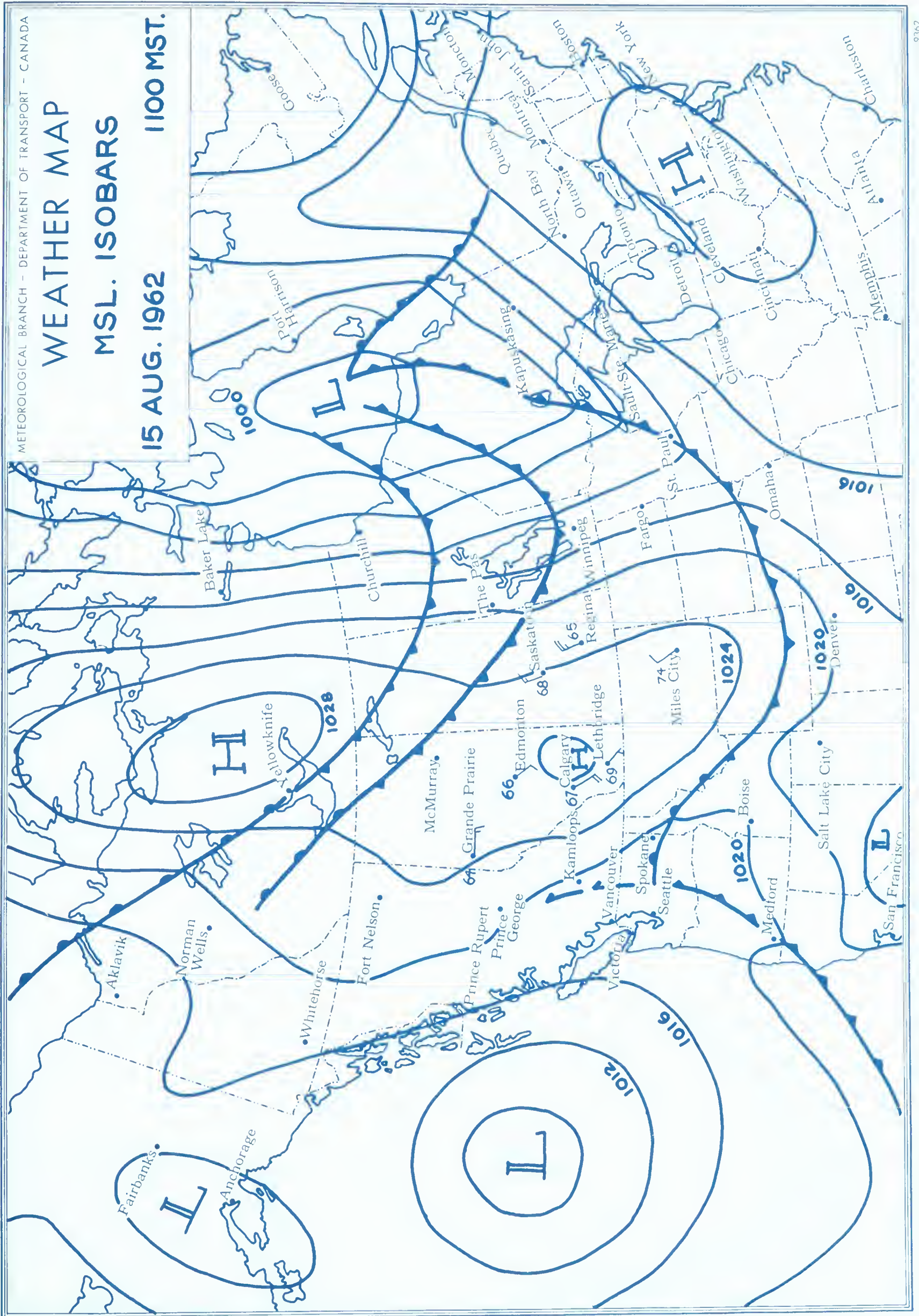


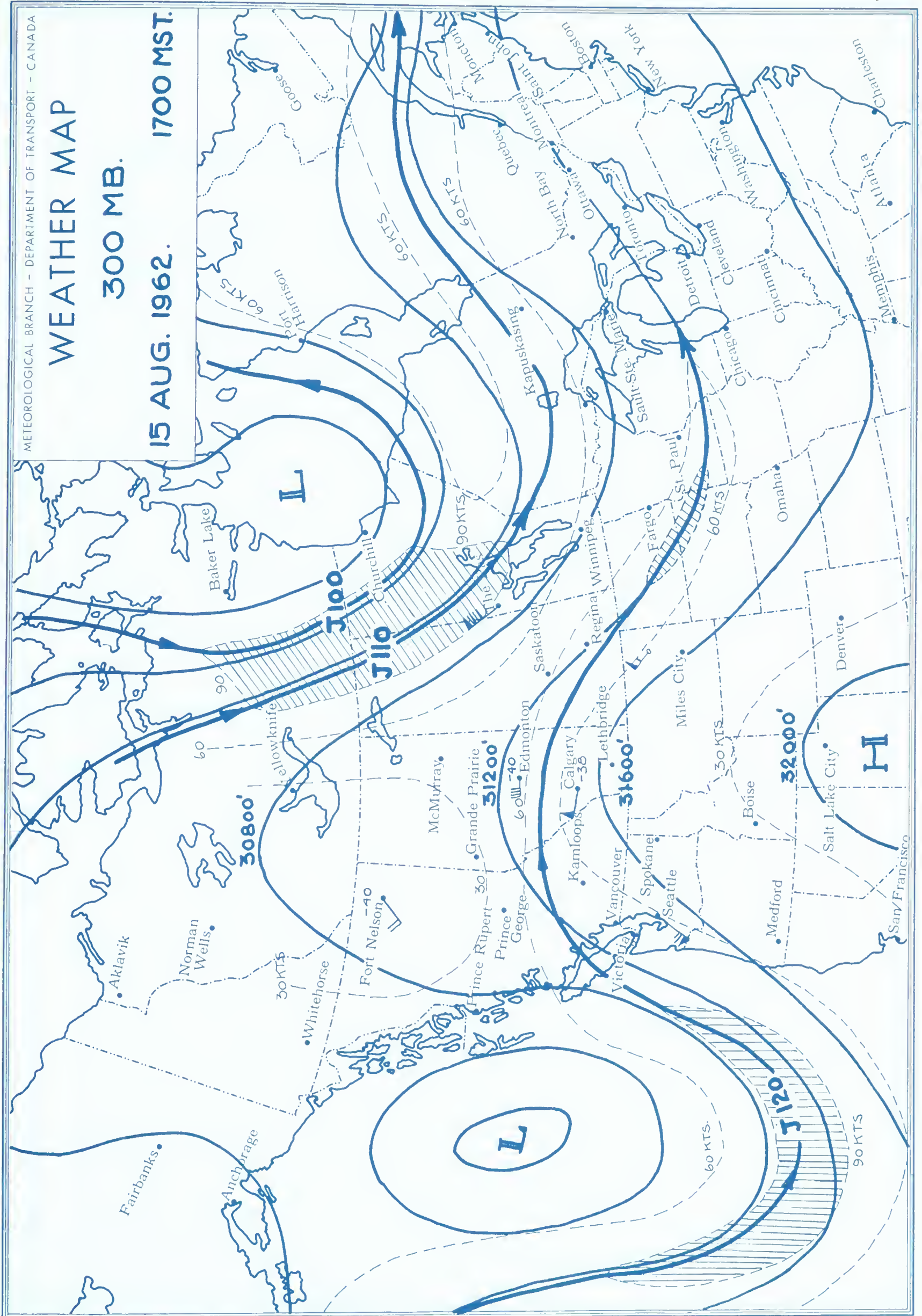


1900-1901

2000-2001

1900-1901





APPENDIX VISome Results of Ray and Absorption Computations

On Pages 215 and 216 are shown the computed ranges, travel times, etc., of rays propagating due west from the Suffield Shot Point. Each set of results consists of two lines: the first line shows the initial conditions at the shot point e.g. zero range, zero time, etc., and the angles of incidence of ray and wave normal. The second line gives the total range and travel time of the ray, the apex height, apex temperature, apex wind component, and the two angles of incidence (both 90°) at the ray apex.

Samples of the absorption computation are shown on pages 217-220. The column headings are self-explanatory.

Range (km)	Travel Time (sec)	Height* (km)	Temper- ature (°K)	Wind (m/sec)	Angle of Incidence:	
					Normal	Ray
0.000	0.000	.778	303.2	1.0	90.00	90.00
198.663	706.451	43.489	275.8	17.1	90.00	90.00
0.000	0.000	.778	303.2	1.0	89.50	89.50
198.151	705.912	43.492	275.8	17.1	90.00	90.00
0.000	0.000	.778	303.2	1.0	89.00	89.00
198.824	705.312	43.503	275.8	17.1	90.00	90.00
0.000	0.000	.778	303.2	1.0	88.50	88.50
197.843	704.711	43.519	275.9	17.2	90.00	90.00
0.000	0.000	.778	303.2	1.0	88.00	88.00
198.173	704.095	43.543	276.0	17.2	90.00	90.00
0.000	0.000	.778	303.2	1.0	87.50	87.50
197.613	703.488	43.573	276.2	17.2	90.00	90.00
0.000	0.000	.778	303.2	1.0	87.00	87.00
197.537	702.785	43.610	276.4	17.2	90.00	90.00
0.000	0.000	.778	303.2	1.0	86.50	86.51
197.355	702.143	43.654	276.6	17.2	90.00	90.00
0.000	0.000	.778	303.2	1.0	86.00	86.01
196.927	701.517	43.704	276.9	17.3	90.00	90.00
0.000	0.000	.778	303.2	1.0	85.50	85.51
196.550	700.862	43.762	277.2	17.3	90.00	90.00
0.000	0.000	.778	303.2	1.0	85.00	85.01
195.629	701.832	43.826	277.6	17.4	90.00	90.00
0.000	0.000	.778	303.2	1.0	84.50	84.51
196.547	699.480	43.897	277.9	17.4	90.00	90.00
0.000	0.000	.778	303.2	1.0	84.00	84.01
195.895	698.852	43.975	278.4	17.5	90.00	90.00
0.000	0.000	.778	303.2	1.0	83.50	83.51
195.532	698.208	44.059	278.8	17.5	90.00	90.00
0.000	0.000	.778	303.2	1.0	83.00	83.02
195.165	697.574	44.151	279.3	17.6	90.00	90.00
0.000	0.000	.778	303.2	1.0	82.50	82.52
195.175	696.913	44.250	279.9	17.7	90.00	90.00
0.000	0.000	.778	303.2	1.0	82.00	82.02
195.105	696.194	44.355	280.4	17.8	90.00	90.00
0.000	0.000	.778	303.2	1.0	81.50	81.52
194.497	695.724	44.468	281.0	17.8	90.00	90.00
0.000	0.000	.778	303.2	1.0	81.00	81.02
194.349	695.098	44.588	281.7	17.9	90.00	90.00
0.000	0.000	.778	303.2	1.0	80.50	80.52
194.464	694.559	44.714	282.4	18.0	90.00	90.00
0.000	0.000	.778	303.2	1.0	80.00	80.02
194.110	694.062	44.849	283.1	18.1	90.00	90.00

* See note on Page 214.

Range (km)	Travel Time (sec)	Height* (km)	Temper- ature (°K)	Wind (m/sec)	Angle of Incidence:	
					Normal	Ray
0.000	0.000	.778	303.2	1.0	79.50	79.53
193.816	693.588	44.990	283.9	18.2	90.00	90.00
0.000	0.000	.778	303.2	1.0	79.00	79.03
193.743	693.029	45.139	284.7	18.3	90.00	90.00
0.000	0.000	.778	303.2	1.0	78.50	78.52
193.617	692.518	45.295	285.5	18.4	90.00	90.00
0.000	0.000	.778	303.2	1.0	78.00	78.03
193.670	692.327	45.458	286.4	18.5	90.00	90.00
0.000	0.000	.778	303.2	1.0	77.50	77.53
193.618	691.996	45.629	287.4	18.7	90.00	90.00
0.000	0.000	.778	303.2	1.0	77.00	77.03
193.499	691.720	45.807	288.4	18.8	90.00	90.00
0.000	0.000	.778	303.2	1.0	76.50	76.53
193.536	691.479	45.993	289.4	18.9	90.00	90.00
0.000	0.000	.778	303.2	1.0	76.00	76.04
193.371	691.147	46.187	290.4	19.1	90.00	90.00
0.000	0.000	.778	303.2	1.0	75.50	75.54
193.288	691.192	46.388	291.6	19.2	90.00	90.00
0.000	0.000	.778	303.2	1.0	75.00	75.04
192.904	691.166	46.598	292.7	19.4	90.00	90.00
0.000	0.000	.778	303.2	1.0	74.50	74.54
193.458	691.113	46.815	293.9	19.5	90.00	90.00
0.000	0.000	.778	303.2	1.0	74.00	74.04
225.914	781.834	47.280	295.2	19.6	90.00	90.00
0.000	0.000	.778	303.2	1.0	73.50	73.54
278.611	925.908	48.878	296.8	19.6	90.00	90.00
0.000	0.000	.778	303.2	1.0	73.00	73.04
310.847	1014.245	50.533	298.5	19.6	90.00	90.00
0.000	0.000	.778	303.2	1.0	72.50	72.55
337.563	1084.900	52.244	300.2	19.6	90.00	90.00
0.000	0.000	.778	303.2	1.0	72.00	72.05
360.238	1148.624	54.013	301.9	19.6	90.00	90.00
0.000	0.000	.778	303.2	1.0	71.50	71.55
382.589	1206.045	55.841	303.8	19.6	90.00	90.00
0.000	0.000	.778	303.2	1.0	71.00	71.05
391.729	1231.936	57.620	303.9	20.7	90.00	90.00
0.000	0.000	.778	303.2	1.0	70.80	70.85
394.363	1240.191	58.276	302.8	21.7	90.00	90.00
0.000	0.000	.778	303.2	1.0	70.60	70.65
398.615	1251.610	58.942	301.7	22.8	90.00	90.00
0.000	0.000	.778	303.2	1.0	70.40	70.45
403.392	1263.212	59.615	300.6	23.9	90.00	90.00

* See note on Page 214.

Absorption Integral* (sec ³ /km ²)	Mean Free Path Parameter** (relative)	Range (km)	Apex Height (km)	Ray Angle (deg)
.51652355E+06	.53989213E+03	198.663	43.486	90.00
.51668396E+06	.54006555E+03	198.151	43.492	89.50
.51354091E+06	.54144724E+03	197.843	43.512	88.50
.52120640E+06	.54421270E+03	197.613	43.572	87.50
.52385496E+06	.54836720E+03	197.555	43.654	86.51
.52804265E+06	.55096608E+03	196.927	43.704	86.01
.53140192E+06	.55391518E+03	196.550	43.762	85.51
.53471607E+06	.55721532E+03	195.620	43.826	85.01
.54754275E+06	.56923202E+03	195.522	44.059	83.51
.55226676E+06	.57394957E+03	195.165	44.151	83.02
.56337236E+06	.58446056E+03	195.105	44.255	82.02
.57650946E+06	.59642120E+03	194.849	44.588	81.02
.58344862E+06	.60295085E+03	194.464	44.714	80.52
.59073945E+06	.60985062E+03	194.110	44.849	80.02
.59865377E+06	.61712389E+03	193.816	44.990	79.53
.60663699E+06	.62477283E+03	193.743	45.139	79.03
.61551363E+06	.63279903E+03	193.617	45.295	78.53
.64467950E+06	.65918009E+03	193.499	45.807	77.03
.66633212E+06	.67872328E+03	193.371	46.187	76.04
.67818155E+06	.68909374E+03	193.288	46.382	75.54
.69044232E+06	.69986824E+03	192.904	46.598	75.04
.69044232E+06	.69986824E+03	192.904	46.598	75.04
.70293152E+06	.71105217E+03	193.458	46.915	74.54
.10093086E+07	.78232282E+03	225.914	47.280	74.04
.17547419E+07	.11340828E+04	278.611	48.878	73.54
.25302751E+07	.14983252E+04	310.847	50.533	73.04
.34049949E+07	.18750554E+04	337.563	52.244	72.55
.44078073E+07	.22645243E+04	360.238	54.013	72.05
.55245482E+07	.26669115E+04	382.589	55.841	71.55
.64770055E+07	.32371968E+04	391.729	57.620	71.05
.69623141E+07	.35710891E+04	394.363	58.276	70.85
.75512049E+07	.39092359E+04	398.615	58.942	70.65
.81965190E+07	.42515612E+04	403.392	59.615	70.45

*The Total Absorption Coefficient is equal to the Absorption Integral multiplied by the factor $0.1948 \times 10^{-8} \text{ km}^2/\text{sec}$.

**To obtain the Mean Free Path in cm at the ray apex, multiply the Parameter by $0.70 \times 10^{-5} \text{ cm}$.

List of names and addresses				
No.	Name	Address	City	State
1	John Doe	123 Main St	New York	NY
2	Jane Smith	456 Elm St	Los Angeles	CA
3	Robert Brown	789 Oak St	Chicago	IL
4	Mary White	101 Pine St	San Francisco	CA
5	James Green	202 Cedar St	Philadelphia	PA
6	Elizabeth Black	303 Birch St	Boston	MA
7	William Gray	404 Spruce St	Seattle	WA
8	Patricia Red	505 Willow St	Portland	OR
9	Richard Blue	606 Ash St	San Diego	CA
10	Susan Yellow	707 Hickory St	Denver	CO
11	Thomas Purple	808 Sycamore St	San Jose	CA
12	Linda Pink	909 Magnolia St	San Antonio	TX
13	Christopher Green	1010 Dogwood St	Fort Worth	TX
14	Amanda White	1111 Redwood St	Columbus	GA
15	Matthew Black	1212 Cypress St	Indianapolis	IN
16	Olivia Gray	1313 Juniper St	San Luis Obispo	CA
17	Ethan Red	1414 Fir St	Providence	RI
18	Sophia Blue	1515 Palm St	San Francisco	CA
19	Benjamin Yellow	1616 Cedar St	San Jose	CA
20	Isabella Purple	1717 Birch St	San Francisco	CA
21	Lucas Pink	1818 Spruce St	San Francisco	CA
22	Charlotte Green	1919 Oak St	San Francisco	CA
23	Henry Black	2020 Elm St	San Francisco	CA
24	Amelia Gray	2121 Pine St	San Francisco	CA
25	Sebastian Red	2222 Cedar St	San Francisco	CA
26	Harriet Blue	2323 Birch St	San Francisco	CA
27	Julian Yellow	2424 Spruce St	San Francisco	CA
28	Esther Purple	2525 Willow St	San Francisco	CA
29	Alfred Green	2626 Ash St	San Francisco	CA
30	Clara White	2727 Hickory St	San Francisco	CA
31	Frederick Black	2828 Sycamore St	San Francisco	CA
32	Ida Gray	2929 Dogwood St	San Francisco	CA
33	Harold Red	3030 Redwood St	San Francisco	CA
34	Gertrude Blue	3131 Cypress St	San Francisco	CA
35	Samuel Yellow	3232 Juniper St	San Francisco	CA
36	Frances Purple	3333 Fir St	San Francisco	CA
37	Charles Green	3434 Palm St	San Francisco	CA
38	Martha White	3535 Cedar St	San Francisco	CA
39	Albert Black	3636 Birch St	San Francisco	CA
40	Elizabeth Gray	3737 Spruce St	San Francisco	CA
41	William Red	3838 Oak St	San Francisco	CA
42	Mary Blue	3939 Elm St	San Francisco	CA
43	James Yellow	4040 Pine St	San Francisco	CA
44	Anna Purple	4141 Cedar St	San Francisco	CA
45	John Green	4242 Birch St	San Francisco	CA
46	Margaret White	4343 Spruce St	San Francisco	CA
47	Robert Black	4444 Willow St	San Francisco	CA
48	Patricia Gray	4545 Ash St	San Francisco	CA
49	Richard Red	4646 Hickory St	San Francisco	CA
50	Linda Blue	4747 Sycamore St	San Francisco	CA
51	Christopher Yellow	4848 Dogwood St	San Francisco	CA
52	Amanda Purple	4949 Redwood St	San Francisco	CA
53	Matthew Green	5050 Cypress St	San Francisco	CA
54	Olivia White	5151 Juniper St	San Francisco	CA
55	Ethan Black	5252 Fir St	San Francisco	CA
56	Sophia Gray	5353 Palm St	San Francisco	CA
57	Benjamin Red	5454 Cedar St	San Francisco	CA
58	Isabella Blue	5555 Birch St	San Francisco	CA
59	Lucas Yellow	5656 Spruce St	San Francisco	CA
60	Charlotte Purple	5757 Willow St	San Francisco	CA
61	Henry Green	5858 Ash St	San Francisco	CA
62	Amelia White	5959 Hickory St	San Francisco	CA
63	Sebastian Black	6060 Sycamore St	San Francisco	CA
64	Harriet Gray	6161 Dogwood St	San Francisco	CA
65	Julian Red	6262 Redwood St	San Francisco	CA
66	Esther Blue	6363 Cypress St	San Francisco	CA
67	Alfred Yellow	6464 Juniper St	San Francisco	CA
68	Clara Purple	6565 Fir St	San Francisco	CA
69	Frederick Green	6666 Palm St	San Francisco	CA
70	Ida White	6767 Cedar St	San Francisco	CA
71	Harold Black	6868 Birch St	San Francisco	CA
72	Gertrude Gray	6969 Spruce St	San Francisco	CA
73	Samuel Red	7070 Oak St	San Francisco	CA
74	Frances Blue	7171 Elm St	San Francisco	CA
75	Charles Yellow	7272 Pine St	San Francisco	CA
76	Martha Purple	7373 Cedar St	San Francisco	CA
77	Albert Green	7474 Birch St	San Francisco	CA
78	Margaret White	7575 Spruce St	San Francisco	CA
79	Robert Black	7676 Willow St	San Francisco	CA
80	Patricia Gray	7777 Ash St	San Francisco	CA
81	Richard Red	7878 Hickory St	San Francisco	CA
82	Linda Blue	7979 Sycamore St	San Francisco	CA
83	Christopher Yellow	8080 Dogwood St	San Francisco	CA
84	Amanda Purple	8181 Redwood St	San Francisco	CA
85	Matthew Green	8282 Cypress St	San Francisco	CA
86	Olivia White	8383 Juniper St	San Francisco	CA
87	Ethan Black	8484 Fir St	San Francisco	CA
88	Sophia Gray	8585 Palm St	San Francisco	CA
89	Benjamin Red	8686 Cedar St	San Francisco	CA
90	Isabella Blue	8787 Birch St	San Francisco	CA
91	Lucas Yellow	8888 Spruce St	San Francisco	CA
92	Charlotte Purple	8989 Willow St	San Francisco	CA
93	Henry Green	9090 Ash St	San Francisco	CA
94	Amelia White	9191 Hickory St	San Francisco	CA
95	Sebastian Black	9292 Sycamore St	San Francisco	CA
96	Harriet Gray	9393 Dogwood St	San Francisco	CA
97	Julian Red	9494 Redwood St	San Francisco	CA
98	Esther Blue	9595 Cypress St	San Francisco	CA
99	Alfred Yellow	9696 Juniper St	San Francisco	CA
100	Clara Purple	9797 Fir St	San Francisco	CA

The above list of names and addresses is for the purpose of the
 study of the distribution of the various colors of the
 human race in the various parts of the world.
 The names are arranged in alphabetical order of the
 surnames, and the addresses are arranged in alphabetical
 order of the cities.

Energy Density (relative)	Fre- quency (cps)	Range (km)	Apex Height (km)	Ray Angle (deg)	Exp(α)
0.3302E+01	5.00	198.663	43.489	90.00	0.9752
0.1095E+02	5.00	198.151	43.492	89.50	0.9752
0.1463E+02	5.00	197.843	43.519	88.50	0.9751
0.5734E+02	5.00	197.613	43.573	87.50	0.9749
0.2671E+01	5.00	197.555	43.654	86.51	0.9747
0.4432E+01	5.00	196.927	43.704	86.01	0.9746
0.1809E+01	5.00	196.550	43.762	85.51	0.9745
0.5125E+02	5.00	195.629	43.826	85.01	0.9743
0.4408E+01	5.00	195.532	44.059	83.51	0.9737
0.5486E+02	5.00	195.165	44.151	83.02	0.9735
0.1282E+02	5.00	195.105	44.355	82.02	0.9729
0.4242E+01	5.00	194.849	44.588	81.02	0.9723
0.4596E+01	5.00	194.464	44.714	80.52	0.9720
0.5403E+01	5.00	194.110	44.849	80.02	0.9716
0.2213E+02	5.00	193.816	44.990	79.53	0.9713
0.1279E+02	5.00	193.743	45.139	79.03	0.9709
0.4086E+02	5.00	193.617	45.295	78.53	0.9705
0.2467E+02	5.00	193.499	45.807	77.03	0.9691
0.1910E+02	5.00	193.371	46.187	76.04	0.9681
0.4115E+01	5.00	193.288	46.388	75.54	0.9675
0.4095E+01	5.00	192.904	46.598	75.04	0.9669

0.3062E+01	10.00	198.663	43.489	90.00	0.9043
0.1015E+02	10.00	198.151	43.492	89.50	0.9042
0.1357E+02	10.00	197.843	43.519	88.50	0.9039
0.5314E+02	10.00	197.613	43.573	87.50	0.9035
0.2473E+01	10.00	197.555	43.654	86.51	0.9026
0.4103E+01	10.00	196.927	43.704	86.01	0.9023
0.1674E+01	10.00	196.550	43.762	85.51	0.9017
0.4740E+02	10.00	195.629	43.826	85.01	0.9011
0.4069E+01	10.00	195.532	44.059	83.51	0.8988
0.5060E+02	10.00	195.165	44.151	83.02	0.8980
0.1180E+02	10.00	195.105	44.355	82.02	0.8961
0.3899E+01	10.00	194.849	44.588	81.02	0.8938
0.4220E+01	10.00	194.464	44.714	80.52	0.8926
0.4957E+01	10.00	194.110	44.849	80.02	0.8913
0.2028E+02	10.00	193.816	44.990	79.53	0.8899
0.1171E+02	10.00	193.743	45.139	79.03	0.8885
0.3734E+02	10.00	193.617	45.295	78.53	0.8870
0.2245E+02	10.00	193.499	45.807	77.03	0.8820
0.1733E+02	10.00	193.371	46.187	76.04	0.8783
0.3727E+01	10.00	193.288	46.388	75.54	0.8762
0.3702E+01	10.00	192.904	46.598	75.04	0.8742

Energy Density (relative)	Fre- quency (cps)	Range (km)	Apex Height (km)	Ray Angle (deg)	Exp(α)
0.2264F+01	20.00	198.663	43.489	90.00	0.6687
0.7507E+01	20.00	198.151	43.492	89.50	0.6686
0.1002E+02	20.00	197.843	43.519	88.50	0.6676
0.3918E+02	20.00	197.613	43.573	87.50	0.6662
0.1819F+01	20.00	197.555	43.654	86.51	0.6638
0.3013E+01	20.00	196.927	43.704	86.01	0.6627
0.1227E+01	20.00	196.550	43.762	85.51	0.6610
0.3468F+02	20.00	195.629	43.826	85.01	0.6593
0.2955E+01	20.00	195.532	44.059	83.51	0.6527
0.3664F+02	20.00	195.165	44.151	83.02	0.6503
0.8492F+01	20.00	195.105	44.355	82.02	0.6447
0.2784F+01	20.00	194.849	44.588	81.02	0.6381
0.3001E+01	20.00	194.464	44.714	80.52	0.6347
0.3510E+01	20.00	194.110	44.849	80.02	0.6311
0.1429E+02	20.00	193.816	44.990	79.53	0.6272
0.8211E+01	20.00	193.743	45.139	79.03	0.6233
0.2606E+02	20.00	193.617	45.295	78.53	0.6190
0.1540E+02	20.00	193.499	45.807	77.03	0.6051
0.1174E+02	20.00	193.371	46.187	76.04	0.5950
0.2508F+01	20.00	193.288	46.388	75.54	0.5895
0.2473E+01	20.00	192.904	46.598	75.04	0.5839

0.2737	50.00	198.663	43.489	90.00	0.8082E-01
0.3068	50.00	198.151	43.492	89.50	0.8076E-01
0.1201E+01	50.00	197.843	43.519	88.50	0.8003E-01
0.4647E+01	50.00	197.613	43.573	87.50	0.7900E-01
0.2116	50.00	197.555	43.654	86.51	0.7723E-01
0.3475	50.00	196.927	43.704	86.01	0.7642E-01
0.1396	50.00	196.550	43.762	85.51	0.7518E-01
0.3891E+01	50.00	195.629	43.826	85.01	0.7397E-01
0.3146	50.00	195.532	44.059	83.51	0.6949E-01
0.3827E+01	50.00	195.165	44.151	83.02	0.6791E-01
0.8475	50.00	195.105	44.355	82.02	0.6434E-01
0.2633	50.00	194.849	44.588	81.02	0.6035E-01
0.2759	50.00	194.464	44.714	80.52	0.5834E-01
0.3131	50.00	194.110	44.849	80.02	0.5631E-01
0.1234E+01	50.00	193.816	44.990	79.53	0.5418E-01
0.6865	50.00	193.743	45.139	79.03	0.5211E-01
0.2101E+01	50.00	193.617	45.295	78.53	0.4991E-01
0.1102F+01	50.00	193.499	45.807	77.03	0.4330E-01
0.7688	50.00	193.371	46.187	76.04	0.3897E-01
0.1565	50.00	193.288	46.388	75.54	0.3678E-01
0.1468	50.00	192.904	46.598	75.04	0.3465E-01

Energy Density (relative)	Fre- quency (cps)	Range (km)	Apex Height (km)	Ray Angle (deg)	Exp(✓)
-0.4845E-01	5.00	193.458	46.815	74.54	0.9663
-0.2839E+01	5.00	192.904	46.598	75.04	0.9669
-0.3424E-01	5.00	225.914	47.280	74.04	0.9520
-0.6641E-01	5.00	278.611	48.878	73.54	0.9181
-0.3414E-01	5.00	310.847	50.533	73.04	0.8841
-0.1050	5.00	337.563	52.244	72.55	0.8472
-0.1080	5.00	360.238	54.013	72.05	0.8068
-0.2648	5.00	382.589	55.841	71.55	0.7641
-0.3582	5.00	391.729	57.620	71.05	0.7295
-0.2179	5.00	394.363	58.276	70.85	0.7124
-0.1903	5.00	398.615	58.942	70.65	0.6923
-0.1863	5.00	403.392	59.615	70.45	0.6709
-0.2566E+01	10.00	192.904	46.598	75.04	0.8742
-0.4372E-01	10.00	193.458	46.815	74.54	0.8720
-0.2955E-01	10.00	225.914	47.280	74.04	0.8215
-0.5139E-01	10.00	278.611	48.878	73.54	0.7105
-0.5814E-01	10.00	310.847	50.533	73.04	0.6109
-0.6385E-01	10.00	337.563	52.244	72.55	0.5152
-0.5670E-01	10.00	360.238	54.013	72.05	0.4237
-0.1181	10.00	382.589	55.841	71.55	0.3409
-0.1390	10.00	391.729	57.620	71.05	0.2832
-0.7879E-01	10.00	394.363	58.276	70.85	0.2576
-0.6313E-01	10.00	398.615	58.942	70.65	0.2297
-0.5627E-01	10.00	403.392	59.615	70.45	0.2026
-0.1714E+01	20.00	192.904	46.598	75.04	0.5839
-0.2899E-01	20.00	193.458	46.815	74.54	0.5783
-0.1638E-01	20.00	225.914	47.280	74.04	0.4555
-0.1843E-01	20.00	278.611	48.878	73.54	0.2548
-0.1325E-01	20.00	310.847	50.533	73.04	0.1392
-0.8728E-02	20.00	337.563	52.244	72.55	0.7043E-01
-0.4314E-02	20.00	360.238	54.013	72.05	0.3224E-01
-0.4679E-02	20.00	382.589	55.841	71.55	0.1350E-01
-0.3157E-02	20.00	391.729	57.620	71.05	0.6429E-02
-0.1347E-02	20.00	394.363	58.276	70.85	0.4405E-02
-0.7651E-03	20.00	398.615	58.942	70.65	0.2784E-02
-0.4677E-03	20.00	403.392	59.615	70.45	0.1684E-02
-0.1017	50.00	192.904	46.598	75.04	0.3465E-01
-0.1635E-02	50.00	193.458	46.815	74.54	0.3261E-01
-0.2638E-03	50.00	225.914	47.280	74.04	0.7333E-02
-0.6366E-10	50.00	360.238	54.013	72.05	0.4758E-09
-0.1406E-04	50.00	278.611	48.878	73.54	0.1944E-03
-0.4236E-06	50.00	310.847	50.533	73.04	0.4451E-05
-0.7791E-08	50.00	337.563	52.244	72.55	0.6286E-07
-0.7165E-12	50.00	382.589	55.841	71.55	0.2068E-11
-0.9820E-14	50.00	391.729	57.620	71.05	0.2000E-13
-0.5756E-15	50.00	394.363	58.276	70.85	0.1882E-14
-0.2939E-16	50.00	398.615	58.942	70.65	0.1069E-15
-0.1282E-17	50.00	403.392	59.615	70.45	0.4616E-17

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